International Journal of Scientific Research in Physics and Applied Sciences
Vol.10, Issue.2, pp.25-30, April (2022)

Research Paper

E-ISSN: 2348-3423

Study of Heavy-Quarkonium Masses with Temperature-Dependent Screened Coulomb Potential

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Available online at: www.isroset.org

Received: 06/Mar/2022, Accepted: 12/Apr/2022, Online: 30/Apr/2022

Abstract: The screened Coulomb potential is adopted to study the heavy meson masses. An effort to make this potential model temperature dependent, modifications have been made by putting Debye mass in place of the screened parameter. An analytical technique is used to solve the Schrödinger equation, and Taylor's series expansion approach is used to calculate the energy eigenvalues. The findings are used to calculate heavy mesons charmonium and bottomonium mass spectra. Our outcomes are in concurrence with the outcomes acquired from experimental and theoretical studies in several reported work. The present work can be used to determine other properties of quarkonium systems in future studies.

Keywords: Heavy quarkonia spectroscopy, Schrödinger equation, screened Coulomb potential.

I. INTRODUCTION

The standard model plays a major role in understanding quantative tests of (QCD) [1]. Schrödinger equation (SE) [2] can be used to study these systems. The spherically symmetric solution of SE is one of the most complex problems in quantum mechanics. It is extremely important for the spectroscopy of atoms and nuclei, and the dynamics of particle interaction [3]. The potential must anticipate two important properties, namely confinement of quark and asymptotic freedom [4],[5],[6] of nuclear interactions. The different method has been applied to solve SE using the asymptotic iteration method (AIM) [7] and super symmetric quantum mechanics.

The most fundamental Cornell potential has been used by the majority of researchers for the study of heavy quarkonium., In the Coulomb field, A. Al-Jamel and H. Widyan [8] analyse in their research the spin-averaged heavy quarkonia masses with asymptotic potential using N-U method. For the formulation of a non-relativistic type potential model, the Schrödinger equation is employed by Al-Oun et al. [9] to study the heavy quarkonia properties. Moreover, for the mass spectra and energy eigenvalues of heavy-heavy and light-heavy mesons, E. Omugbe group. [10] solved SE using a Killing beck potential model combined with an inversely quadratic potential model.

The Yukawa [11], also known as the screened coulomb potential, is extremely significant in many branches of Physics. It's only useful for short-range interactions [12],[13]. The screened-Coulomb potential is a well-known potential for the state of non-ideal plasma state.
The heavy quark-antiquark systems such as $c\bar{c}$ and $b\bar{b}$ are being studied; we are attempting to determine the implications of the time variable screened Coulomb potential as Eq. 1.

$$V_{SCP}(r) = -\frac{\alpha}{r} + \frac{\beta e^{-\delta r}}{r} - \frac{\gamma e^{-2\delta r}}{r^2} + \xi \tag{1}$$

Where, α, β and γ are potential strength parameters. *Screening parameter* δ , Controls the shape as in figure 1. and ξ is an arbitrary parameter introduced to improve the accuracy of the potential model. when $\gamma = 0$ and also $\beta = \gamma = \delta = 0$ it reduces to Hellman and Coulomb potential as well.

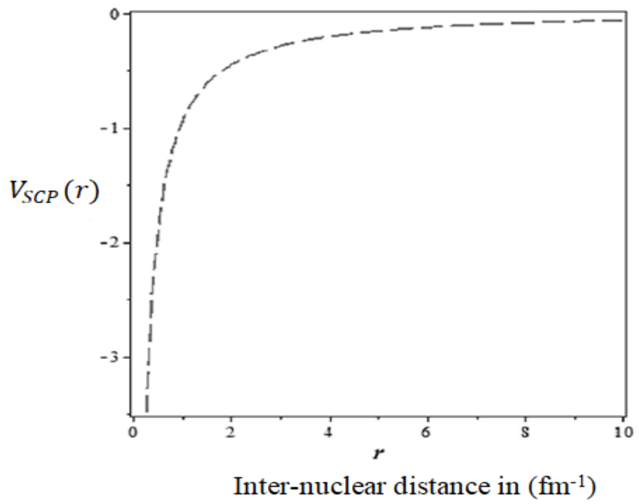

Figure 1.Variation of the potential with inter-nuclear distance r

The motive of the work is to obtain masses of quarkonium systems. For this, we study the SE in screened Coulomb potential framework model with an expansion of the series.

II. SCHRODINGER EQUATION SOLUTION WITH TEMPERATURE-DEPENDENT SCREENED COULOMB POTENTIAL

For this, choose the radial SE of the following type [6].

$$\frac{d^2\cup(r)}{dr^2}+\frac{2}{r}\frac{dU^2}{dr}+\left[\frac{2\mu}{\hbar^2}\left(E_{nl}-V(r)\right)-\frac{l(l+1)}{r^2}\right]\cup(r)=0 \tag{2}$$

In above equation, the terms l, μ, r *and* $\hbar$ represents angular momentum quantum number, systems reduced mass, inter-particle distance and Plank's constant respectively.

To make temperature -variable potential model, screening factor δ is displace by the Debye mass $[m_D(T)]$, and eliminates at $T \to 0$, we obtain

$$V_{SCP}(r,T)=-\frac{\alpha}{r}+\frac{\beta e^{-m_D(T)r}}{r}-\frac{\gamma e^{-2m_D(T)r}}{r^2}+\xi \tag{3}$$

Expand the exponential part of Eq.(3) up to order three by using Taylor's series expansion method and substitute it into the Eq.(3) to make the potential interact with quark anti-quark system.

$$V_{SCP}(r,T)=-\frac{a_0}{r}+a_1r+a_2r^2+\frac{a_3}{r^2}\quad+\ a_4 \tag{4}$$

Where $a_{1,}a_2, a_3$ *and* a_4 have the values taken from series expansion

We may get the following by putting Eq. (4) into Eq. (2).

$$\frac{d^2\cup(r)}{dr^2}+\frac{2}{r}\frac{dU^2}{dr}+\left[\kappa+\frac{P}{r}-Qr-Rr^2-\frac{L(L+1)}{r^2}\right]\cup(r)=0 \tag{5}$$

Where

$$L(L+1)=\frac{2\mu a_3}{\hbar^2}+l(l+1) \tag{6}$$

Making an Anzats wave function[14]

$$\cup(r)=e^{-ar^2-br}\mathrm{f}(r) \tag{7}$$

Where a and b are positive constants to be determined, and then we find the function as in series form. By differentiating Eq. (7) twice, we get the following by putting the results into Eq. (5). Then arrives the function (r) in series form.

$$\mathrm{f}(r) = \sum_{n=0}^{\infty} a_n r^{2n+L} \tag{8}$$

We get the following by holding the first two derivatives of Eq. (8), and after that, collecting the powers of r. We have a linearly independent equation, which indicates that all of the terms except r are separate and equal to zero. As a result, it's a zero-valued r coefficient. Having this in mind, we can assess the relationship with each term.

$$(2n+L)[(2n+L-1)] + [2(2n+L) - L(L+1)] = 0 \tag{9}$$

$$-2b(2n+L) + P - 2a = 0 \tag{10}$$

$$-4a(2n+L) + k + b^2 - 6a = 0 \tag{11}$$

$$4ab - Q = 0 \tag{12}$$

$$4a^2 - R = 0 \tag{13}$$

From Eq.(10) the value of b is

$$b = \frac{P}{4n+2L+2} \tag{14}$$

Similarly from Eq.(13) the value of a is

$$a = \frac{\sqrt{R}}{2} \tag{15}$$

We use Eq. (11) to get the energy eigenvalues equation, and we have

$$k = 2a(4n+2L+3) - b^2 \tag{16}$$

After substituting the values of $a, b, k\ and\ L$ from Eq. (5),(6),(14), and (15) into Eq.(16), we obtain

$$E_{nl} = \sqrt{\frac{\hbar^2 a_2}{2\mu}}\left(4n + 2\sqrt{(2l+1)^2 + \frac{8\mu a_3}{\hbar^2}}\right) - \frac{2\mu a_0^2}{\hbar^2}\left(\sqrt{(2l+1)^2 + \frac{8\mu a_3}{\hbar^2}}\right)^{-2} + a_4 + \xi \tag{17}$$

Substituting Debey mass values into Eq.(17) we find the temperature variable energy eigen values for the potential as:

$$E_{nl} = \left.\begin{matrix} \sqrt{\dfrac{-\hbar^2 \beta m_D^3(T)}{12\mu}\left(4n + 2\sqrt{(2l+1)^2 + \dfrac{8\mu\gamma}{\hbar^2}}\right)} \\ -\dfrac{2\mu}{\hbar^2}(\beta - \alpha + 2\gamma m_D)^2\left(4n + 1\sqrt{(2l+1)^2 + \dfrac{8\mu\gamma}{\hbar^2}}\right)^{-2} \\ -\beta m_D(T) - 2\gamma m_D^2(T) + \xi \end{matrix}\right\} \tag{18}$$

For the flavour of heavy quark antiquark systems, we calculate the spectra of masses of quarkonia, using the below relation [15].

$$M = 2m + E_{nl} \tag{19}$$

We find the masses spectra for screened Coulomb potential by putting Eq.(18) into Eq. (19).

$$M = 2m + \left\{ \sqrt{\frac{-\hbar^2 \beta m_D^3(T)}{12\mu}} \left(4n + 2\sqrt{(2l+1)^2 + \frac{8\mu\gamma}{\hbar^2}}\right) - \frac{2\mu}{\hbar^2}(\beta - \alpha + 2\gamma m_D)^2 \left(4n + 1\sqrt{(2l+1)^2 + \frac{8\mu\gamma}{\hbar^2}}\right)^{-2} - \beta m_D(T) - 2\gamma m_D^2(T) + \xi \right\} \quad (20)$$

Table 1: The $(c\bar{c})$ system masses (in GeV) [(m_c=1.207, $\delta = 0.245$, $\mu = 0.6135$ $A = 0.6046$, $\beta = -0.656$, $\gamma = 5.680$, $m_D(T) = 1.49$, $\hbar = 1$, $\xi = -0.001$]

State	Present work	[16]	[17]	[18]
1S	3.097	3.096	3.096	3.096
2S	3.678	3.686	3.686	3.686
1P	3.524	3.433	3.255	3.525
2P	3.769	3.910	3.779	3.773
3S	4.041	3.984	4.040	4.040
4S	4.239	4.150	4.269	4.263
1D	3.504	3.767	3.504	3.770
2D	4.147	-	-	4.159

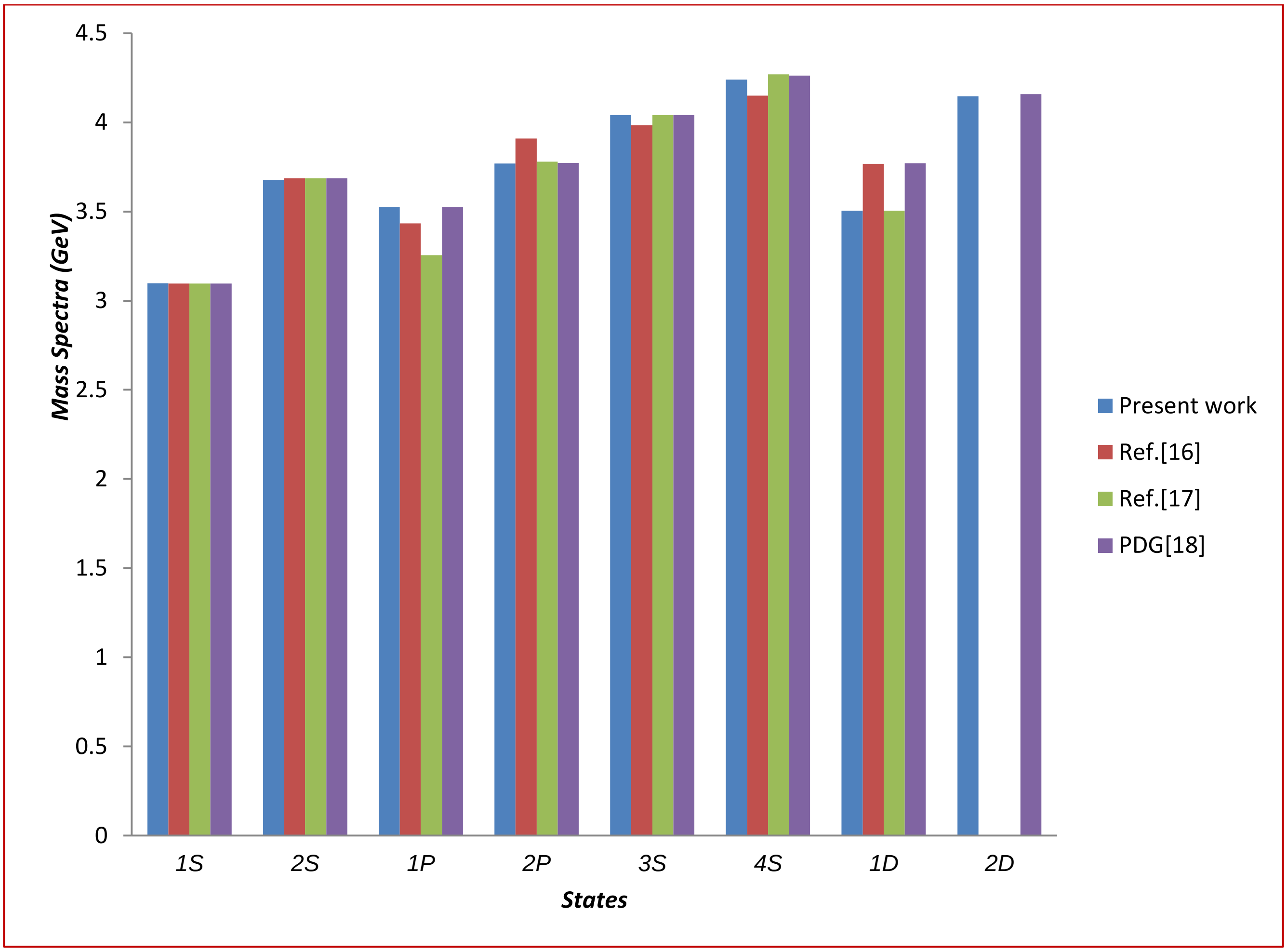

Figure 2. Mass Spectra of Charmonium for different states

Table 2: The ($b\bar{b}$)system masses (in GeV) [(m_b=4.824, $\delta = 0.255$, $\mu = 2.5115$
$A = 2.4112$, $\beta = .996$, $\gamma = 3.876$, $m_D(T) = 1.54$, $\hbar = 1$, $\xi = -0.01$]

State	Present Work	[16]	[17]	[18]
1S	9.460	9.460	9.460	9.460
2S	10.023	10.023	10.023	10.023
1P	9.898	9.840	9.619	9.899
2P	10.179	10.160	10.114	10.260
3S	10.355	10.280	10.355	10.355
4S	10.577	10.420	10.567	10.580
1D	10.167	10.140	9.864	10.164
2D	10.297	-	-	-

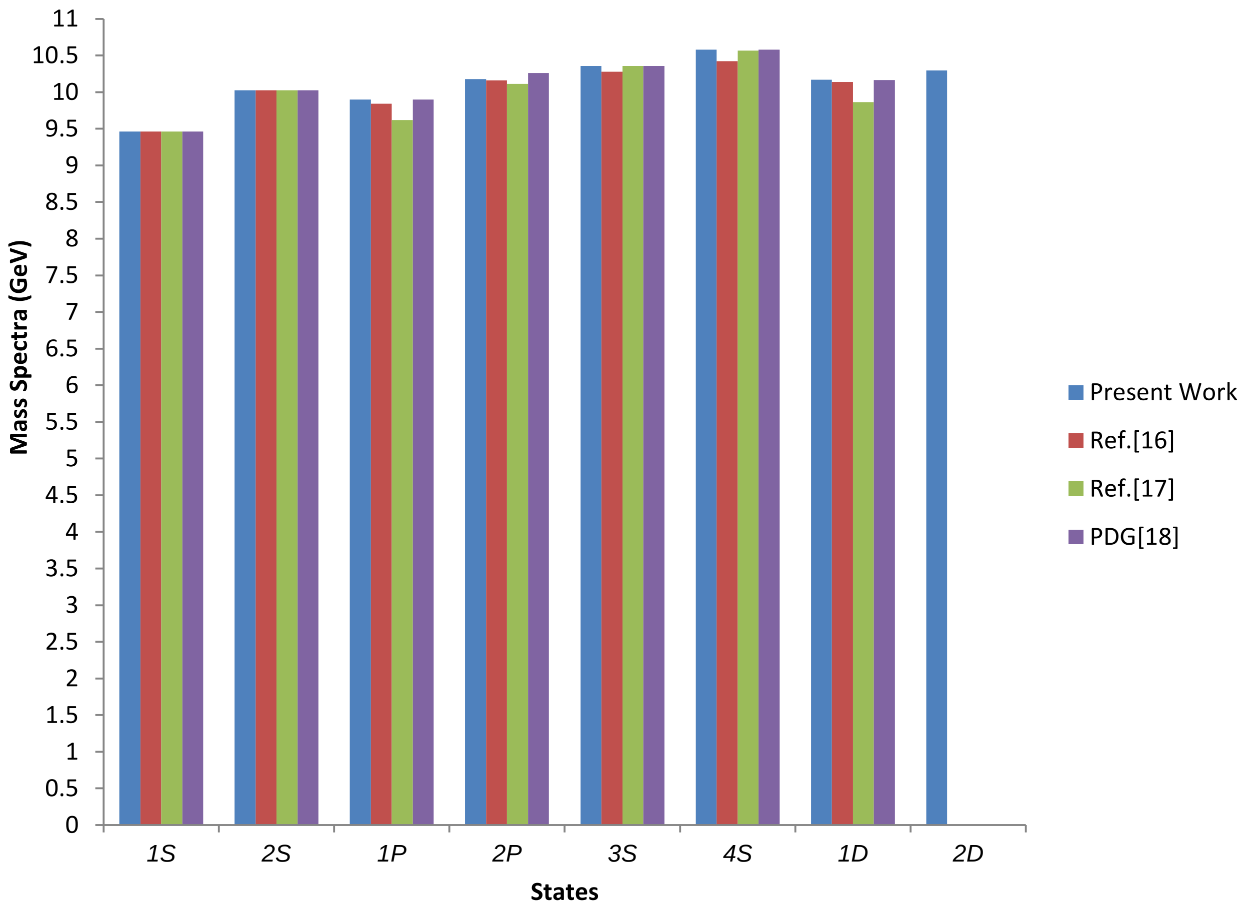

Figure 3. Mass Spectra of bottomonium for different states

III. RESULTS AND DISCUSSION

In this paper, we find the functioning of quarkonia masses is qualitatively consistent with the QCD sum rule and it is a tool for determining quarkonium masses. As a result, the current method determined the quarkonium states for the given potential successfully. We calculated the masses of heavy quarkonia by using approximate solutions for different states, shown in 1 and 2, tables respectively. In figures 1 and 2, we also plotted mass spectra against different states of bottomonium and charmonium. The masses of charmonium increase in higher states except for the 1D state, whereas the masses of bottonomium constantly increase except for the 1P state. To assess the correctness of the current finding, A analytically derived Chi-square function was used to evaluate the error between the calculated worth by theoretically and experimentally. The highest error in the values calculated in comparison to experimental findings is 0.00128 GeV. This shows an improvement in the present model. The findings show that the spectrum of mass agree with theoretical studies [16],[17] and the values obtained through experiments [18] of other research work. Finally, we believe that our findings may be useful in determining other properties of quarkonium systems in future studies.

IV. CONCLUSIONS

In this work, we employed screened coulomb potential for quarkonium interactions. Simply by substituting the Debye mass for the screening framework, the potential model was made temperature variable. For the energy eigen values, the expansion of the series technique is used to solve Schrödinger equation. The findings is used to obtain the masses for different states. The results supported experimental values and theoretical research. We also discussed the graphs that we obtained. The mass spectra of heavy mesons have been accurately estimated using the exponential type potential. The SE solutions can also be used to explain other properties of the quarkonium system.

V. ACKNOWLEDGMENT

I'd like to express my deepest appreciation to the referees and editorial board for devoting thei time and effort to assessin g the aforementioned work.

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