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Research Article

Plasma Density Effects on EMIC Wave Evolution: A Kappa Distribution Approach

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Abstract— This study delves into the impact of varying plasma particle density on Electromagnetic Ion Cyclotron (EMIC) waves within a multi-ion magneto-plasma composed of hydrogen (H+), helium (He+), and oxygen (O+) ions. Unlike traditional models that assume Maxwellian distributions, we employ the Kappa distribution to more accurately represent the non-Maxwellian nature of space plasmas. By incorporating the Kappa distribution, we examine how alterations in plasma density affect crucial EMIC wave properties, including dispersion relations, growth rates, and resonance energies. Our findings reveal that the Kappa index, which characterizes the high-energy tail of the distribution, significantly influences these wave characteristics. Moreover, the presence of multiple ion species introduces intricate interactions that further modify EMIC wave behaviour. This research offers valuable insights into wave-particle interactions in space plasmas and has potential implications for space weather modelling and magnetospheric dynamics.

Keywords— EMIC waves, multi-component plasma, Kappa distribution function, Particle aspect analysis, Plasma Particle Density, auroral acceleration

1. Introduction

EMIC waves are low-frequency transverse waves, oscillating between 0.1 and 5 Hz, and are detected on Earth's surface as Pc1-Pc2 pulsations [1]. Originating in the equatorial zone of Earth's magnetosphere, these waves travel along magnetic field lines to the ionosphere as left-handed circularly polarized waves [2] highlighted the significance of ion cyclotron instability in the generation of EMIC waves, laying the groundwork for understanding their behaviour in space plasmas. EMIC waves are a significant aspect of plasma physics, particularly in space environments where they influence particle dynamics and energy distribution. These waves are critical in understanding various plasma phenomena, including wave-particle interactions, energy transfer, and space weather dynamics. In a multi-ion plasma environment, EMIC waves interact with different ion species, leading to complex wave behaviours that are essential for accurate modelling and interpretation of space plasma systems [3][4].

The plasma particle density significantly influences the dispersion and propagation characteristics of EMIC waves. For instance, the density of different ion species determines the cyclotron resonance frequency [5], at which ions interact most strongly with the waves. For hydrogen, the cyclotron frequency is higher than for heavier ions like helium and oxygen, leading to different resonance behaviours across species. As the density of heavier ions like oxygen increases, the cyclotron resonance shifts to lower frequencies, thereby altering the wave properties and the regions in the plasma where wave-particle interactions occur [6][7].

Moreover, in a non-Maxwellian plasma, where the velocity distribution of particles deviates from thermal equilibrium, the propagation of EMIC waves is further influenced. In these cases, the classical Maxwellian distribution fails to capture the presence of high-energy particles, which are often present in space plasmas. This deviation is commonly described by the Kappa distribution function[8], which accounts for a more realistic particle distribution in environments like the Earth's magnetosphere [9].

Traditional models of EMIC waves often use Maxwellian distributions to describe plasma particles. However, space plasmas, especially in regions like the auroral acceleration zone [10], frequently deviate from Maxwellian behaviour. The Kappa distribution function, which accounts for non-

Maxwellian characteristics with a high-energy tail, offers a more accurate representation of such plasmas [11][12]. This distribution is particularly relevant for modelling plasma with steep loss-cone distributions and varying particle densities [13].

This study investigates the combined effect of particle density and the Kappa distribution function on EMIC wave in a multi-ion plasma environment consisting of H^+, He^+, and O^+. We focus on how varying plasma densities, coupled with the Kappa distribution, influence EMIC wave properties, including dispersion relations, growth rates, and resonance energies. By analysing these effects, we aim to elucidate how the Kappa distribution enhances or alters wave dynamics compared to traditional Maxwellian models [14]. The findings are evaluated in relation to space plasma settings, especially within the auroral acceleration zone, where non-Maxwellian characteristics and varying ion densities are prevalent[15].

Understanding these combined effects provides crucial insights into wave-particle interactions and their implications for space weather phenomena. This research contributes to a more comprehensive understanding of EMIC wave dynamics and their role in magnetospheric processes.

The rest of the paper is organized as follows -Section 1 introduces the background and motivation for the study. Section 2 provides a review of related work on EMIC wave propagation in multi-ion plasmas, with particular focus on the roles of plasma density and non-Maxwellian (Kappa) distributions. In Section 3, we describe the key plasma parameters considered and outline the density variation function used in our analysis. In Section 4, the **Kappa distribution function** is expressed to characterize the non-thermal velocity distribution of plasma particles, which is often observed in space environments such as the auroral acceleration region .Section 5 is dedicated to the EMIC wave growth rate, while Section 6 discusses the formulation of growth length. Section 7 outline the methodology and Section 8 presents and analyses the numerical results, highlighting how variations in plasma density and the Kappa index affect EMIC wave behaviour. Finally, Section 9 conclude the main findings of the study and suggests potential directions for future research.

2. Related Work

The propagation and evolution of Electromagnetic Ion Cyclotron (EMIC) waves in multi-ion plasmas have been extensively explored due to their pivotal role in wave-particle interactions in space plasmas. Early studies by Cornwall et al. [2] and Horne and Thorne [7] investigated the resonant heating of ions, emphasizing the importance of heavy ions like He^+ and O^+ in modifying EMIC wave characteristics. Denton et al. [4] and Engebretson et al. [5] explored field-aligned propagation and density distribution effects, providing foundational insights into EMIC wave dispersion in the magnetosphere.

The influence of non-Maxwellian velocity distributions has also garnered attention, with several works proposing the Kappa distribution as a realistic model for suprathermal particle populations in space [8, 11]. Livadiotis [8] and Vasyliūnas et al. [11] provided comprehensive theoretical frameworks for the use of Kappa functions in space physics. Summers and Thorne [9] extended this to wave dispersion modelling, integrating modified plasma dispersion functions that account for high-energy tails.

More recent studies have applied the Kappa distribution to EMIC wave analysis. Mitchell and Baines [14] and Meda and Ahirwar [15] showed how lower Kappa indices significantly amplify wave growth due to enhanced high-energy resonances. These studies demonstrated that deviations from Maxwellian assumptions are essential to accurately predict wave behaviour in auroral and radiation belt regions.

Heavy ion contributions, particularly from O^+ and He^+, have been shown to alter the instability thresholds and resonance conditions for EMIC waves [6, 16, 17]. Andre and Yau [18] linked these ion populations to ion outflows, especially in auroral regions. Jordanova et al. [16] provided simulation evidence that heavy ions lead to increased EMIC wave activity, reinforcing observational data from missions like Cluster and THEMIS.

Despite these contributions, limited studies have jointly investigated the simultaneous impact of plasma density variation and the Kappa distribution in a multi-ion framework. This gap is addressed in the present study, which integrates both aspects to assess EMIC wave behaviour in a cold, magnetized plasma. The combined analysis of ion composition and suprathermal particle effects enables a more holistic understanding of wave evolution under realistic space plasma conditions.

Density variation

Perturbation associated with particle density is evaluated; to find various quantity we consider kappa distribution and density distribution as [15]

$$n_l = \frac{hV_{\perp H^+}+\Omega_{H^+}+K_\Pi F_k\left(V_{\Pi H^+}\right)}{\left[K_\Pi V_{\Pi H^+}-\left(\omega-\Omega_{H^+}\right)\right]}\times[\cos(\alpha_o)-\varepsilon\cos(\alpha+\varepsilon t\beta\sin(\alpha-\beta t)] + \frac{hV_{\perp He^+}+\Omega_{He^+}+K_\Pi F_k\left(V_{\Pi He^+}\right)}{\left[K_\Pi V_{\Pi He^+}-\left(\omega-\Omega_{He^+}\right)\right]}\times[\cos(\alpha_o)-\varepsilon\cos(\alpha+\varepsilon t\beta\sin(\alpha-\beta t)] + \frac{hV_{\perp O^+}+\Omega_{O^+}+K_\Pi F_k\left(V_{\Pi O^+}\right)}{\left[K_\Pi V_{\Pi O^+}-\left(\omega-\Omega_{O^+}\right)\right]}\times[\cos(\alpha_o)-\varepsilon\cos(\alpha+\varepsilon t\beta\sin(\alpha-\beta t)] \quad (1)$$

Where

$\alpha = K_\Pi z - \omega t - \Psi$ $\quad (\beta = K_\Pi V_{\Pi l} - (\omega - \Omega_l)$

Distribution function

To accurately describe the non-Maxwellian characteristics of space plasmas, particularly in the auroral acceleration region, we employed the **Kappa distribution function**. This distribution effectively captures the presence of suprathermal particles, which are not well represented by traditional Maxwellian assumptions.

The isotropic **Kappa distribution function** is given by:

$$F_k(V_l) = \frac{1}{\pi^{3/2}} \frac{1}{V_{\perp H^+}^2 + V_{\Pi H^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \times \left\{1 + \frac{V_{\Pi H^+}^2}{k_p V_{\perp H^+}^2} + \frac{V_{\perp H^+}^2}{k_p V_{T\perp H^+}^2}\right\}^{-k_p-1} + \frac{1}{\pi^{3/2}} \frac{1}{V_{\perp He^+}^2 + V_{\Pi He^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \times \left\{1 + \frac{V_{\Pi He^+}^2}{k_p V_{\perp He^+}^2} + \frac{V_{\perp He^+}^2}{k_p V_{T\perp He^+}^2}\right\}^{-k_p-1} + \frac{1}{\pi^{3/2}} \frac{1}{V_{\perp O^+}^2 + V_{\Pi O^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \times \left\{1 + \frac{V_{\Pi O^+}^2}{k_p V_{\perp O^+}^2} + \frac{V_{\perp O^+}^2}{k_p V_{T\perp O^+}^2}\right\}^{-k_p-1} \tag{2}$$

$$l = H^+/He^+/O^+.$$

k_p is the kappa distribution index

bi-kappa distribution is implemented as

$$F_k(V_{I\Pi l}) = \frac{1}{\pi^{1/2}} \frac{1}{V_{T\Pi H^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \left\{1 + \frac{V_{\Pi H^+}^2(\omega-\Omega_{H^+})^2}{K_\Pi V_{T\Pi H^+}^2}\right\}^{-k_p-1} + \frac{1}{\pi^{1/2}} \frac{1}{V_{T\Pi He^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \times \left\{1 + \frac{V_{\Pi He^+}^2(\omega-\Omega_{He^+})^2}{K_\Pi V_{T\Pi He^+}^2}\right\}^{-k_p-1} + \frac{1}{\pi^{1/2}} \frac{1}{V_{T\Pi O^+}^2} \frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)} \times \left\{1 + \frac{V_{\Pi O^+}^2(\omega-\Omega_{O^+})^2}{K_\Pi V_{T\Pi O^+}^2}\right\}^{-k_p-1} \tag{3}$$

In above equation $V_{T\perp l}^2$ and $V_{T\Pi l}^2$ are thermal velocity.

$$V_{T\perp l}^2 = \left[\frac{k_p-3/2}{k}\frac{2k_pT_{\perp H^+}}{m_{H^+}}\right] + \left[\frac{k_p-3/2}{k}\frac{2k_pT_{\perp He^+}}{m_{He^+}}\right] + \left[\frac{k_p-3/2}{k}\frac{2k_pT_{\perp O^+}}{m_{O^+}}\right] \tag{4}$$

$$V_{T\Pi l}^2 = \left[\frac{k_p-3/2}{k_p}\frac{2k_pT_{\Pi H^+}}{m_{H^+}}\right] + \left[\frac{k_p-3/2}{k_p}\frac{2k_pT_{\Pi He^+}}{m_{He^+}}\right] + \left[\frac{k_p-3/2}{k_p}\frac{2k_pT_{\Pi O^+}}{m_{O^+}}\right] \tag{5}$$

The kappa distribution function is represented as[9]

$$Z_k(\xi) = \frac{1}{\pi^{1/2}} \frac{1}{k_p^{1/2}} \frac{\Gamma(k_p+1)}{\Gamma(k_p-1/2)} \int_{-\infty}^{\infty} \frac{\left(1+\frac{x^2}{k_p}\right)^{-k_p} dx}{(x-\xi)} \tag{6}$$

Growth Rate for EMIC waves

The growth rate is obtained as-

$$\frac{\gamma}{\omega_l} = \frac{\frac{\pi^{3/2}\Omega_{H^+}}{K_\Pi V_{T\Pi H^+}}\left[\frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)}\left(1-\frac{\omega}{\Omega_{H^+}}\right)\left(\frac{T_{\perp H^+}}{T_{\Pi H^+}}\right)-1\right]\times\left[1+\frac{(\omega-\Omega_{H^+})^2}{K_\Pi^2 V_{T\Pi H^+}^2}\right]^{-k_p-1}}{\left(\frac{CK_\Pi}{\omega_{pH^+}^2}\right)^2\left(\frac{2\Omega_{H^+}-\omega}{\Omega_{H^+}-\omega}\right)+\frac{1}{2}\frac{\omega^2}{(\Omega_{H^+}-\omega)^2}} + \frac{\frac{\pi^{3/2}\Omega_{He^+}}{K_\Pi V_{T\Pi He^+}}\left[\frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)}\left(1-\frac{\omega}{\Omega_{He^+}}\right)\left(\frac{T_{\perp He^+}}{T_{\Pi He^+}}\right)-1\right]\times\left[1+\frac{(\omega-\Omega_{He^+})^2}{K_\Pi^2 V_{T\Pi He^+}^2}\right]^{-k_p-1}}{\left(\frac{CK_\Pi}{\omega_{pHe^+}^2}\right)^2\left(\frac{2\Omega_{He^+}-\omega}{\Omega_{He^+}-\omega}\right)+\frac{1}{2}\frac{\omega^2}{(\Omega_{He^+}-\omega)^2}} + \frac{\frac{\pi^{3/2}\Omega_{O^+}}{K_\Pi V_{T\Pi O^+}}\left[\frac{\Gamma(k_p+1)}{k_p^{3/2}\Gamma(k_p-1/2)}\left(1-\frac{\omega}{\Omega_{O^+}}\right)\left(\frac{T_{\perp O^+}}{T_{\Pi O^+}}\right)-1\right]\times\left[1+\frac{(\omega-\Omega_{O^+})^2}{K_\Pi^2 V_{T\Pi O^+}^2}\right]^{-k_p-1}}{\left(\frac{CK_\Pi}{\omega_{pO^+}^2}\right)^2\left(\frac{2\Omega_{O^+}-\omega}{\Omega_{O^+}-\omega}\right)+\frac{1}{2}\frac{\omega^2}{(\Omega_{O^+}-\omega)^2}} \tag{7}$$

It is noted that the k-Lorentz distribution impacts the growth rate through plasma density and modifications to the electromagnetic wave moving parallel to the ambient magnetic field.

Growth length

The growth length of the electromagnetic ion cyclotron wave is derived from

$$\boldsymbol{L_g = \frac{V_g}{\gamma}}$$

Where, $\boldsymbol{\gamma}$ is growth rate, Vg is group velocity of the wave

So now

$$L_g = \frac{1}{\gamma\omega_{pH^+}^2}\left(-C^2K_\Pi\Omega_{H^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pH^+}^2K_\Pi\Omega_{H^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pH^+}^2K_\Pi^2\Omega_{H^+}}}\right) + \frac{1}{\gamma\omega_{pHe^+}^2}\left(-C^2K_\Pi\Omega_{He^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pHe^+}^2K_\Pi\Omega_{He^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pHe^+}^2K_\Pi^2\Omega_{He^+}}}\right) - + \frac{1}{\gamma\omega_{pO^+}^2}\left(-C^2K_\Pi\Omega_{O^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pO^+}^2K_\Pi\Omega_{O^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pO^+}^2K_\Pi^2\Omega_{O^+}}}\right) \tag{8}$$

Thus, the growing length of EMIC waves propagating parallel to the magnetic field has been influenced by the kappa distribution function.

$$L_g = \frac{1}{\gamma\omega_{pH^+}^2}\left(-C^2K_\Pi\Omega_{H^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pH^+}^2K_\Pi\Omega_{H^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pH^+}^2K_\Pi^2\Omega_{H^+}}}\right) + \frac{1}{\gamma\omega_{pHe^+}^2}\left(-C^2K_\Pi\Omega_{He^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pHe^+}^2K_\Pi\Omega_{He^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pHe^+}^2K_\Pi^2\Omega_{He^+}}}\right) - + \frac{1}{\gamma\omega_{pO^+}^2}\left(-C^2K_\Pi\Omega_{O^+} + \frac{C^4K_\Pi^3+2C^2\omega_{pO^+}^2K_\Pi\Omega_{O^+}}{\sqrt{C^4K_\Pi^4+4C^2\omega_{pO^+}^2K_\Pi^2\Omega_{O^+}}}\right) \tag{9}$$

3. Methodology

This study investigates the effect of plasma particle density and the Kappa distribution function on the growth of Electromagnetic Ion Cyclotron (EMIC) waves in a multi-ion plasma environment composed of H^+, He^+, and O^+ ions. The analysis assumes a cold, collisionless, and magnetized plasma where waves propagate parallel to the ambient magnetic field, a common condition in the auroral acceleration region [7, 10].

To incorporate non-Maxwellian particle behavior, we adopted the Kappa distribution function, which has been widely used to model the suprathermal features of space plasmas [8, 11, 12]. Unlike the classical Maxwellian distribution, the Kappa function accounts for high-energy tails in the velocity distribution, making it more suitable for space environments with deviations from thermal equilibrium.

The study examines both the temporal growth rate and growth length of EMIC waves under varying plasma densities and different Kappa indices (e.g., k_p=2 and k_p=6). These parameters help evaluate how non-thermal particle populations and ion composition affect wave-particle interactions. The presence of heavier ions like O^+ and He^+ is especially important, as they resonate at lower frequencies and significantly alter wave dispersion and amplification [4, 5, 16].

A bi-Kappa distribution approach was implemented to reflect anisotropies in plasma environments, capturing differences in parallel and perpendicular particle dynamics relative to the magnetic field [9, 14]. This approach enables an accurate representation of particle behavior in space plasmas, particularly in magnetospheric regions.

Numerical analysis were performed using Mathcad to solve the dispersion relation and calculate the growth rate and growth length as functions of wave vector and ion density. The calculation workflow included:

- Inputting plasma parameters (magnetic field, ion densities, temperatures),
- Applying the Kappa distribution to model velocity space,
- Solving wave growth conditions under different ion compositions,
- Plotting the resulting wave characteristics for analysis.

The outcomes reveal how density variation and non-Maxwellian effects jointly influence EMIC wave evolution, offering a more comprehensive picture compared to Maxwellian-based models. These findings align with earlier theoretical frameworks [6, 13, 15] and are relevant to magnetospheric studies and space weather prediction models.

4. Result and discussion

The results show the impact of different kappa distribution indices on the characteristics of EMIC waves. The study provides analytical expressions for various parameters, helping to understand the behaviour of these waves in different plasma conditions. Considering the steepness of the kappa distribution function, the plasma parameters to the auroral acceleration zone are utilized in the numerical analysis of the dispersion relation, resonant energies, growth rate, and growth length.

B_O=4300nT $\Omega_{H^+} = 412S^{-1}$ $\Omega_{He^+} = 102.5S^{-1}$

$\Omega_{O^+} = 25.625S^{-1}$ $\frac{V^2_{T\perp e}}{V_{T\Pi e}} = .10 - 02$ $\frac{V^2_{T\perp i}}{V_{T\Pi i}} = 10 - 15$

$T_{\perp i} = 25 - 50eV$ $V_{T\Pi i} = 6.41 \times 10^8 cm/s$

$\omega^2_{pH^+} = 3.18 \times 10^8 S^{-2}$ $\omega^2_{pHe^+} = 2.156 \times 10^5 S^{-2}$

$\omega^2_{pO^+} = 2.156 \times 10^4 S^{-2}$

$k_{II} = 10^{-10} cm^{-1}, k_\perp = 10^{-6} cm^{-1}, v_A = 3 \times 10^{10} cms^{-1}$, $\Omega_{H^+} = 412 s^{-1}, \Omega_{He^+} = 103 s^{-1}, \Omega_{O^+} = 26 s^{-1}, v_{TIIe^+} = 8.38 \times 10^7 cms^{-1}$, $\omega_{PH^+} = 9.31 \times 10^4 s^{-1}, \omega_{PHe^+} = 3.292 \times 10^4 s^{-1}, \omega_{Po^+} = 1.646 \times 10^4 s^{-1}$, $v_{TH^+} = 4.37 \times 10^7 cms^{-1}, v_{THe^+} = 4.01 \times 10^6 cms^{-1}, v_{TO^+} = 3.9 \times 10^6 cms^{-1}$

The graphs presented in Figures 1-4 demonstrate the correlation between the K_Π and the growth rate of EMIC waves propagation within a multi-ion plasma. These graphs examine how various ion compositions and Kappa (k_p) distribution values influence wave growth rates. This analysis is crucial in the investigation of space plasmas, where non-thermal particle distributions are prevalent, and $k_p$effectively represent the presence of energetic particles in the plasma environment

Graph (3 & 4) shows how the growth rate of EMIC waves decreases as K_Π increases for different ion compositions, particularly with varying hydrogen (H^+) and helium (He^+) concentrations. The higher hydrogen concentration corresponds to a reduction in the EMIC wave growth rate, suggesting that an increased density of lighter ions (H^+) leads to weaker wave-particle interactions and, therefore, lower amplification of the waves. The higher concentration of heavier ions (He^+) in the curve enhances the growth rate, which aligns with the understanding that heavier ions contribute more significantly to EMIC wave growth compared to lighter ions like hydrogen[16][17].

Graph (1 and 2) represents the case with k_p=2 & 6, indicating a significant deviation from a thermal Maxwellian distribution. The overall growth rates in this graph are higher than in graphs (3) and (4) for the same K_Π values. This highlights the role of a lower k_p value, which allows for a greater number of high-energy particles that interact more effectively with the EMIC waves. The heavier oxygen ions (O^+) contribute more to the wave instability compared to lighter ions, resulting in higher growth rates for the same K_Π [18].

The Kappa distribution is critical in plasma physics as it characterizes how far the plasma distribution deviates from thermal equilibrium. A lower Kappa value (e.g., k_p =2) implies a more significant presence of high-energy particles, which enhances wave-particle interactions and increases the EMIC wave growth rate. Conversely, higher Kappa values (e.g., k_p =6) correspond to distributions closer to thermal equilibrium, where fewer energetic particles exist, leading to weaker wave growth [10].

The presented analysis emphasizes the significant role of ion composition and Kappa distribution in regulating the growth rate of EMIC waves within multi-ion plasmas. Heavier ions, like oxygen and helium, coupled with a lower Kappa index (indicating a non-thermal particle population), enhance wave-particle interactions and consequently boost EMIC wave growth.

The four graphs figs. 5-8 represent the relationship between growth length and K_Π with varying values of density of H^+, O^+, and He^+, as well as different values of k_p (6 and 2). Figures 5 and 6 illustrate the relationship between growth length and K_Π for various H^+ densities and Kappa values (k_p =6 and k_p =2). As K_Π increases, growth length rises sharply, especially for higher H^+ densities. Lower H^+ densities result in smaller growth lengths but still exhibit a steady increase with K_Π. The growth rate is slower at k_p=6, where distributions approach thermal equilibrium, compared to k_p =2, which enhances non-thermal interactions. The presence of O^+ slightly reduces growth length. Figures 7 and 8 introduce He^+ ions, which further moderate growth length compared to H^+-dominant scenarios[19]. However, H^+ remains the primary driver of higher growth lengths. Fig.(7) and (8), indicating that the presence of He^+ might slow down the growth length[20].

So overall A higher hydrogen ion concentration (H^+) correlates with higher growth lengths. The reduction of k_p suppresses overall growth length. The presence of heavier ions appears to have a small mitigating effect on growth length, slightly reducing the growth length for each K_Π values. These trends can be linked to theoretical frameworks in ionospheric and plasma physics, where the combination of parameters such H^+, O^+, and He^+ influence plasma growth dynamics[21]. The parameter k_p, which is often related to geomagnetic activity, plays a crucial role in moderating these effects [22][23].

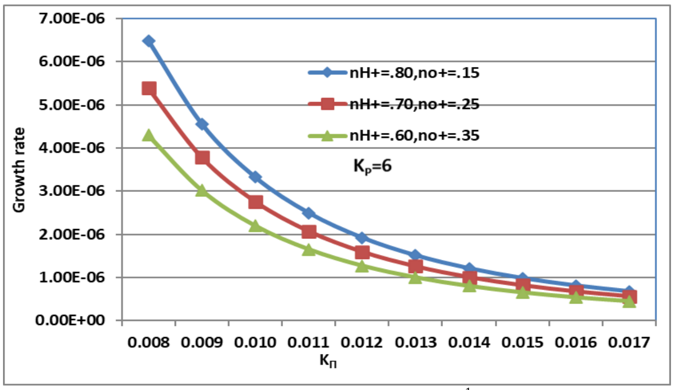

Figure 1. Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & O^+ ion density at k_p= 6.

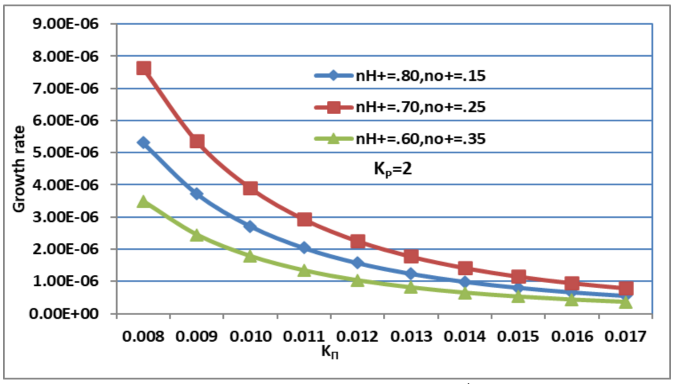

Figure 2. Change of the growth rate (γ/ω) vs K_Π (cm^{-1}) for varying values of the H^+ & O^+ ion density at $\boldsymbol{k_p}$= 2.

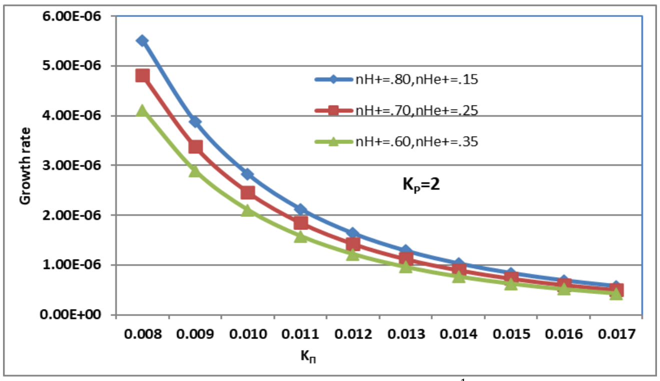

Figure 3. Change of the growth rate (γ/ω) vs K_Π (cm^{-1}) for varying values of the H^+ & He^+ ion density at $\boldsymbol{k_p}$= 2.

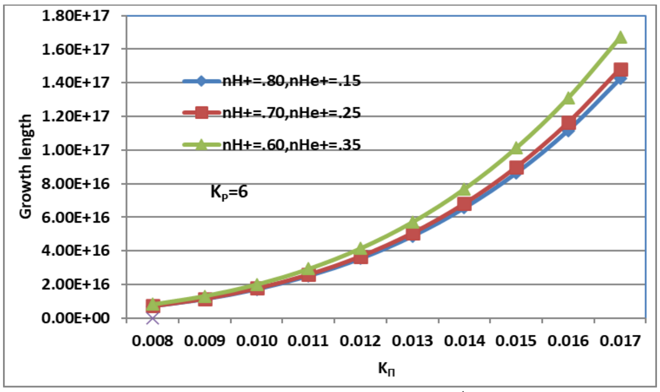

Figure 4. Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & He^+ ion density at $\boldsymbol{k_p}$= 2.

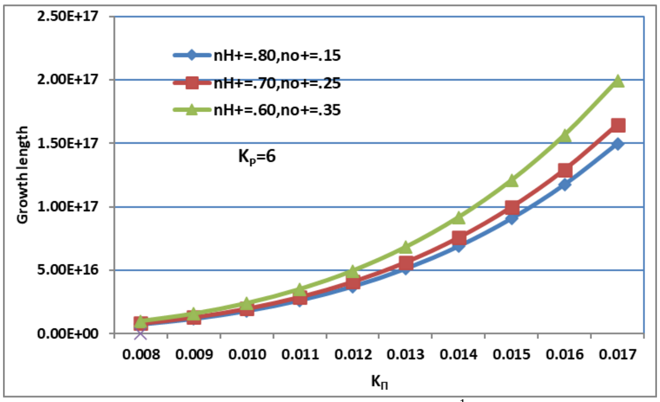

Figure 5. Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & O^+ ion density at k_p= 6.

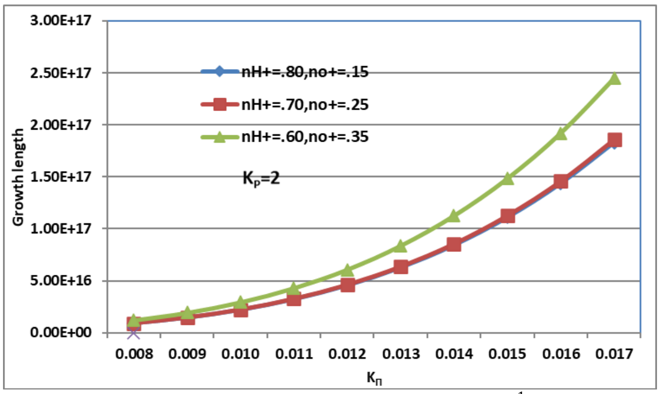

Figure 6. Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & O^+ ion density at k_p= 2.

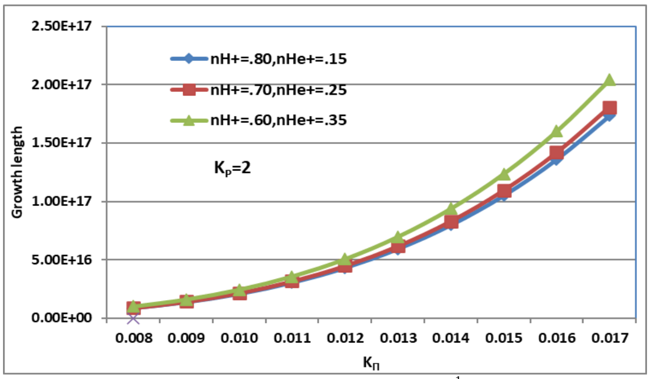

Figure 7. Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & He^+ ion density at $k_p = 2$.

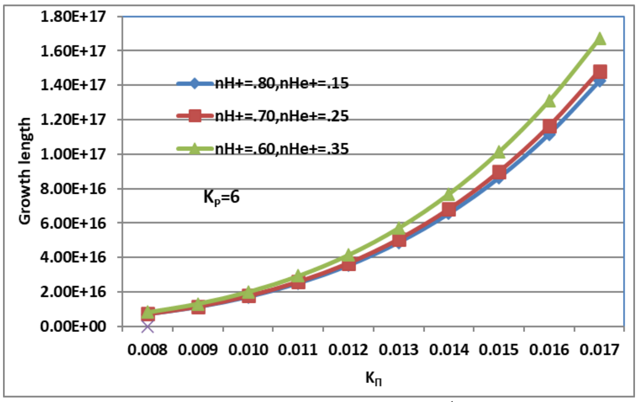

Figure 8 Change of the growth length L_g vs K_Π (cm^{-1}) for varying values of the H^+ & He^+ ion density at $k_p = 2$.

5. Conclusions

The findings of this study provide compelling insights into the fundamental processes that govern the behavior of EMIC waves in multi-ion magneto-plasma environments. It is evident that higher plasma particle densities play a pivotal role in significantly increasing wave growth rates, driven by intensified ion-wave interactions. This discovery emphasizes the critical importance of understanding how plasma density shapes the dynamics of space plasma systems, opening new avenues for deeper exploration of wave-particle interactions in complex environments.

The influence of heavier ions, such as O^+ and He^+, further underscores the complexity and richness of these systems. These ions resonate more effectively with EMIC waves at lower frequencies, enhancing wave growth and making them indispensable in the overall wave dynamics. Their role highlights the importance of multi-ion effects in shaping the energy transfer processes within plasma, providing a more holistic understanding of how diverse ion populations influence space plasma behavior.

The Kappa distribution, particularly at lower values of k_p, unveils the presence of super thermal particles, facilitating broader velocity distributions and more intricate resonant interactions. This, in turn, drives wave growth, especially in plasmas dominated by heavier ions. The multi-ion effects further complicate wave dispersion characteristics, reinforcing the need for a more sophisticated approach to studying wave-particle interactions in these environments.

These findings are not only vital for advancing our theoretical understanding of space plasmas but also hold significant practical implications. By enhancing our models of space weather dynamics, this research lays the groundwork for better space weather forecasting and more resilient space missions. Understanding how ion composition and wave-particle interactions influence space weather phenomena is crucial for predicting their impact on Earth's technological systems, from satellite communication to power grids. As we continue to unravel the complexities of space plasmas.

Future Scope: Future research should extend this work by incorporating oblique wave propagation to explore how non-parallel EMIC waves interact with multi-ion plasmas, as real magnetospheric conditions often involve complex wave angles. Validating our findings with observational data from missions like THEMIS or Cluster could confirm the predicted growth rate enhancements and refine Kappa distribution parameters for specific plasma regions. Additionally, investigating temperature anisotropies and their interplay with plasma density could provide deeper insights into wave instability triggers. Developing three-dimensional simulations that account for spatial variations in ion composition would further enhance model accuracy, paving the way for comprehensive space plasma frameworks. These directions promise to advance our understanding of magnetospheric dynamics and improve predictive capabilities for space weather phenomena.

Data Availability
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest
The authors declare that there is no conflict of interest regarding the publication of this paper.

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Author's Contribution
Rahul Bhaisaniya: Conceptualization, Methodology, Data Analysis, Writing – Original Draft, Investigation, Visualization, Validation, Writing – Review & Editing.

Dr G Ahirwar: Resources, Supervision, Project Administration.

We, the undersigned authors, declare that the manuscript titled "Investigate the Influence of Plasma Particle Density Described by the Kappa Distribution Function on EMIC Waves within a Multi-Ion Magneto-Plasma" is our original work and has not been submitted elsewhere for publication. Rahul Bhaisaniya, as the primary author and research scholar, conducted the theoretical analysis, performed the numerical simulations, and interpreted the results presented in this work.

He also prepared the manuscript draft and ensured its alignment with the research objectives. Dr. Ganpat Ahirwar, as the research supervisor, provided continuous guidance throughout the study, offered critical insights into the methodology and theoretical framework, and contributed to refining the manuscript for clarity and scientific rigor. We confirm that all necessary permissions and ethical considerations have been adhered to during the course of this research.

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