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Research Article

Numerical Results on the Radiative MHD Squeeze Flow of Vanadium Pentoxide (V_2O_5)-Based Jeffrey Hybrid Nanofluid through Porous Parallel Plates

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Abstract— our exploration is mainly concerned with the properties of the heat and mass transfer in Jefferies hybrid nanofluids over the squeezing tunnel that travels across a porous material. We also studied the nature of magnetohydrodynamics (MHD) fluid flow. Thermal radiation parameters effects together with other pertinent parameters were studied. Suitable similarity transformation is applied while discretizing the system of dimensionless equations. Vanadium Pentoxide (V2O5) dispersions are deliberated in the base fluid. Validation of this research is achieved by relating to published results. Graphs and tables are used to discuss the momentum, temperature and Nusselt number profiles. The graphical results show that the momentum profile, as well as temperature profile, are decreased with an increase in the Squeeze number, Deborah number and Hartman number. The Temperature and Nusselt number profiles increase with an increase in the Squeeze parameter, heat source/ sink parameter, Eckert number and radiation parameter, respectively.

Keywords— MHD, Squeeze flow, Jeffrey hybrid nanofluid, Thermal radiation, Eckert number, Squeezing flow, Hartman number, heat source/ sink

1. Introduction

Due to its wide range of physical applications, the investigation of the transfer of heat and flow for squeezing unstable kinematic flow of fluid via disparate plates has proved a fascinating field of study over millennia. For example, food processing, hydrodynamic machinery, lubrication setup, compression, and crop damage from freezing emergence, along with variability, amongst other things. The volatile flow in two dimensions of a viscous magnetohydrodynamic MHD fluid across two parallel inconceivable plates was investigated and analyzed by Akbar et al. [1]. A review of the literature reveals that the squeeze Jeffrey flow across two parallel plates is the subject of the majority of studies in these days. Squeezing flow is the term used to describe the constriction of fluid across two separate plates that results in the fluid beginning to flow. Because it is used in the geometric model of lubricant flow in pneumatic elevators, molding with injections, and bearings as well, numerous scientists performed their researches on this area; such as Madaki et al. [2], [3], [4], Hussaini et al. [5], [6]. Stefan [7] proposes to explore the squeezing of viscosity over a horizontal tube. Cameron [8] subsequently investigated the squeezing lubricant fluid review beyond two unbounded sheets for viscous fluid of squeezing flow. Wang [9] found a completely new similarity transformation that converts Navier-Stokes towards ordinary differential equations. Bujurke et al. [10] then looked at an analytical solution in accordance with Wang's findings. In addition to Khan et al. [11], Rashidi et al. [12]. Scientists are quite interested in Jeffrey fluid because it exhibits both relaxation and retardation characteristics. How the fluid behaves in response to applied shear stress. Because it can change from a shear thinning flow to a Newtonian fluid when a high force is applied to the fluid, it is categorized as such. Noor et al. [13]. Matinfar et al. [14] went into great detail into heat radiation effects on Jeffery fluid flows during squeezing. The impact of thermal radiation and heat generation/absorption on the squeezing of unstable Cu as well as TiO_2-nanofluid was investigated quantitatively as well as computationally by Madaki et al. [15]. The influence of electromagnetic radiation on unsteadies the free convection magnetohydrodynamic flows of Brinkman-type systems over a medium that is porous with Newtonian heat has been examined by Mamatha et al. [16]. Thermal radiation influence on squeezing flow fluid from Casson between parallel plates was reported by Khan et al. [17[. Makinde et al. [18] examined the effects of Brownian motion and thermophoresis on MHD bioconvection of nanofluid through quartic chemical process along with the non- linear thermal radiation beyond an upper horizontal surface of a paraboloid of rotation. The influence of an integrated electromagnetic field upon radiant bioconvection flow via a plate that is vertical having thermophoresis along with Brownian motion was studied by Avinash et al. [19]. The impacts of suspended nanoparticles and non-linear thermal radiation on the convection and heat transfer boundary layer flow of nanofluids on a heated vertical sheet were examined by Mahanthesh et al. [20]. The purpose of a heat generation/absorption is to decrease and increase a fluid's thermal conductivity, correspondingly. The temperature of the high conductivity fluid rises, while the low conductivity fluid exhibits the reverse behavior. Noor et al. [21] used the power series technique, Qasim [22] examined the effects of the heat generation/absorption around temperature along mass movement over a vertical stretching plate.

Ahmed et al. [23] stated that several studies have demonstrated the usage of composite nanofluid in technological advancement programs, including refrigerant for engines for vehicles, nuclear energy facilities, and electrical devices. Numerous physical models have demonstrated the flow of hybrid nanofluids. Alghamdi et al. [24] deliberated on the Magnetohydrodynamic nanofluid flow and thermal transmission at an inertia plug over a horizontal stretchable sheet for the Casson hybrid nanofluid. Subsequently, the effects of MHD, thermal slip, viscous dissipation, and an exponential convective heat exchange on stationary point movement at vertically stretched surfaces were examined by Abbas et al. in [25]. Dual sheets as well as multi-wall nanotubes made of carbon with conventional fluid are considered. Mahabaleshwar et al. [26] investigated the MHD flow of Copper- Aluminum- oxide composites with water as the base fluid on a porous stretch/shrink medium through slip in velocity conditions, in addition to radiative heat transfer.

There has not yet been any research done on the mass along with heat transmission of the MHD composite nanofluid through the squeezing of a pair of parallel plates acting as thermal radiation sources, with vanadium pentoxide (V_2O_5) acting as the solid material (nanoparticle), to the best of our knowledge. Squeezing Jeffrey's hybrid nanofluid flow due to its complexity in terms of governing equations has generally received little attention. Therefore, this research is aimed at obtaining the mathematical solutions of the Mass and heat transfer in radiant-MHD vanadium pentoxide (V_2O_5)-based squeezing flow Jeffrey nanofluid hybrids are equipped with heat generation/ absorption (i.e., filling the void left by Noor and Shafie [27]), and hence compare the present result with the existing ones.

2. Description of the Problem

This research will contain a numerical solution describing the behaviour of the hybrid nanofluid Jeffrey squeezed between a pair of parallel plates. However, our study would concentrate towards the effects of radiant heating, magnetic field on the fluid flow and the heat source/sink on the general flow pattern. Thus, the fluid flow would consider to be squeezing, Newtonian and incompressible.

We studied the unstable squeezed Jeffrey hybrid nanofluid flow across a pair of plates over a porous material with magnetohydrodynamic along heat generation/absorption together with chemical reaction. Vanadium peroxide hybrid nanoparticles are taken into consideration. There is an expanse of $y = \pm h(t) = l(1 - \alpha t)^{1/2}$ between the two plates. Both the top and bottom plates are subject to the external velocity $v_2(t) = \frac{\partial h(t)}{\partial t}$. Up to t = 1/α, the two separate plates shift away when $\alpha < 0$ but nearer when $\alpha > 0$. Just as shown on fig. 1 below. The bottom surface is exerted with magnetic field, $B(t) = B_0(1-\alpha t)^{-\frac{1}{2}}$ vertically as in Noor et al. [28].

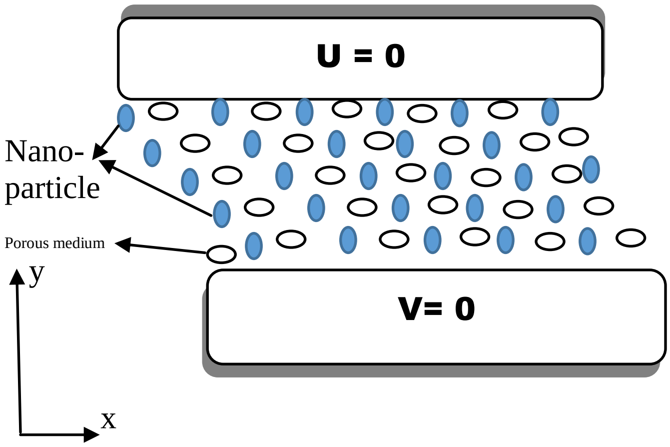

Figure 1. Schematic diagram of the problem.

Muhammad et al. ([29], [30]) contained a more general formulation of the Jeffrey hybrid nanofluid's. With momentum, temperature, nanoparticle concentration, and continuity equations just as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = \frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{\partial^2 v}{\partial y^2} + \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\lambda_2}{1+\lambda_1}$$
$$\left(\frac{\partial^3 v}{\partial t \partial y^2} + u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right) \tag{2}$$
$$-\frac{\sigma_{hnf}B(t)}{\rho_{hnf}}u - \frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{\varphi}{k_1(t)}u = 0,$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{hnf}}{(\rho Cp)_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]-\frac{1}{(\rho Cp)_{hnf}}\frac{\partial q_r}{\partial y}+\frac{Q(T-T_\infty)}{\rho_{hnf}}=0, \quad (3)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-k_c\left(C-C_\infty\right)=0, \quad (4)$$

Subject to the boundary conditions

$$y=h(t):u=0,C=C_\infty,v=v_w=\frac{\partial h(t)}{\partial t},T=T_w, \quad (5)$$

$$y=0:\frac{\partial u}{\partial y}=0,\frac{\partial^3 u}{\partial y^3}=0,\frac{\partial T}{\partial y}=0,\frac{\partial C}{\partial y}=0,v=0, \quad (6)$$

Table 1. Thermophysical properties of the fluid and solid particles

Physical Properties	Fluid phase (water)	Vanadium Pentoxide (V_2O_5)
$C\rho$ (j/Kg k)	4179	127.7
ρ (Kg/m^3)	997.1	3.35
K (W/mK)	0.613	4.22
$\sigma(\mu S/cm)$	0.05	13.0

According to the Stefan-Boltzmann rule, the amount of radiation absorbed q_r over period of time departing an object corresponds with a fourth of a degree of the temperature in absolute terms, or $q_r=\sigma T^4 A$. This represents the radiation heat fluctuation. Where T is the absolute temperature, A is the emitting body's area, σ is the Stefan Boltzmann constant, and q is the heat transfer per unit time. Nevertheless, Roseland also clearly expressed the radiative heat flux factor that appears in equation (4) above as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (7)$$

In reference to certain research conducted by Kothandapani and Prakash [31] as well as Akbar et al. [1], it is believed for the temperature difference inside the flow is significantly limited, and the term T^4 can be expressed as a function of temperature. Because of this, T^4 is expanded using the Taylor series expansion around T_∞ ignoring the higher-order constants. As a result, we devise

$$T^4=T_\infty^4+4T_\infty^3T-4T_\infty^4$$

$$T^4=4T_\infty^3T-3T_\infty^4 \quad (8)$$

Substituting eqs. (7) and (8) into (3), gives

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{hnf}}{(\rho Cp)_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]+\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\frac{Q(T-T_\infty)}{\rho_{hnf}}=0, \quad (9)$$

3. Method of solution with Runge-Kutta fourth order along with shooting technique

We used the same non-dimensional variables as Shafie and Noor [27] to transform the mechanism of partial differential system (PDEs) into ordinary differential system (ODEs).

$$u=\frac{ax}{2(1-at)}f'(\eta),v=-\frac{al}{2\sqrt{(1-at)}}f(\eta),$$

$$\eta=\frac{y}{l\sqrt{(1-at)}}f'(\eta),\theta=\frac{T}{T_w},\varphi=\frac{C}{C_w} \quad (10)$$

When the similarity variables (10) are substituted into equations (1)–(4), the resulting non-dimensional ODE is:

$$\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)f^{iv}-S\left(\eta f'''+3f''+f'f''-ff'''\right)+\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{De}{2}\left(\eta f^{v}+5f^{iv}+2f''f'''-ff^{iv}-ff^{v}\right)-\frac{\sigma_{hnf}}{\sigma_f}\frac{\rho_f}{\rho_{hnf}}Ha^2f''-\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{1}{Da}f''=0, \quad (11)$$

$$\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}\frac{k_{hnf}}{k_f}\frac{1}{\Pr}\left(1+\frac{4}{3}Rd\right)\theta''+S\left(f\theta'-\eta\theta'\right)+\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}\gamma\theta+\frac{\mu_{hnf}}{\mu_f}\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}Ec\left[\left(1+\frac{1}{\lambda_1}\right)\left[(f'')^2+4\delta^2(f')^2\right]\right]=0, \quad (12)$$

$$\frac{1}{Sc}\varphi''+Sc\left(f\varphi'-\eta\varphi'\right)-R\varphi=0, \quad (13)$$

These are the situations at the boundary

$$\eta=0:\varphi'(\eta)=0,\ f(\eta)=0,\ f''(\eta)=0,\theta(\eta)=0,\ f^{iv}(\eta)=0,$$

$$\eta=1:\varphi(\eta)=1,\ f(\eta)=1,\theta(\eta)=1,\ f'(\eta)=0, \quad (14)$$

Thus, following lists are correlations between thermophysical characteristics of the hybrid nanofluid:

$$\frac{\mu_{hnf}}{\mu_f}=\frac{1}{\left(1-\phi_{hnf}\right)^{2.5}}$$ Dynamic viscosity (μ) (15)

$$\rho_{hnf}=\left(1-\phi_{hnf}\right)_{\rho f}+\phi_{V_2O_5} \quad (16)$$

Density of hybrid nanofluid (ρ)

$$\left(\rho Cp\right)_{hnf}=\left(1-\phi_{hnf}\right)\left(\rho Cp\right)_f+\phi_{v_2O_5}\left(\rho Cp\right)_{v_2O_5}$$

Heat capacity $\left(\rho Cp\right)$ (17)

$$\frac{\sigma_{hnf}}{\sigma_f}=\left[1+\frac{3\phi_{hnf}\left(\phi_{v_2O_5}\sigma_{v_2O_5}\right)}{\phi_{v_2O_5}\sigma_{v_2O_5}+2\phi_{hnf}\sigma_{hnf}-\phi_{hnf}\sigma_f \left(\phi_{v_2O_5}\sigma_{v_2O_5}-\sigma_f\left(\phi_{v_2O_5}\sigma_{v_2O_5}\right)\right)}\right]$$

Electrical conductivity (σ) (18)

$$\frac{k_{hnf}}{k_f}=\frac{\frac{\left(\rho_{V_2O_5}k_{V_2O_5}+\phi_{hnf}k_{V_2O_5}\right)}{\phi_{hnf}}+2k_f+\left(\phi_{V_2O_5}k_{V_2O_5}-2\phi_{V_2O_5}k_f\right)}{\left(\frac{\phi_{V_2O_5}k_{V_2O_5}}{\phi_{V_2O_5}}\right)+2k_f-\left(\phi_{V_2O_5}k_{V_2O_5}\right)+\phi_{V_2O_5}k_f}$$

Thermal conductivity (k) (19)

The substantial expressions from the governing equations are defined as

$$Rd=\frac{4\sigma^{*}T_{\infty}^{3}}{k^{*}k_f},S=\frac{\alpha l^2}{2v_f},Ha=lB_0\sqrt{\frac{\sigma}{\rho_f v_f}},\ R=\frac{ak_2l^2}{v_f},Sc=\frac{v_f}{D_m},$$

$$\Pr=\frac{v_f}{\alpha_f},Da=\frac{k_0}{\varphi l^2},\gamma=\frac{Q_0 l^2}{v_f(\rho Cp)_f},Ec=\frac{\alpha^2x^2}{4C_pT_w(1-at)^2},$$

$$De=\frac{\alpha\lambda_2}{1-at},\delta=\frac{1}{x}\left(1-at\right)^{\frac{1}{2}},$$

4. Results and Discussion

In this part, we comprehensively explained the influence of relevant parameters as graphical representations of the characteristics of fluid profiles. For hybrid nanofluid containing vanadium pentoxide, the important features of the heat radiation parameter, squeezing number, Eckert number, chemical reaction, ratio of retardation time parameter, and some other relevant factors on fluid profiles are described in profundity. Additionally, table 1 above discusses the thermophysical characteristics of hybrid nanoparticles. By applying appropriate similarity transformations, the flow's governing equations are converted into ODEs. Using a shooting technique, a transformed collection of ODEs is deciphered by the use of Runge–Kutta–Fehlberg's fourth–fifth-order (RKF 45) technique. To validate the current finding for −f (1), Table 2 has been produced using published results from Jyothi et al. [32] as well as Shafie and Noor [27]. Whereas, table 3, shows the computed values for the Skin friction coefficient concerning the thermal radiation parameter and the heat source/ sink parameter. It is clear that irrespective of the change of values in the heat source/ sink the fact remained the same for any rise in the radiation parameter, the Skin friction coefficient is decreased. The momentum profile f (η) for a range of Squeeze number values is shown in Fig. 2. It shows that when the Squeeze number increases, the momentum profile is greatly improved.

Table 2. Numerical results of - $f''(1)$ for Squeeze number (S)
With $\lambda_1\to\infty$, Da→∞, De→∞, Ha = Ec = δ = γ = R = φ_2 = **0** and Sc = Pr = 1.

Squeeze num **(s)**	Jyothi et al. (2021) $-f''(1)$	Shafie and Noor (2023) $-f''(1)$	Present result $-f''(1)$
-1.0	2.170090	2.170255	2.170223
-0.5	2.617403	2.617512	2.617391
0.01	3.007133	3.007208	3.007001
0.5	3.336449	3.336504	3.336620
2.0	4.167389	4.167411	4.167389

Table 3. Computations showing the Skin friction coefficient $\left(C_f\right)$
When $\Pr=1,S=0.5,\mathrm{De}=0.12,Da=2.5,Ha=7.$

Thermal radiation **(Rd)**	C_f $\gamma=0.1$	C_f $\gamma=0.3$	C_f $\gamma=0.5$	C_f $\gamma=0.7$	C_f $\gamma=0.9$
0	0.0842	0.0842	0.0842	0.0842	0.0842
1	0.0841	0.0841	0.0841	0.0841	0.0841
3	0.0840	0.0840	0.0840	0.0840	0.0840
5	0.0839	0.0839	0.0839	0.0839	0.0839
7	0.0838	0.0838	0.0838	0.0838	0.0838

The consequences of the Deborah number parameter on the momentum profile are displayed in Fig. 3. It indicates that, as the De values are enhanced, the fluid's momentum rises.

When De is elevated, the Lorentz force in fluid flow is induced, which strengthens the adhesion force at the outermost layer boundary. Conversely, the fluid near the exterior surface moves faster as the permeability of the porous material increases. This means that the fluid is accelerating at the boundary region because the friction force within the fluid is the consequence of the Deborah number parameter on the Temperature profile displayed in Fig. 4. It indicates that, as the De values are enhanced, the fluid's temperature rises. When De is elevated, the Lorentz force in fluid flow is induced, which strengthens the adhesion force at the outermost layer boundary. Furthermore, Fig. 5 illustrates the temperature inclining with a higher Hartmann number. It is demonstrated that the fluid's temperature inclines when a magnetic field is applied to its lower surface. The flow's resistance is increased by the Lorentz force that MHD produces strengthening. The squeezing parameter is an indicator of the motility between two plates/ Surfaces that are travelling apart when S < 0 and closer together when S > 0. Whether S > 0 or S < 0, Fig. 6 depicts the fluid's temperature as it slows down. The physical interpretation states that whereas surface compression causes the fluid to travel slowly in the thin channel, substantial resistance causes the fluid to flow more slowly in the larger channel. Likewise, Fig. 7 illustrates how the heat source/ sink parameter affects the Nusselt number profile, showing that as the parameter increases, the axial Nusselt number decreases. Additionally, it is seen that the addition of the values of the parameter increases the particle's intermolecular forces, causing a drop in temperature at the bottom plate and an increase in flow viscosity. The effects of Eckert number Ec on the Nusselt number profile are illustrated on Fig. 8, which it demonstrates that the Nusselt number profile becomes less as the Eckert number rises.

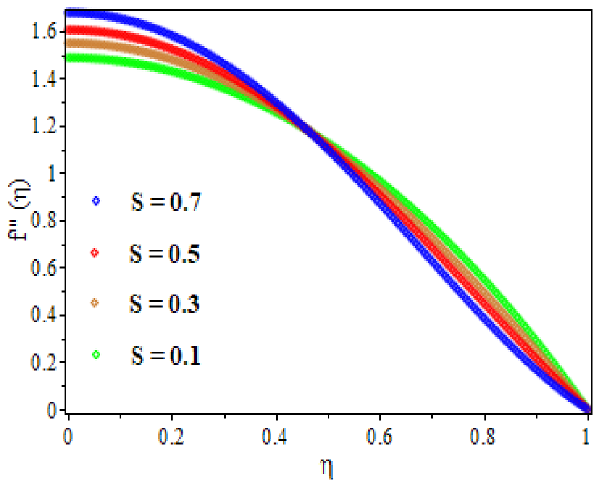

Fig. 2: Effects of Squeeze number (S) on Momentum profile

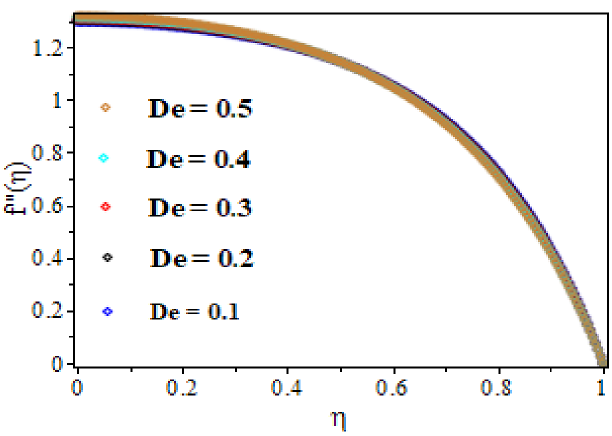

Fig. 3: Effects of Deborah Number (De) on Momentum Profile

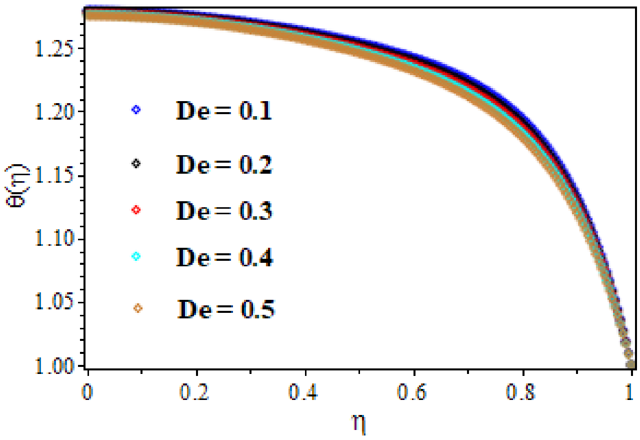

Fig. 4: Effects of Deborah number (De) on Temperature Profile

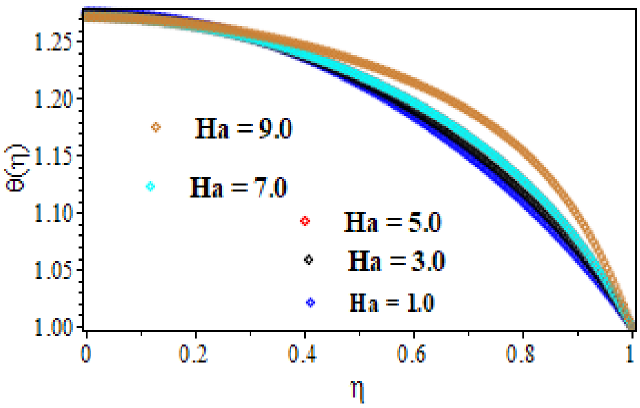

Fig 5: Effects of Hartmann number (Ha) on Temperature Profile

This is because heat energy is released into the fluid when frictional forces are present, which lowers the Nusselt number in the flow zone. Furthermore, viscous dissipation reduces momentum in the presence of viscous dissipation of the Eckert number. Finally, Fig. 9, depicted the influence of radiation parameter over the Nusselt number profile, it can be observed here that, for any increase in the radiation parameter it yields a significant decrease in the Nusselt number profile.

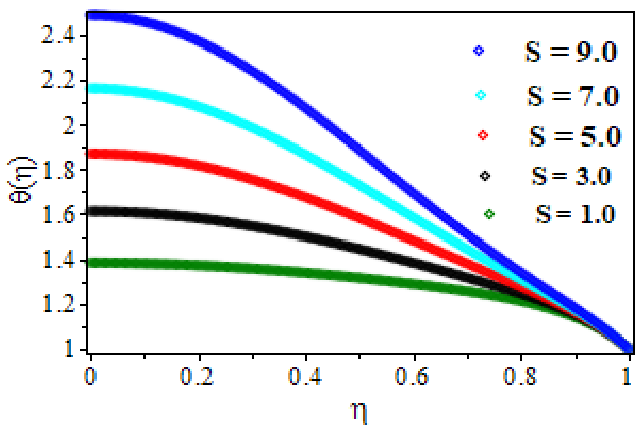

Fig. 6: Effects of Squeeze number (S) on Temperature profil

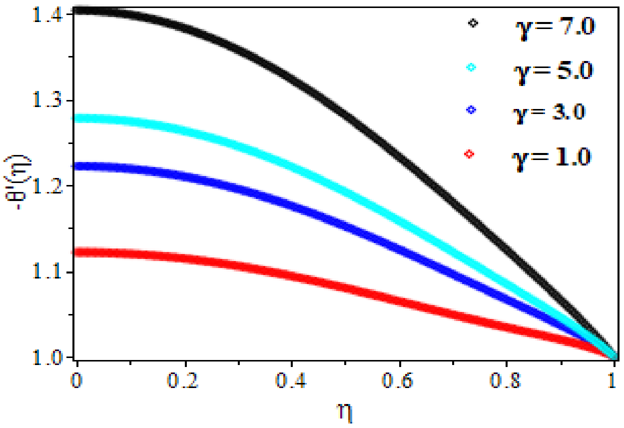

Fig 7. Effects of Heat source/ Sink on Nusselt Number Profile

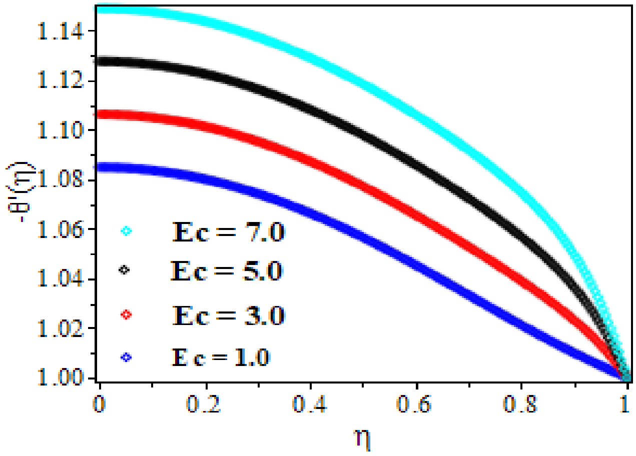

Fig. 8: Effects of Eckert Number (Ec) on Nusselt Number Profile

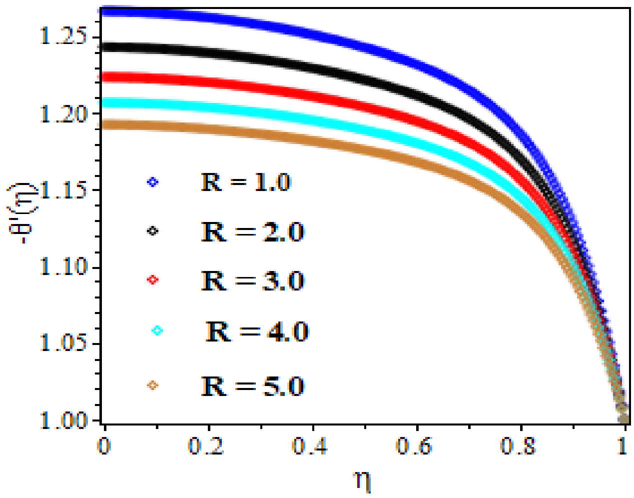

Fig. 9: Effects of Radiation Parameter on Nusselt Number Profile

4. Conclusion and Future Scope

In this study, we investigated the Casson hybrid nanofluid flow behaviour between two parallel plates while taking heat radiation and particle deposition of vanadium pentoxide (V2O5) into account. The governing equations that illustrate the fluid flow are then converted into non-linear ODEs with the support of appropriate similarity variables. The resulting equations are then numerically solved using the shooting technique and the RKF 45 scheme. Subsequently, Graphs are used to show how some relevant parameters affect various fluid profiles. The following is a summary of the findings above:

1. The momentum and temperature profiles accelerate with an increase in Deborah's number.
2. The Momentum profile is enhanced with an increase in the Squeeze number while for temperature profile the reverse is recorded.
3. The Nusselt number is seen to decrease with an increase in the Eckert number.
4. The temperature profile is seen to increase with an increase in Hartman number parameter
5. The Nusselt number profile is decreased with an increase in the heat source/ sink.
6. The Nusselt number profile is decreased with an increase in the radiation parameter.

Data Availability
None.

Conflict of Interest
The authors declare that they have no competing interests.

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Authors' Contributions
The authors contributed correspondingly towards the research. They unanimously reviewed the literature and conceived the study. They harmoniously contributed in writing the first draft of the manuscript, reviewed and edited the manuscript hence approved the final camera-ready manuscript.

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