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Study of Stabilizing a Wormhole with Negative Casimir Effect of Fermion

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Abstract— This work presents a review on making a traversable wormhole with the help of the negative Casimir effect produced by fermions. According to Morris-Thorn traversable wormhole conditions, we need negative mass-energy density to stabilize a wormhole. So we are creating this local negative energy density with the interaction of fermions with two magnetically charged Blackhole. This interaction produced negative Casimir-like energy. This local negative energy density can be used to stabilize a wormhole and make it traversable.

***Keywords*—** Negative mass-energy, Traversable Wormhole, etc.

I. INTRODUCTION

Wormholes are pathways connecting two points a billion light-years apart or even a few meters [1],[3]. They could be in the same universe as we know or maybe between our universe and another parallel universe. The primary concern regarding the Wormhole is its stability. The wormholes collapse as soon as they are formed due to a strong gravitational pull at the mouths [1]. Later, physicists suggested that exotic matter could keep the mouth open, thereby making the wormhole stable for travel [7],[8]. Exotic matter is hypothetical, and its properties will be against the laws of physics that we know. They have negative mass-energy density; therefore, they account for repulsion, causing the space-time to curve oppositely. Thereby the tendency of Wormhole to pull on itself is violated [26]. However, exotic matter is yet to be found, although possibilities exist in a small amount. The pseudo-negative-pressure density produced by the Casimir effect is the known closest representation for an exotic matter [5]. But that still won't be enough to make the Wormhole humanly traversable.

According to Morris-Thorn traversable wormhole conditions, it is challenging to construct as it needs negative mass-energy density [7],[8],[28]. As the Wormhole's throat is very unstable and collapses under its gravity, negative mass-energy can create negative gravity to keep the throat stable. Negative mass-energy density can be found in exotic matter, which has negative mass. But the exotic matter is not yet discovered, and the possibility of its existence is also significantly less. So to make a wormhole traversable[17],[18], we need other kinds of matter that have already been discovered and can produce local negative energy density. So after reviewing many excellent papers on stabilizing a wormhole [2],[3],[5],[6],[7],[8],[9],[10],[11],[26],[27], this paper gives a brief idea of the current development of this field. In this, we are using the negative Casimir effect produced by fermions to stabilize a wormhole. Negative mass-energy density produced by the Casimir effect of fermions can stabilize a wormhole.

Section I contains the Introduction of How a wormhole can be stabilized, Section II has the methodology of General Relativity to construct a wormhole, Section III includes the dynamics of Fermion and their interaction with two magnetically charged Blackholes, section IV describes results and discussion, section V describes the Review of the result, and Section VI concludes research work with future directions).

II. METHODOLOGY

The major limitation in creating a wormhole or even discovering it is the stability and the need for exotic matter. Without exotic matter, wormholes could be created using electrically charged fermions with small mass enough not to collapse. If we can find a solution to use fermions in creating a wormhole that is large enough, then that will significantly contribute to wormhole studies that are still in their early stages.

Our Method starts from Einstein Field Equation [12], and then the interaction of Fermion with two magnetically charged black holes leads to local negative mass-energy density.

The Einstein Field equation is,

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor

Einstein Field Equation describes gravity as a result of space-time curved by mass and energy. $G_{\mu\nu}$ Is determined by the curvature of space-time at a particular point in space-time equated with energy and momentum [4,15,16]. Solution components of $g_{\mu\nu}$ specify space-time geometry. Initial trajectory - particles found using geodesic equation.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Here, the L.H.S. tells the curvature of space-time, and R.H.S. gives the mass and energy.

Mass tells space-time how to curve, and space-time tells mass how to move [12],[29].

Solving Einstein field equation, we can find the Metric equation of Static spherically symmetric Wormhole and describe it in [5].

Space-time Metric of Spherically symmetric Static Wormholes [10],[11] is

$$ds^2 = -e^{2\phi(r)}c^2dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta + \sin^2\theta d\phi^2\right)$$

$e^{\phi(r)}$ And $b(r)$ are arbitrary functions of the radial coordinate. In the case of wormholes, these functions are referred to as the redshift function and shape function [2], respectively. The essential characteristics of a wormhole geometry are encoded in these functions, so to have a wormhole, these two functions must satisfy some general constraints discussed by Morris and Thorne.

Morris-Thorne Metric

Based on the Schwarzschild metric

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

the space-time metric as proposed by Morris-Thorne can be written in general as,

$$ds^2 = -e^{2\Phi(l)}c^2dt^2 + dl^2 + r^2(l)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Where,

$$l \in (-\infty, +\infty)$$

The two asymptotically flat regions occur at $l \approx \pm\infty$ and the radius of the Wormhole $r_0 = \min\left[r(l)\right]$

Using the Reimann, Ricci, and Einstein tensors, this can be reduced to

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Here $\Phi(r)$ is gravitational redshift, and b(r) is arbitrary functions of radius only and is the speed of light. The radial component l is related to r by $l \pm \left(r^2 - r_o^2\right)$ and the functions $\Phi(r)$ and $b(r)$ are $\Phi = 0, b = \frac{r_o^2}{r}$

For the following non-zero elements,

$$G_{tt} = e^{-2\phi}$$

$$G_{rr} = \left(1 - \frac{b}{r}\right)^{-1}$$

$$G_{\theta\theta} = r^2$$

$$G_{\phi\phi} = r^2\sin^2\theta$$

$$G_{\mu\nu} = \begin{bmatrix} e^{-2\Phi} & 0 & 0 & 0 \\ 0 & \left(1-\frac{b}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}$$

According to Morris-Thorne's condition, the tension should be more than the total energy density in a traversable wormhole throat. That is $\tau_o > \rho_o$ [7],[8],[9]. This is called exotic matter property. Its property is that when an observer falls through the Wormhole with a radial velocity close to the speed of light will measure a negative mass-energy density. This violates many energy conditions of general relativity [19],[20],[21],[22],[23],[24],[25]. According to which mass-energy density should be positive. So, this violates weak energy conditions, Null energy conditions, and dominant energy conditions.

So, constructing a traversable wormhole whose throat is stable satisfies a traversable wormhole's Morris-Thorne property $\tau_o > \rho_o$. It should violate many energy conditions. So, local mass-energy density becomes negative. This should be done by using exotic matter. So, How to construct a stable wormhole.

III. MODELLING AND ANALYSIS

Here, critical violation of the Average Null Energy Condition is done by negative Casimir-like energy produced by the charged fermions. In this solution, a pair of entangled near extremal black holes interacts by exchanging fermion fields [5]. For this, at first, we will analyze magnetically charged black holes [13],[14] and their interactions with charged fermions. As a result of these interactions, there will be negative Casimir-like energy. Then the construction of the Wormhole and its metric will be done afterward.

The metric of a single magnetically charged black hole is

$$ds^2 = -\left(1-\frac{2MG}{r}+\frac{r_e^2}{r^2}\right)dr^2 + \left(1-\frac{2MG}{r}+\frac{r_e^2}{r^2}\right)^{-1} dr^2$$
$$+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

where, $r_e^2 = \frac{\pi q^2 G}{g^2}$, M=mass of the black hole, q=integer representing charge of the black hole, g=coupling constant of u(1) gauge field.

The horizon is localized at $r = r_+$
where, $r\pm = MG \pm \sqrt{m^2G^2 - r_e^2}$

Here, relations for temperature and entropy are

$$T = \frac{r_+ - r_-}{4\pi r_+^2}$$

$$S = \frac{\pi r_+^2}{G}$$

At extreme points, $T \to 0$ and $r_+ = r_- = r_e$.

Here, M and S can be expanded from small T to get -

$$M = \frac{r_e}{G} + \frac{2\pi^2 r_e^2 T^2}{G} + \ldots$$
$$S = \frac{\pi r_e^2}{G^2} + \frac{4\pi^2 r_e^3 T}{G^2} + \ldots$$

And, $AdS_2 \times S^2$ geometry [5] can approximate nearly the geometry near the horizon. Therefore,

$$ds^2 = r_e^2\left[-d\tau_r^2\left(\rho_r^2-1\right)+\frac{d\rho_r^2}{\rho_r^2-1}+\left(d\theta^2+\sin^2\theta d\phi^2\right)\right]$$

where, $\tau_r = 2\pi T t$ and $\rho_r = \frac{r-r_e}{2\pi T r_c^2}$ for $r - r_e << r_e$

This metric is a good approximation when, $r - r_e << r_e$ and $\mathrm{Tr}_e << 1$. This geometry connects the flat space-time through a region located around $r \approx r_e$.

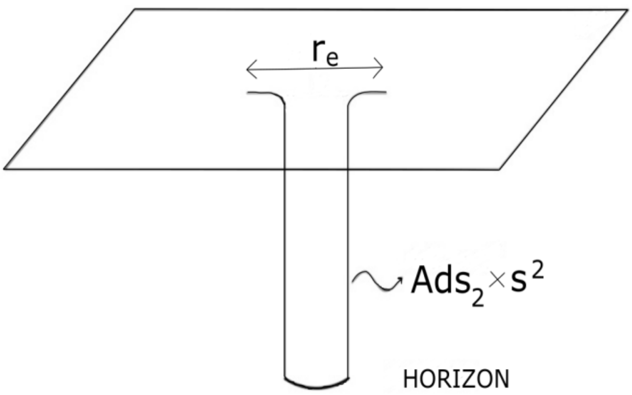

Figure 1: A drawing representing a near extremal black hole geometry [5].

Dynamics of Fermion

The action equation of a single charged Dirac fermion [6] with charge Q=1 is

$$I = \int d^4x\sqrt{g}\left[\frac{R}{16\pi G} - \frac{1}{4g^2}F^2 + i\vec{x}\left(X - iA\right)\chi\right]$$

Here, A is the vector potential. We can get this from the magnetically charged black hole.

$$A = \frac{q}{2}\cos d\phi$$

In this magnetic field, massless charged Fermion gives a series of Landau levels. The energy of Landau levels has an orbital contribution and a magnetic dipole contribution. For the lowest Landau level, these two contributions of Fermion cancel each other, giving rise to a zero-energy state [5]. This state has degeneracy q, related to corresponding angular momentum through the relation 2J+1=q.

Therefore four-dimensional chiral Fermion gives rise to the q massless two-dimensional chiral Fermion [6]. This is important for construction.
Four-dimensional spinor is

$$\chi = \psi \otimes \eta$$

Here, $\eta \to$ Spinor on the S^2 sphere $\psi \to$ spinor in the other two directions.
The lowest Landu level corresponds to a negative chirality spinor η_- that obeys the two-dimensional massless Dirac equation with a magnetic field on the two-sphere.

$$\nu^\alpha\left(\Delta_\infty - iA_\alpha\eta\right) = 0$$

The solution of this equation is

$$\eta_- \alpha e^{im\phi}\left(\sin\frac{\theta}{2}\right)^{j-m}\left(\cos\frac{\theta}{2}\right)^{j+m}$$

Where, $j = \frac{q-1}{2}$ and $-j \le m \le j$.

For, $q >> 1$ these solutions are localized around $\theta_m - \frac{m}{i}$.

Each of these modes on the sphere gives [5] a two-dimensional massless method ψ in the r and t directions. These equivalent fermions propagate through space with metric.

$$ds_2^2 = \left|g_{tt}\right|\left(-dt^2 + dx^2\right)$$

Here, $dx = \sqrt{\frac{g_{rr}}{-g_{tt}}}dr$.

These are the creators of negative Casimir-like energy.

Two interacting magnetic Black Holes

If we see two oppositely charged black holes from a significant distance $d >> r_e$, they will behave like a magnetic dipole.
The vector potential of this magnetic dipole is given by

$$A = \frac{q}{2}\left(\cos\theta_1 - \cos\theta_2\right) d\phi$$

Where θ_1 and θ_2 angles are represented in the figure.

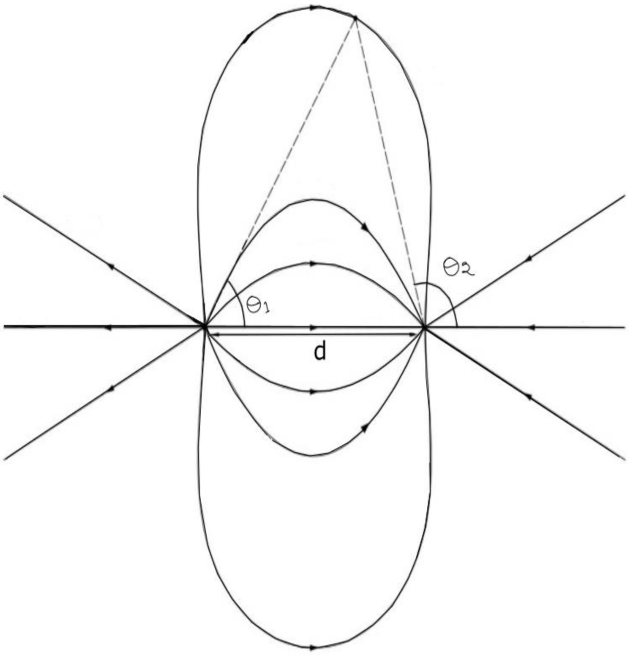

Figure 2: Magnetic sources at distance d from each other and their field lines [5].

This configuration is rotational invariant around the axis connecting the two sources. Considering it in Z-axis and angel around it to be spatial metric in cylindrical coordinate is

$$ds^2 = d\rho^2 + dz^2 + \rho^2 d\phi^2$$

In these coordinates, components of the magnetic fields are-

$$B_\rho = \frac{\delta_z A\phi}{\rho}$$

$$B_z = -\frac{\delta_\rho A\phi}{\rho}$$

$$B_\phi = 0$$

The tangent vector along the field lines has to be along B. If the solution is $A\phi =$ constant, the gradient will be normal, and the condition will be fulfilled.

Now, the equations of the field are -

$$\cos\theta_1 - \cos\theta_2 = V, 0 \le V \le 2$$

$$V = \frac{j+m}{m}$$

Here, we connected the geometry of the magnetic field lines to the fermions fields. Now, the length of the trajectory traveled by a fermion along a field line will be dependent on V and the distance d between the two mouths of the Wormhole,

$$L_{fieldline} = df\left(V\right)$$

Here, shape function $f\left(V\right)$ is determined by coordinate transformation of and ρ to θ_1 and θ_2.

IV.RESULTS AND DISCUSSION

Assembly of Wormhole

All the above elements describe the traversable wormholes created by joining the throats of two magnetically charged black holes [5],[17],[18].

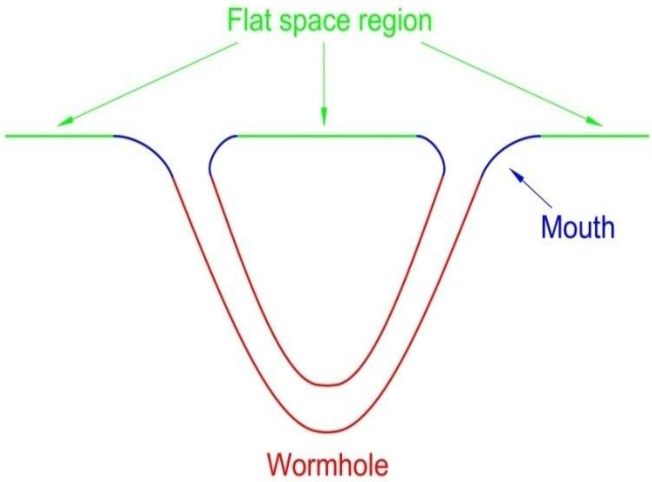

Figure 3: Assembly of the Wormhole [5].

This picture represents the Splitting the geometric configuration by paper into three different regions, the actual Wormhole, the two mouths, and the flat space around them

Actual Wormhole is characterized by the metric same as that of the throat of a near extremal black hole.

$$ds^2 = r_e^2\left[-dt^2\left(\rho^2+1\right) + \frac{d\rho^2}{\rho^2+1} + \left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

These coordinates are not similar to those used in the single magnetic black hole. This is a coordinate system covering the Penrose Diagram AdS_2. The metric of the magnetically charged black hole $\left(r - r_e << r_e\right)$ will be matched to that of asymptotic regions of the metric $(\rho >> 1)$.

Math happens via the following $\tau = \frac{t}{l}, \rho = \frac{\left(r - r_e\right)}{r_e^2}$.

Given two conditions for the overlapping section $\rho >> 1$ from wormhole metric and $r - r_e << 1$ from mouth metric, finding inequality which involves 1 :

$$\rho = \frac{\left(r - r_e\right)}{r_e^2} >> 1 \text{ and } \frac{r - r_e}{r_e} << 1 \text{ gives } \frac{l}{r_e} >> 1 .$$

There exists a cut-off point past which overlap will not make sense anymore. The throat of a wormhole opens around $r - r_e \approx r_e$.

Calculating the rescaled length of the Wormhole,

$$L_{throat} = \int_{\rho_{cutoff}}^{\rho_{cutoff}} d\rho \frac{l}{\rho^2+1}$$

$$L_{throat} = l\left[\arctan\left(\rho_{cutoff}\right) - \arctan\left(-\rho_{cutoff}\right)\right]$$

$$L_{throat} = l\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = l\pi$$

For calculating negative energy density, we are required to calculate the length of the fermions trajectory.

The 2-D fermions exist on a circular curve exiting from one mouth and traveling with the magnetic field line until they reach the other mouth, entering the second mouth and traveling through a Wormhole. They once again go to the original mouth.

The whole curve length is the sum between the Wormhole length of the field line and two small transitions segments around the mouth.

Analyzing the system means the length of the throat is much larger than that of the distance between the mouth $L_{throat} - l\pi$.

The Casimir - like energy given by

$$E_{wormhole} = E_{gorundstate} + E_{conformalanomaly}$$

q fermions moving in a circle of length , energy is

$$E_{wormhole} = -\frac{q2\pi}{12L} + \frac{q\pi}{24L} = -\frac{q\pi}{8L} = -\frac{q}{8l}$$

As we know that $L = \pi l$.

Now, we have to consider two regions: one for the asymptotically flat space-time outside Wormhole and the other for the throat of Wormhole, and these two solutions have to overlap at mouth region.

$$ds^2 = -A(r)dt^2 + B(r)dt^2 + r^2 d\Omega^2$$

We get,

$$A = \frac{1}{B} = \left(1-\frac{r}{r_e}\right)^2 - \frac{2\grave{o}}{r_e}$$

$$where, \grave{o} = GM - r_e$$

The stress-energy has quantum contribution as,

$$\hat{T}_{tt} = \hat{T}_{xx} = -\frac{q}{8\pi l^2}\frac{1}{4\pi r_e^2}$$

$$T_{\alpha}^{\beta} = T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T_{\grave{o}}^{\grave{o}}$$

$$where, \alpha, \beta = t, x$$

This also contains classical contributions from the magnetic field

$$R_{\rho\rho} - \frac{1}{2} g_{\rho\rho} R - 8\pi G\left(T_{\rho\rho}^{\text{mag}} + \hat{T}_{\rho\rho}\right) = 0$$

$$T_{\rho\rho} = -\frac{q}{8\pi 4\pi r_e^2 \left(1+\rho^2\right)^2}$$

Then from above einstein's equation, we get

$$\rho\phi' - \phi = -\frac{qG}{4\pi r_e^2\left(1+\rho^2\right)}$$

Since $\rho\phi' - \phi = 8\pi G\left(1+\rho^2\right)\hat{T}_{\rho\rho}$

We move on to overlap two metric and matching sphere part two matrices we get,

$$\phi = \frac{\left(r-r_e\right)\left(r_e+r\right)}{r_e^2}$$

Approximating as $r + r_e \approx 2r_e\left(r-r_e\right)$ is very small in this limit)

$$\phi \approx \frac{2\left(r-r_e\right)}{r_e}$$

Comparing the leading order time component for l we get,

$$l = \frac{16 r_e^3}{qG}$$

Therefore,

$$E = \frac{r_e^3}{Gl^2} + E_{wormhole} = \frac{r_e^3}{Gl^2} - \frac{q}{8l}$$

Putting l, we get $E = -\frac{q^2 G}{256 r_e^3}$ also expressed in the form of mass correction

$$E = \frac{2\grave{o}}{G}$$

But what if this is not the case $d <<< l$, then

$$L = \pi L + df\left(\nu\right)$$

and

$$E = \frac{r_e^3}{Gl^2} + \frac{q}{24l} - \frac{q\pi}{6}\int_0^2 \frac{d\nu}{2}\frac{1}{(l\pi + f\left(\nu\right)}$$

$$E = \frac{r_e^3}{Gl^2} + \frac{q}{24l} - \frac{q\pi}{6}\frac{1}{\pi l + d}$$

The minimal energy will be,

$$E = -\frac{q\pi}{24d}$$

The low limit of l gives the above approximation. Using these values of l, we obtain $L_{throat\,\min} = d$

V. REVIEW OF RESULT

This model gives rise to negative energy density at the throat. That is,

$$E = -\frac{q\pi}{24d}$$

Which is the key to creating traversable Wormholes. This negative energy can prevent the Wormhole's throat from collapsing under its gravity [5],[27].

From a significant distance $r >> r_e$ looks like we are looking at two oppositely magnetic charged black holes. They were eventually attracting each other and colliding in one. But to stabilize this system, we must consider two black holes rotating about their center of mass. Where angular velocity calculated by Kepler's 3rd law

$$\Omega = 2\sqrt{\frac{r_e}{d^3}}$$

A small orbital eccentricity would result in a small perturbation of the throat.

There are few concerns for mouth while rotating

- Calculated trajectories of fermions disturb, as they feel the additional force by rotation
- As they accelerate, the radiation is emitted as they are charged particles.
- The Unruh-like temperature will occur, making energy inside the throat less negative.
- Another way of stabilizing is by AdS_4 space-times. These can be the rotation of black holes in AdS_4 spaces through boundary conditions that allow fermions from one space-time.

VI. CONCLUSION AND FUTURE SCOPE

Therefore we conclude, that the negative mass-energy density we found using $L_{throat_{\min}} = d$

$$E = -\frac{q\pi}{24d}$$

This negative energy density can Stabilize the Wormhole and can hold the throat open. The length of Fermion's trajectory is also estimated; the throat's minimum length will equal the distance between the mouths, validating our hypothesis that this is a long wormhole. Negative charge densities are produced by varying the l, which is the free line parameter. This is the prerequisite for making a wormhole that can be traversed.

Yet, none of the proposed solutions have been experimentally observed and are still restricted to mathematical calculations to date. There is more scope of development in theoretical as well as experimental in this field of research.

VII. RELATED WORK

To Stabilize a wormhole, we need local negative mass-energy density to achieve this; Juan Maldacena, Alexey Milekhin, and Fedor Popov used the Casimir effect of Fermion particles in their paper on the Traversable wormholes in four dimensions [5].

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