World Academics Journal of Engineering Sciences
Research Paper
Vol.8, Issue.3, pp.06-10, September (2021)
E-ISSN: 2348-635X

Linguistic Based Pareto Guided Fuzzy Multi-Objective Optimization

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Available online at: www.isroset.org

Received: 22/Jul/2021, Accepted: 20/Sept/2021, Online: 30/Sept/2021

Abstract— In principle, a problem consists of mutually conflicting objectives in an engineering system design generates multiple solutions instead of a single solution. A set of solutions is required to satisfy such types of problems. These solutions are popularly known as Pareto-optimal solutions. An engineer always interests in reducing the cost and improving the reliability of the system simultaneously. This paper proposes linguistic fuzzy multi-objective models where reliability and cost are involved in the system design. Such types of formulations give an idea to find out the best compromise solution as per the demand of the decision-maker. At the same time, it removes the uncertainty confronted by a system design engineer in formulating the system design at the initial stage. The Pareto-optimal solutions are obtained by an efficient multi-objective evolutionary technique, namely NSGA-II. A designer finds the guided Pareto-optimal solutions as per the formulated models of the problem in linguistic forms. The best compromise solution is found by the decision-maker while analyzing the different cases of fuzzy models. The proposed approach is shown by taking a numerical example.

Keywords— Reliability, Fuzzy multi-objective optimization problem (FMOOP), Linguistic hedges, NSGA-II

I. INTRODUCTION

Multi-objective optimization is a field of multi-criteria decision-making [1] in which two or three objective functions are simultaneously optimized. It occurs in many branches of science, engineering, economics, and logistics. Generally, the performance of the system is measured by “Reliability” [2]. So, it is considered as one of the important characteristics of the system design. A design engineer always desires to reduce the cost and improve the reliability of the system simultaneously. The mathematical formulation of such type of problem is known as a multi-objective optimization problem (MOOP). A MOOP generates a set of optimal solutions known as “Pareto-optimal solutions” [3] instead of a single solution. The weighted sum approach [4] is a popular mathematical treatment to the MOOP. But such an approach depends highly on users as well as it requires multiple runs to generate multiple solutions [5]. Computer scientists have done excellent works in this direction and developed several generations of multi-objective evolutionary algorithms (MOEAs) [6]. Non-dominated sorting genetic algorithm-II (NSGA-II) is one of such techniques to find Pareto-optimal solutions with good diversity and better convergence [3]. It is a popular algorithm for finding multiple solutions in a single simulation run. Many engineering design problems have been evaluated successfully by NSGA-II [7]. It has enough capacity to handle a MOOP with mutually conflicting objectives. A decision-maker chooses the best solution from multiple solutions as per his/her choice [4]. Apart from this, the formulation of a reliability-based system design needs to be tackled with uncertainties [8]. It may be due to lack of judgment, expert’s information character, vagueness, unaware of environment, qualitative statements, etc. Fuzzy set theory [9] is an effective technique to resolve these issues at the initial stage of designing [4]. In this context, many papers can be looked into the research literature [4], [8], [10]. The mathematical modeling of the system design problems is an art where the models are built more flexibly and adaptable to the human decision-making process [4].

This paper gives multiple fuzzy models by using linguistic hedges [11]. NSGA-II is employed for solving all mathematical models of the problems. It finds multiple Pareto-optimal fronts to each combination of linguistic hedges as per the requirement of the decision-maker. Now, a decision-maker has multiple options to choose the Pareto as per his/her choice. Finally, the fuzzy ranking method helps in deciding the best compromise solution from multiple optimal fronts. The proposed approach is illustrated through a numerical example of a life support system in a space capsule. The rest of the paper is organized as follows. Section II contains a mathematical model of the problem. Section III contains the proposed approach with a numerical example. Section IV gives the results and discussion. Section V concludes the conclusion of the proposed work with future directions.

II. MATHEMATICAL MODEL OF THE PROBLEM

Let a system be of n components, the reliability of each component is given by R_i , $i = 1, 2, ..., n$ and its corresponding cost is denoted by $C_i(R_i)$. The mathematical model of the problem is given as follows [4].

$$Maximize\, R_S(R_1, R_2, ..., R_n) = \begin{cases} \prod_{i=1}^{n} R_i \text{ for series system} \\ or \\ 1 - \prod_{i=1}^{n}(1 - R_i) \text{ for parallel system} \\ or \\ \text{Combination of series and parallel system} \end{cases} \quad (1)$$

$$Minimize\; C_S(R_1, R_2, ..., R_n) = \sum_{i=1}^{n} C_i(R_i) \quad (2)$$

$$subject\ to\ 0 \le R_{i,\min} \le R_i \le 1, R_{S,\min} \le R_S \le 1, i = 1, 2, ..., n \quad (3)$$

where n is the total number of components in the system, and $R_{i,\min}$ and $R_{S,\min}$ are minimum values of the i^{th} component and the system respectively.

III. Methodology

Let us consider the Mathematical model of the life-support system [6] using a block diagram (see Figure 1) given as follows:

$$Max. R_S = 1 - R_3\left[(1-R_1)(1-R_4)\right]^2 - (1-R_3)\left[1 - R_2\{1-(1-R_1)(1-R_4)\}\right]^2 \quad (4)$$

$$Min.\ C_S = 2 * \sum_{i=1}^{4} K_i R_i^{\alpha_i} \quad (5)$$

$$subject\ to\ 0.5 \le R_i \le 1, 0.5 \le R_S \le 1; i = 1, 2, ..., n \quad (6)$$

where different parameters values of K_i, as $K_1 = 100, K_2 = 100, K_3 = 200, K_4 = 150$ and α_i as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.6$

The problem is stated as “Maximize system reliability as close as possible to 1 with an approximate system cost of 641.8 (cost units)”.

FMOOP (9) is reformulated in various forms by linguistic hedges [11] that modifies the meaning of the term as per the requirements of the decision-maker. The role of the decision-maker plays an important role in reaching the desired goal. Keeping this viewpoint in mind, linguistic hedges are used to make interactions with the decision-maker. These interactions are developed in form of various models listed in Table 1, Table 2 and Table 3.

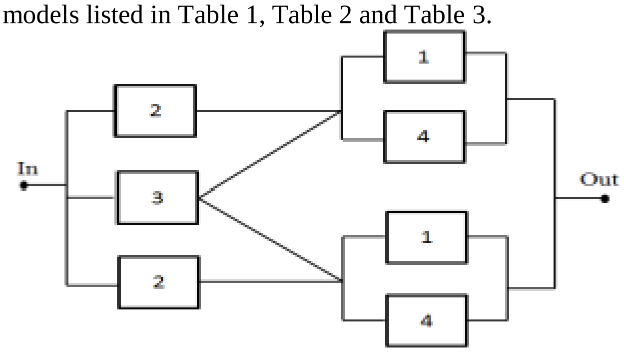

Figure 1. Block diagram of the life-support system in a space capsule

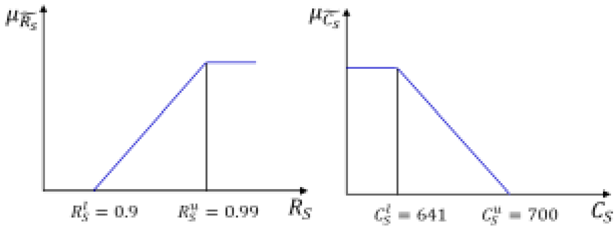

Figure 2. Linear membership function for system reliability R_S and System cost C_S

Linear membership models of R_S and C_S are shown in Figure 2 and formulated as follows:

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le 0.9 \\ \dfrac{R_S - 0.9}{0.99 - 0.9}, & 0.9 \le R_S \le 0.99 \\ 1, & R_S = 1 \end{cases} \quad (7)$$

$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \le 641 \\ \dfrac{700 - C_S}{700 - 641}, & 641 \le C_S \le 700 \\ 0, & C_S \ge 700 \end{cases} \quad (8)$$

Now, FMOOP is modeled as follows:

$$Max.\left(\mu_{\tilde{R}_S}, \mu_{\tilde{C}_S}\right) s.t.\ 0.5 \le R_i \le 1;\ i = 1, 2, 3, 4 \quad (9)$$

Here, $\mu_{\tilde{R}_S}$ and $\mu_{\tilde{C}_S}$ are called degree of satisfaction of system reliability and its cost respectively.

To find the Pareto-optimal solutions interactively, NSGA-II is applied to the proposed fuzzy models. The best Pareto-fronts are obtained based on rigorous experimentation and tuning of the parameters. The parameters settings for the NSGA-II are given in Table 4. The best compromise solution is obtained as follows [7]:

$$\mu_{best} = \max_P \left[\min\left\{\mu_{\tilde{R}_S}, \mu_{\tilde{C}_S}\right\}\right] \quad (10)$$

where min is the minimum degree of membership values of objective functions, max is the maximum membership value among the calculated minimum, P is the number of obtained Pareto-optimal solutions.

Table 1. A decision-maker gives importance to reliability more than the cost

R_S	C_S	**Fuzzy model**
Very very	Somewhat	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^4, \left(\mu_{\tilde{C}_S}\right)^{0.5}\right)$ $s.t.\ 0.5 \le R_i \le 1;\ i = 1, 2, 3, 4$
Extremely	A little	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^3, \left(\mu_{\tilde{C}_S}\right)^{1.3}\right)$ $s.t.\ 0.5 \le R_i \le 1;\ i = 1, 2, 3, 4$
Very	Slightly	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^2, \left(\mu_{\tilde{C}_S}\right)^{1.7}\right)$ $s.t.\ 0.5 \le R_i \le 1;\ i = 1, 2, 3, 4$

Table 2. A decision-maker gives importance to cost more than reliability

R_S	C_S	**Fuzzy model**
Somewhat	Very very	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{0.5},\left(\mu_{\tilde{C}_S}\right)^{4}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$
A little	Extremely	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{1.3},\left(\mu_{\tilde{C}_S}\right)^{3}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$
Slightly	Very	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{1.7},\left(\mu_{\tilde{C}_S}\right)^{2}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$

Table 3. A decision-maker gives importance both in the same manner

R_S	C_S	**Fuzzy model**
Very very	Very very	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{4},\left(\mu_{\tilde{C}_S}\right)^{4}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$
Extremely	Extremely	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{3},\left(\mu_{\tilde{C}_S}\right)^{3}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$
Very	Very	$Max.\left(\left(\mu_{\tilde{R}_S}\right)^{2},\left(\mu_{\tilde{C}_S}\right)^{2}\right)$ $s.t.\ 0.5\le R_i\le 1;\ i=1,2,3,4$

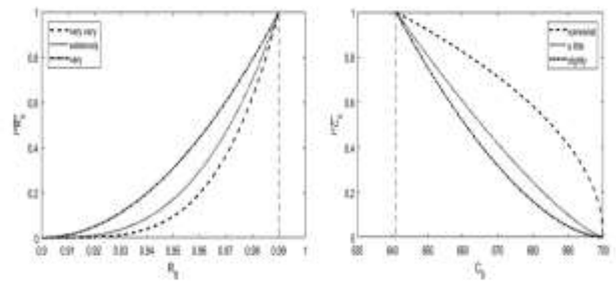

Figure 3. The shape of the degree of satisfaction for (a) R_S ; (b) C_S based on Table 1

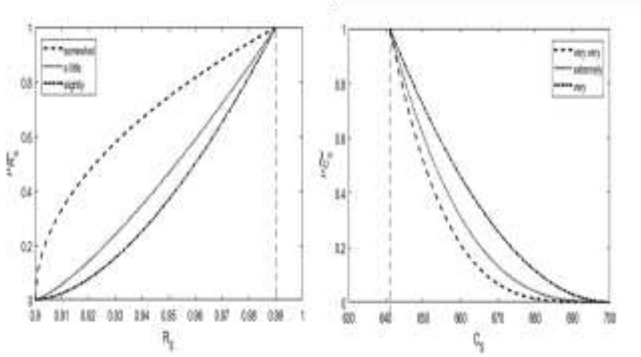

Figure 4. The shape of the degree of satisfaction for (a) R_S ; (b) C_S based on Table 2

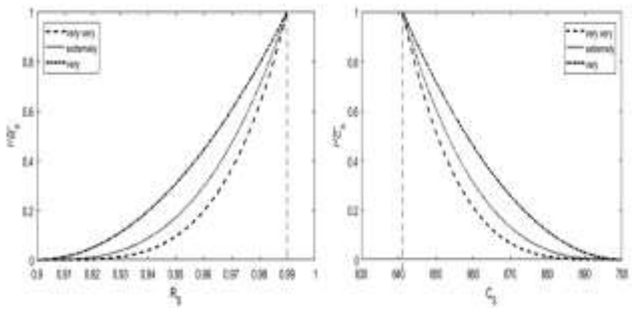

Figure 5. The shape of the degree of satisfaction for (a) R_S ; (b) C_S based on Table 3

Table 4. Parameters settings for the NSGA-II

Population size	40
Maximum number of generations	100
Crossover probability	0.9
Mutation probability	0.1
Distribution index for crossover	20
Distribution index for mutation	80
Random seed	0.1234
Number of objective functions	2
Number of evaluations	4000
Lower limit on system reliability	0.9
Upper limit on system reliability	0.99
Lower limit on system cost	641
Upper limit on system cost	700

Table 5. The best compromise solutions from different Pareto-guided solutions based on linguistic hedges

	Fuzzy model	R_1	R_2	R_3	R_4	R_S	C_S	μ_{best}
Case-I	(Very very, Somewhat)	0.622374	0.999749	0.500007	0.500020	0.964273	657.13	0.714146
	(Extremely, A little)	0.665425	0.976357	0.500011	0.500040	0.968498	654.72	**0.761089**
	(Very, Slightly)	0.639872	0.999328	0.500007	0.500020	0.967541	656.37	0.739459
Case-II	(Somewhat, Very very)	0.629725	0.998838	0.500006	0.500020	0.965632	657.45	0.721159
	(A little, Extremely)	0.643283	0.998932	0.500013	0.500041	0.968014	654.53	**0.755713**
	(Slightly, Very)	0.642878	0.998537	0.500007	0.500020	0.967913	656.43	0.738550
Case-III	(Very very, Very very)	0.713329	0.9936377	0.500002	0.500011	0.978684	648.58	**0.871562**
	(Extremely, Extremely)	0.6904878	0.9932626	0.500007	0.500009	0.975237	650.88	0.832526
	(Very, Very)	0.647889	0.998427	0.500011	0.500042	0.968790	655.18	0.759620

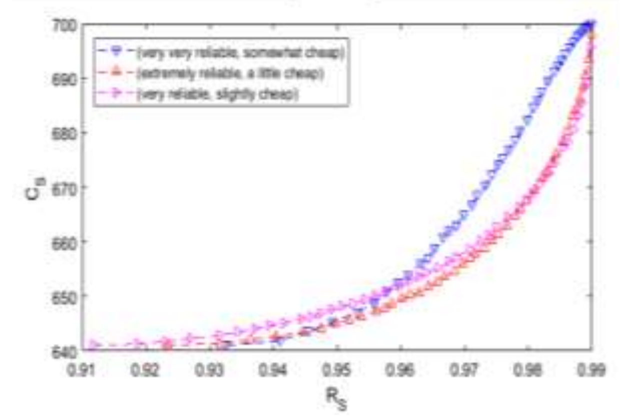

Figure 6. The Pareto-optimal fronts based on the fuzzy model in Table 1

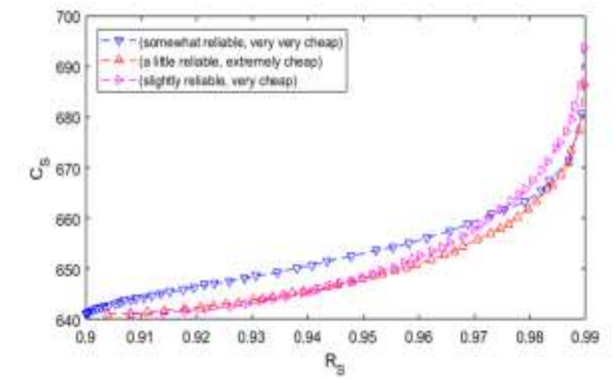

Figure 7. The Pareto-optimal fronts based on the fuzzy model in Table 2

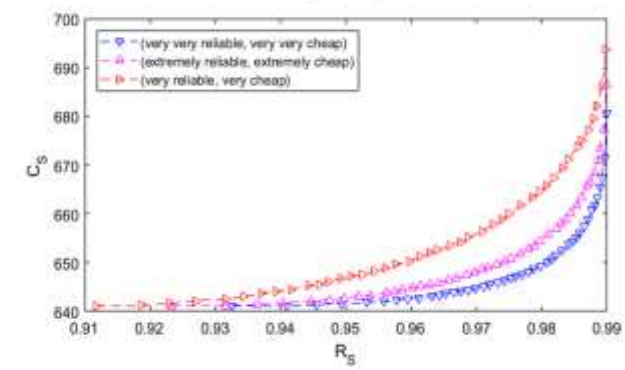

Figure 8. The Pareto-optimal fronts based on the fuzzy model in Table 3

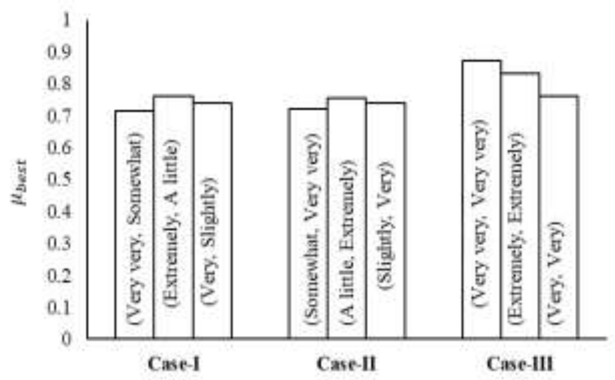

Figure 9. The maximum degree of satisfaction is based on Table 1, Table 2, and Table 3.

IV. RESULTS AND DISCUSSION

After applying the proposed approach to a numerical example of a life support system in a space capsule, a set of Pareto-optimal solutions is generated for each case. Simulation results are shown in Figure 6, Figure 7, and Figure 8 based on Table 1, Table 2, and Table 3 respectively. These are Pareto-guided solutions that are obtained after the interactions with the decision-maker. Figure 6 shows the Pareto-guided solutions towards the system reliability with different linguistic hedges. Here, the case (Extremely, A little) reaches maximum satisfaction level as 0.761089 where system reliability is 0.968498 and system cost is 654.72. Figure 7 shows the Pareto-guided solutions towards system cost with different linguistic hedges. Here, the case (A little, Extremely) reaches maximum satisfaction level as 0.755713 where system reliability is 0.968014 and system cost is 654.53. Figure 8 shows the Pareto-guided solutions towards the system reliability and its cost in the same manner with different linguistic hedges. In this case, the maximum satisfaction level reaches up to 0.871562 with system reliability 0.978684 and system cost 648.58. In Figure 9, a comparative analysis is given to all combinations of hedges that are used in formulating the Pareto-guided solutions. The best compromise solution is obtained by the fuzzy ranking method. Finally, Table 5 gives the results of the proposed approach. After observing all of the cases, case-III dominates as compared to others due to its convergence towards maximum satisfaction level rapidly and reaches up to the highest as 0.871562 with hedge combination (very very, very very).

V. CONCLUSION AND FUTURE SCOPE

In this piece of work, an attempt is made to obtain the best optimal system design interactively for an FMOOP. For this purpose, linguistic modifiers are used in keeping the views of the decision-maker. Algorithm NSGA-II is employed to solve the problem effectively. Simulation results are shown in terms of multiple fronts. The present approach is successfully applied to the engineering system design problem where two mutually conflicting objectives are involved. It is found that Case-III (approaching both the objectives in the same manner) gives better results. Linguistic hedges can provide the Pareto-guided solutions to the decision-maker in a more concise way. After that, the fuzzy ranking method is used to find out the best compromise solution to each case. The proposed approach can be effective in multi-objective decision-making of reliability-based system design problems. In the future, higher level of uncertainty techniques of fuzzy set theory such as type-2 fuzzy set, intuitionistic fuzzy set, inter-valued intuitionistic fuzzy set, etc. can be used to tackle the problem in an engineering system design.

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