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A Generalization to Integral e^-x. (f(x)-f'(x))dx

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Abstract—with the help of this paper, the author is providing a generalization related to the integral of $e^{-x}\left(f(x)-f'(x)\right)dx$ for bringing the main result author used concept of integral by parts, the concept of mathematical induction, integration, differentiation etc. by applying author's generalization we can easily solve many time-consuming integrals which may a good thing for solvers to find the given integrals in less duration of time respectively. Most of the times peoples can't think about founding generalizations of those problems that they solved. Those who tries, loose their motive due to complexities in calculations and that's why only some of them found generalization to a specific expression or problem respectively.

Keywords—Induction, Integral by Parts, Integral, derivative, Function etc.

I. INTRODUCTION

Some weeks ago, In Toyesh paper on A generalization to the integral of $e^{x}\left(f(x)+f'(x)\right)$[1]. Toyesh introduced his way or method to solve integrals like $e^{x}sinx\ dx$, $e^{x}cosx\ dx$ etc same as in this paper we are taking generalization to the integral $e^{-x}\left(f(x)-f'(x)\right)dx$ as both of them are likely same that's why now also in this paper we will use mathematical induction [2] that used to prove given mathematical statement perfectly may that's why mathematicians used to tell mathematical induction is just a "Domino tile fall" but here we will use induction in such a manner which should also be correct in which firstly we will prove some theorems on the basis of which we conclude main result respectively. There we will use Integration, Differentiation, Integral by parts [3][4] etc for proving different theorems.

Overview
We can divide this paper on the basis of different sections are as follows: -

Abstract and keywords

1. **Introduction**: in this section author briefly discuss meaning of the title and basic knowledge required
   And **Overview**: in this section we can observe content of the paper.
2. **Related Work**: this section deals with the published work related to the main title.
3. **Main Result**: this section is the main section which carried new theorems, work etc.
4. O**pen problem:** in this section there are some open problems for renders to found and further discussion
5. **Conclusion**: this section deals with uses of introduced method, further scope of the method, and summary of the work.

Acknowledgement
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II. RELATED WORK

Generally, in most of the books we find integral of $e^{x}\left(f(x)+f'(x)\right)$ as Integral Calculus by Amit M Agrawal, Mathematics class 12 by R.D. Sharma [5], Mathematics of class 12 by R.S. Aggarwal, Mathematics of class 12- NCERT

[6] etc. that's why in [1] Toyesh provide generalization of it only. as far integral of $e^{-x}\big(f(x) - f'(x)\big)$ likely above integral but there is a difference of (-ive) sign above e and before differentiation of function i.e., f'(x). we know that integral of $e^{-x}\big(f(x) - f'(x)\big)$ is not soo! popular that's why first here taken it as theorem no. 1.

III. MAIN RESULT

We are proving some theorems are as follows: -

Theorem 1.

$$\int e^{-x}(f(x) - f'(x))dx = -e^{-x}f(x) + C$$

Proof:

$$\int e^{-x}(f(x) - f'(x))dx = \int e^{-x}f(x)dx - \int e^{-x}f'(x)dx$$

$$= f(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f(x)\cdot\int e^{-x}\,dx\right)dx - \int e^{-x}f'(x)dx$$

$$= -e^{-x}f(x) + \int e^{-x}f'(x)dx - \int e^{-x}f'(x)dx + C = -e^{-x}f(x) + C$$

Theorem 2.

$$\int e^{-x}(f(x) - f''(x))dx = -e^{-x}\big(f(x) + f'(x)\big) + c$$

Proof:

$$\int e^{-x}(f(x) - f''(x))dx = \int e^{-x}f(x)dx - \int e^{-x}f''(x)dx$$

1. $\int e^{-x}f(x)dx = f(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f(x)\cdot\int e^{-x}\,dx\right)dx$

$$= -e^{-x}f(x) + \int e^{-x}f'(x)dx$$

2. $\int e^{-x}f'(x)dx = f'(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f'(x)\cdot\int e^{-x}\,dx\right)dx$

$$= -e^{-x}f'(x) + \int e^{-x}f''(x)dx$$

Then,

$$\int e^{-x}(f(x) - f''(x))dx = -e^{-x}f(x) - e^{-x}f'(x) + \int e^{-x}f''(x)dx - \int e^{-x}f''(x)dx$$

$$\int e^{-x}(f(x) - f''(x))dx = -e^{-x}\big(f(x) + f'(x)\big) + c$$

Theorem 3.

$$\int e^{-x}(f(x) - f'''(x))dx = -e^{-x}\big(f(x) + f'(x) + f''(x)\big) + C$$

Proof:

$$\int e^{-x}(f(x) - f'''(x))dx = \int e^{-x}f(x)dx - \int e^{-x}f(x)dx$$

1. $\int e^{-x}f(x)dx = f(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f(x)\cdot\int e^{-x}\,dx\right)dx$

$$= -e^{-x}f(x) + \int e^{-x}f'(x)dx$$

2. $\int e^{-x}f'(x)dx = f'(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f'(x)\cdot\int e^{-x}\,dx\right)dx$

$$= -e^{-x}f'(x) + \int e^{-x}f''(x)dx$$

3. $\int e^{-x}f''(x)dx = f''(x)\int e^{-x}\,dx - \int\left(\frac{d}{dx}f''(x)\cdot\int e^{-x}\,dx\right)dx$

$$= -e^{-x}f''(x) + \int e^{-x}f'''(x)dx$$

Then,

$$\int e^{-x}(f(x)-f'''(x))dx = -e^{-x}f(x)-e^{-x}f'(x)-e^{-x}f''(x)+\int e^{-x}f'''(x)dx-\int e^{-x}f(x)dx+C$$
$$\int e^{-x}(f(x)-f'''(x))dx = -e^{-x}\cdot\left(f(x)+f'(x)+f''(x)\right)+C$$

Theorem 4.

$$\int e^{-x}(f(x)-f''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)\right)+C$$

Proof:

$$\int e^{-x}(f(x)-f''''(x))dx = \int e^{-x}f(x)dx-\int e^{-x}f(x)dx$$

1. $\int e^{-x}f(x)dx = f(x)\int e^{-x}\,dx-\int\left(\frac{d}{dx}f(x)\cdot\int e^{-x}\,dx\right)dx$
$= -e^{-x}f(x)+\int e^{-x}f'(x)dx$
2. $\int e^{-x}f'(x)dx = f'(x)\int e^{-x}\,dx-\int\left(\frac{d}{dx}f'(x)\cdot\int e^{-x}\,dx\right)dx$
$= -e^{-x}f'(x)+\int e^{-x}f''(x)dx$
3. $\int e^{-x}f''(x)dx = f''(x)\int e^{-x}\,dx-\int\left(\frac{d}{dx}f''(x)\cdot\int e^{-x}\,dx\right)dx$
$= -e^{-x}f''(x)+\int e^{-x}f'''(x)dx$
4. $\int e^{-x}f'''(x)dx = f'''(x)\int e^{-x}\,dx-\int\left(\frac{d}{dx}f'''(x)\cdot\int e^{-x}\,dx\right)dx$
$= -e^{-x}f'''(x)+\int e^{-x}f''''(x)dx$

Then,

$$\int e^{-x}(f(x)-f'''(x))dx = -e^{-x}f(x)-e^{-x}f'(x)-e^{-x}f''(x)-e^{-x}f'''(x)+\int e^{-x}f''''(x)dx$$
$$-\int e^{-x}f(x)dx+C$$

That's why

$$\int e^{-x}(f(x)-f''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)\right)+C$$

Likewise, we can also prove some other identities as follows: -

$$\int e^{-x}\left(f(x)-f'(x)\right)dx = -e^{-x}f(x)+C$$
$$\int e^{-x}(f(x)-f''(x))dx = -e^{-x}\left(f(x)+f'(x)\right)+c$$
$$\int e^{-x}(f(x)-f'''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)\right)+C$$
$$\int e^{-x}(f(x)-f''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)\right)+C$$
$$\int e^{-x}(f(x)-f'''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)+f''''(x)\right)+C$$
$$\int e^{-x}(f(x)-f''''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)+f''''(x)+f'''''(x)\right)+C$$
$$\int e^{-x}(f(x)-f'''''''(x))dx = -e^{-x}\left(f(x)+f'(x)+f''(x)+f'''(x)+f''''(x)+f'''''(x)+f''''''(x)\right)+C$$

On behalf of above results we are concluding our main result or theorem that is

Theorem 4.

$$\int e^{-x}(f(x)-f^{n}(x))dx = -e^{-x}\left(\sum_{k=0}^{n-1}f^{k}(x)\right)+C$$

Here, nth power denoted for nth order derivative.

IV. OPEN PROBLEMS

Some open problems are as follows: -

1. Prove the main generalization without using induction.
2. Does there possible some other generalizations to $\int e^{-x}\big(f(x)-f'(x)\big)dx$?
3. Found the applications of the provided generalizations.

V. CONCLUSION AND FUTURE SCOPE

In this paper there is a generalization to $\int e^{-x}\big(f(x)-f'(x)\big)dx$. Have been proposed with the help of notable theorems, concept of induction (Mathematical Induction), General Observation and problems. After its author asks some problems in the section open problem. Many time taking problems can easily solve by using provided generalization, in short duration of time and also this method is very straight forward and easy way to solve given integral but note that given integral should be in the form of provided generalization. Further, secondly finding what is given function and what order of derivative of given function is used in problem then it's very useful to apply given generalization.

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