Bianchi Type I Bulk Viscous Fluid String Dust Magnetized Cosmological Model with $\Lambda$-Term in Bimetric Theory of Gravitation

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Abstract—Bianchi Type I bulk viscous fluid string dust magnetized cosmological model with cosmological constant term have been evaluated by solving Rosen’s field equations. The cosmological constant plays the important role and it affects the physical behavior of the model in the presence as well as in the absence of both the magnetic field and the bulk viscosity. The model is studied for the positive, negative and zero values of cosmological constant. Further in special cases, the physical and geometrical aspects of the model are studied in the absence of magnetic field and in the absence of bulk viscosity.

Keywords—Bimetric Theory, Bulk Viscous Fluid, Cosmic String, Magnetic Field, Bianchi Type I.

I. INTRODUCTION

The matter distribution in the universe is satisfactorily described by the perfect fluid. However, in the universe, when neutrino decoupling occurred, then the matter behaved like viscous fluid in the early stage of the universe. The magnetic field is present in the galactic and intergalactic system. In the early phase of the universe, there is no definite evidence that the present day universe (FRW universe) was of the same type. Therefore it is important to study different Bianchi Type models in the context of the early phase of the universe. Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein’s theory of gravitation. The one among them is Rosen’s bimetric theory of gravitation [1], [2]. The Rosen’s bimetric theory is the theory of gravitation based on two metrics. One is the fundamental metric tensor $g_{ij}$ which describes the gravitational potential and the second metric $\gamma_{ij}$ refers to the flat space–time and it describes the inertial forces associated with the acceleration of the frame of reference. The metric tensor $g_{ij}$ determines the Riemannian geometry of the curved space time which plays the same role as given in Einstein’s general relativity and it interacts with matter. The background metric $\gamma_{ij}$ refers to the geometry of the empty universe (no matter but gravitation is there) and describes the inertial forces. The metric tensor $\gamma_{ij}$ has no direct physical significance but appears in the field equations. Therefore it interacts with $g_{ij}$ but not directly with matter. One can regard $\gamma_{ij}$ as giving the geometry that would exist if there were no matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$. Thus at every point of space–time, there are two metrics

$$ds^2 = g_{ij}dx^i dx^j$$

$$d\eta^2 = \gamma_{ij}dx^i dx^j$$

The field equations of Rosen’s bimetric theory of gravitation are

$$N^i_j - \frac{1}{2} N \delta^i_j + \Lambda \ g^i_j = -8\pi k T^i_j$$

where $N^i_j = \langle 1/2 \rangle \gamma^{pr} (g^{ij} g_{uj}) \psi^p \ , \ N = N^i_i \ , \ k = \sqrt{8\pi/\gamma}$

together with $g = \det (g_{ij})$ and $\gamma = \det (\gamma_{ij})$. Here the vertical bar ($\mid$) stands for $\gamma$–covariant differentiation and $T^i_j$ is the energy–momentum tensor of matter fields.

Several aspects of bimetric theory of gravitation have been studied by Rosen [1], [2]. Karade [3], Katore, Rane [4], Israelit [5] and Khadekar, Tade [6]. In particular, Reddy, Rao [7] have obtained some Bianchi Type cosmological models in bimetric theory of gravitation. The purpose of Rosen’s bimetric theory is to get rid of the singularities that occur in general relativity that appears in the big–bang in cosmological models and therefore, recently there has been a lot of interest in cosmological models in related to Rosen’s bimetric theory of gravitation.
In the context of general relativity cosmic strings do not occur in Bianchi Type models. Two Bianchi Type cosmological models in four dimensions and one Bianchi Type cosmological model in higher dimensions have been studied by Krori, Choudhuri, Mahanta [8]. They have shown that the cosmic strings do not occur in Bianchi type V cosmology. Borkar, Dhongle [9] have deduced the Bianchi type I string dust model with magnetic field in stationary space-time in general relativity. Bali, Dave [10], Bali, Upadhyaya [11] and Bali, Singh [12] have investigated Bianchi type IX, I and V string cosmological models respectively under different physical conditions in general relativity. Bali, Anjali [13] have investigated Bianchi type I bulk viscous fluid string dust magnetized cosmological model in general relativity, by introducing the condition \(A=(BC)^n\) where \(n>0\) in Einstein field equations, whereas Bali, Pareek [14] have deduced Bianchi type I string dust cosmological model with magnetic field in general relativity by imposing the condition \(A=N(BC)^n\) in Einstein field equations, where \(n>0\) and \(N\) is proportionality constant. Further Borkar, Charjan [15], [16], [17], [18], [19] and Gaikwad, Borkar, Charjan [20] have investigated many magnetized cosmological models in bimetric theory of gravitation by using the techniques of Letelier [21], [22] and Stachel [23].

Recently people like Bali, Dave [10], Pradhan, Yadav, Rai [24], Pradhan, Yadav, Singh [25], Pradhan [26], Wang [27], [28] and Borkar, Charjan [29] have developed the models in the field of bulk viscous fluid solutions and Bianchi type string models which are very interesting and useful models in general relativity. In an attempt to achieve our bulk viscous model with and without magnetic field, we use the terminology and the notations of Bali, Anjali [13].

In this paper, Bianchi Type I bulk viscous fluid string dust magnetized cosmological model with \(\Lambda\) - term have been evaluated by solving Rosen’s field equations. To get determinate solution we have assumed that \(\sigma\) is proportional to 0 and \(\zeta = 0 = \text{constant}\) where \(\sigma\) is shear, 0 is the expansion in the model and \(\zeta\) is the coefficient of bulk viscosity. The cosmological constant \(\Lambda\) plays the important role and it affects the physical behaviour of the model in the presence as well as in the absence of both the magnetic field and the bulk viscosity. The model is studied for the positive, negative and zero values of cosmological constant \(\Lambda\). Further in special cases, the physical and geometrical aspects of the model are studied in the absence of magnetic field or (and) in the absence of bulk viscosity.

The paper is organized as follows, Section I contains the introduction of our work and it includes the references of the work done earlier in the related field. Section II contains the solutions of Rosen’s field equations for our model, Section III describes the physical quantities with their significance, Section IV contains discussions on some special cases and Section V contains the conclusion of the research work.

II. SOLUTIONS OF ROSEN’S FIELD EQUATIONS

We consider Bianchi Type I metric in the form

\[ ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \]  

(4)

where \(A, B\) and \(C\) are functions of \(t\) alone. Here \(B \neq C\) otherwise, we get LRS Bianchi Type I model. The flat metric corresponding to metric (4) is

\[ d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \]  

(5)

The energy momentum tensor \(T_{ij}\) for string dust is given by

\[ T_{ij} = \nu_i \nu_j - \lambda x_i x_j - \xi \lambda^2 (g_i^j + \nu_i \nu_j) + E_{ij} \]  

(6)

with

\[ \nu_i \nu_j = -x_i x_j = -1 \]  

(7)

and

\[ \nu^i x_i = 0 \]  

(8)

In this model \(\epsilon\) is the rest energy density for a cloud of strings and is given by \(\epsilon = \epsilon_p + \lambda\) where \(\epsilon_p\) and \(\lambda\) denote the particle density and the string tension density of the system of strings respectively, \(x^i\) is the direction of strings and \(\zeta\) is the coefficient of bulk viscosity.

The electromagnetic field \(E_{ij}\) is (given by Lichnerowicz [30])

\[ E_{ij} = \mu \left[ \beta \left( \nu_i \nu_j + \frac{1}{2} g_{ij} \right) - \epsilon h_j \right] \]  

(9)

where four velocity vector \(\nu_i\) is given by

\[ g_{ij} \nu^i \nu^j = -1 \]  

(10)

and \(\mu\) is the magnetic permeability and the magnetic flux vector \(h_i\) defined by

\[ h_i = \frac{\sqrt{-g}}{2\mu} \epsilon_{ijkl} F^{kl} \nu^j \]  

(11)

where \(F_{ij}\) is the electromagnetic field tensor and \(\epsilon_{ijkl}\) is the Levi-Civita tensor density. Assume the comoving coordinates system, so that \(\nu^1 = \nu^2 = \nu^3 = 0, \nu^4 = 1\). Further we assume that the incident magnetic field is taken along \(x\) - axis so that \(h_1 \neq 0\) and \(h_2 = h_3 = h_4 = 0\). The first set of Maxwell’s equation

\[ F_{[ij,k]} = 0 \]  

(12)

yield \(F_{23} = \text{constant} H\) (say). Due to the assumption of infinite electrical conductivity, we have \(F_{14} = F_{24} = F_{34} = 0\). The only non-vanishing component of \(F_{ij}\) is \(F_{23}\). So that

\[ h_1 = \frac{AH}{\mu BC} \]  

(13)
and
\[ |q|^2 = \frac{H^2}{\mu B^2 C^2}. \]  

(14)

From equation (9), we obtain
\[ -E_1^2 = E_2^2 = E_3^2 = -E_4^2 = \frac{H^2}{2\mu B^2 C^2}. \]  

(15)

From equation (6), we get
\[ T_1^2 = \left( -\lambda - \frac{H^2}{2\mu B^2 C^2} - \zeta \nu_{\theta,\theta} \right), \]
\[ T_2^2 = T_3^2 = \left( \frac{H^2}{2\mu B^2 C^2} - \zeta \nu_{\theta,\theta} \right), \]
\[ T_4^2 = \left( \epsilon + \frac{H^2}{2\mu B^2 C^2} \right). \]  

(16)

Using equation (16), Rosen’s field equation (3) gives
\[ \frac{-A_{44}}{A} + \frac{B_{44}}{B} + C_{44} + \frac{A_{44}^2}{A^2} - \frac{B_{44}^2}{B^2} - \frac{C_{44}^2}{C^2} = \]
\[ 16\pi ABC \left( \lambda + \frac{H^2}{2\mu B^2 C^2} + \zeta \nu_{\theta,\theta} \right) - 2\Lambda \]  

(17)

\[ \frac{A_{44}}{A} - \frac{B_{44}}{B} + C_{44} - \frac{A_{44}^2}{A^2} + \frac{B_{44}^2}{B^2} - \frac{C_{44}^2}{C^2} = \]
\[ 16\pi ABC \left( -\frac{H^2}{2\mu B^2 C^2} + \zeta \nu_{\theta,\theta} \right) - 2\Lambda \]  

(18)

\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + C_{44} - \frac{A_{44}^2}{A^2} - \frac{B_{44}^2}{B^2} - \frac{C_{44}^2}{C^2} = \]
\[ 16\pi ABC \left( \epsilon + \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda \]  

(19)

\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + C_{44} + \frac{A_{44}^2}{A^2} - \frac{B_{44}^2}{B^2} - \frac{C_{44}^2}{C^2} = \]
\[ 16\pi ABC \left( \epsilon + \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda \]  

(20)

where \( A_4 = \frac{dA}{dt}, \ B_4 = \frac{dB}{dt}, \ C_4 = \frac{dC}{dt} \) etc.

Equations (17) to (20) are four equations in six unknowns \( A, B, C, \lambda, \epsilon \) and \( \Lambda \) therefore to deduce a determinate solution, we assume two extra conditions. First is that the shear tensor \( \sigma_{ij} \) is proportional to the expansion \( (\theta) \) which leads to
\[ A = (BC)^n, \ \text{where} \ n > 0 \]  

(21)

and second is string dust Zel’ dovich [31] condition
\[ \epsilon = \lambda \]  

(22)

\( \text{i.e.}, \) the rest energy density is equal to the string tension density.

From equations (19) and (20), we obtain
\[ 2\frac{C_{44}}{C^2} - 2\frac{C_{44}^2}{C^4} = 16\pi ABC \left( \zeta \nu_{\theta,\theta} - \epsilon - \frac{H^2}{\mu B^2 C^2} \right) \]  

(23)

Adding equation (17) and (23) and using the condition \( \epsilon = \lambda \), we get
\[ \frac{B_{44}}{B} \frac{A_{44} - C_{44}}{A} + \frac{A_{44}^2}{A^2} - \frac{B_{44}^2}{B^2} + \frac{C_{44}^2}{C^2} = \]
\[ 16\pi ABC \left( 2\zeta \nu_{\theta,\theta} - \frac{H^2}{2\mu B^2 C^2} \right) - 2\Lambda \]  

(24)

From equations (21) and (24), we write
\[ (n-1)\frac{B_{44}}{B^2} + (n+1)\frac{C_{44}}{C^2} + (1 - n)\frac{B_{44}}{B} - (n + 1)\frac{C_{44}}{C} = \]
\[ -16\pi K(BC)^n - 32\pi(BC)^{n+1} \zeta \nu_{\theta,\theta} - 2\Lambda \]  

(25)

where \( K = \frac{H^2}{2\mu} \).

From equations (18) and (19), we obtain
\[ \frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_{44}^2}{C^2} - \frac{B_{44}^2}{B^2} \]  

(26)

On simplifying above equation, we get
\[ \frac{(CB_4 - BC_{44})}{BC} \left( \frac{BC_4}{C} \right) = \]  

(27)

which on integrating yield
\[ \frac{(B_4}{C} \left( \frac{B_4}{C} \right) = LBC \]  

(28)

where \( L \) is the constant of integration.

Using assumptions \( BC = \mu \) and \( \frac{B}{C} = v \), the above equation (28) leads to
\[ \frac{v_4}{v} = L \]  

(29)

Now using equation (21) and the condition \( BC = \mu \) and \( \frac{B}{C} = v \), the equation (25) gives
\[ -n \left( \frac{\mu_{44}}{\mu} \right) + n \left( \frac{\mu_{44}^2}{\mu^2} \right) = -16\pi K \mu^{n-1} + 32\pi\mu^{n+1} \zeta \nu_{\theta,\theta} - 2\Lambda \]  

(30)

Applying the condition \( \zeta = \text{constant} \) to the above equation, we get
\[ \mu_{44} - \frac{\mu_{44}^2}{\mu} + \beta \mu^{n+2} = \frac{16\pi K}{n} \mu^n + \frac{2\Lambda \mu}{n} \]  

(31)

where \( \beta = \frac{32\pi}{n} \zeta \nu_{\theta,\theta} \), which reduces to
\[ \frac{d}{d\mu} \left( f^2 \right) + \left( -\frac{2}{\mu} \right) f^2 = 2 \left[ \frac{16\pi K}{n} - \beta \mu^2 \right] \mu^n + \frac{4\Lambda \mu}{n} \]  

(32)

where \( \mu_4 = f(\mu) \).

The differential equation (32) has solution
\[ f^2 = P\mu^2 + \frac{32\pi K}{n(n-1)} \mu^{n+1} - \frac{2\beta}{n+1} \mu^{n+3} + \frac{4\Lambda}{n} \mu^2 \log \mu \]  

(33)

where \( P \) is the constant of integration.
From equation (29) we write
\[
\log v = \frac{L}{\sqrt{P \mu^2 + \frac{32 \pi K}{n(n-1)} \mu^{n+1} - \frac{2 \beta}{(n+1)} \mu^{n+3} + \frac{4 \Lambda}{n} \mu^2 \log \mu}} + \log b
\]
Using \( \mu = f(\mu) \) and expression (33), the metric (4) will be
\[
ds^2 = -\frac{d\mu^2}{P \mu^2 + \frac{32 \pi K}{n(n-1)} \mu^{(n+1)} - \frac{2 \beta}{(n+1)} \mu^{(n+3)} + \frac{4 \Lambda}{n} \mu^2 \log \mu + \mu^2 \nu d\nu^2 + \frac{\mu}{v} dv d\nu + \frac{1}{v} d\nu^2}
\]
where \( v \) is determined by equation (34).
After suitable transformation of coordinates i.e., putting \( \mu = T, x = X, y = Y, z = Z \) the above metric (35) takes the form
\[
ds^2 = -\frac{dT^2}{P T^2 + \frac{32 \pi K}{n(n-1)} T^{(n+1)} - \frac{2 \beta}{(n+1)} T^{(n+3)} + \frac{4 \Lambda}{n} T^2 \log T} + T^2 dX^2 + T \nu dY^2 + \frac{T}{v} dZ^2
\]
Now choosing the cosmic time \( u = \pm \log T \). For convenience, we can select \( u = -\log T \), then the model (36) goes over to
\[
ds^2 = -\frac{du^2}{P + \frac{32 \pi K}{n(n-1)} e^{-u(n-1)} - \frac{2 \beta}{(n+1)} e^{-u(n+1)} - \frac{4 \Lambda}{n} u} + e^{-u} \left( e^{-u(2n-1)} dX^2 + \nu dY^2 + \frac{1}{v} dZ^2 \right)
\]
This is the Bianchi Type I bulk viscous fluid string dust magnetized cosmological model with \( \Lambda \)-term in bimetric theory of gravitation.

### III. PHYSICAL QUANTITIES WITH THEIR SIGNIFICANCE

The energy density \( \varepsilon \), the string tension density \( \lambda \) and cosmological term \( \Lambda \) for the model (37), (in terms of cosmic time \( u \)) is given by
\[
\varepsilon(= \lambda) = \left( -2 K e^{2u} + \frac{(2n-1) \beta}{64 \pi} \right)
\]
\[
\Lambda = \left( -8 \pi K e^{-u(n-1)} + \frac{3}{8} n \beta e^{-n(n+1)} \right)
\]
It is noticed that the energy density \( \varepsilon \), the string tension density \( \lambda \) and cosmological term \( \Lambda \) all are positive if \( n > (64 \pi K e^{2u})/3\beta \), which suggests the existence of the model for \( n > (64 \pi K e^{2u})/3\beta \) with positive cosmological constant \( \Lambda \). Further \( \varepsilon(= \lambda) = \Lambda = 0 \), yield \( n = -1/4 \) which is not possible, since \( n > 0 \) and hence we never get the dust model with zero value of cosmological constant \( \Lambda \). The model is dusty for \( n = ((64 \pi K e^{2u})/\beta) + (1/2) \) with \( \Lambda \neq 0 \).

It is seen that \( \varepsilon(= \lambda) > 0 \) and \( \Lambda < 0 \), for \( n > 1/8 \) and thus the model exist with negative values of cosmological constant \( \Lambda \). The scalar expansion \( \theta \) has value
\[
\theta = (n+1) \left( \frac{P + \frac{32 \pi K}{n(n-1)} e^{-u(n-1)}}{n(n-1)} - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-u(n+1)} \right)^{1/2}
\]
The components of shear tensor \( \sigma_{ij} \) are given by
\[
\sigma_1^1(= \sigma_{11}) = \frac{2(n-1)}{3} \left( \frac{P + \frac{32 \pi K}{n(n-1)} e^{-u(n-1)}}{n(n-1)} - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-u(n+1)} \right)^{1/2}
\]
\[
\sigma_2^2 = \frac{1}{2} \sigma_1^1 + \frac{L}{2}
\]
\[
\sigma_3^3 = -\frac{1}{2} \sigma_1^1 - \frac{L}{2}
\]
\[
\sigma_4^4 = 0
\]
and its magnitude is
\[
\sigma^2 = \frac{(2n-1)^2}{12} \left( \frac{P + \frac{32 \pi K}{n(n-1)} e^{-u(n-1)}}{n(n-1)} - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-u(n+1)} \right)^{1/2} + \frac{L^2}{4}
\]
The scalar expansion \( \theta \) and the components of shear tensor \( \sigma_{ij} \) are the decreasing functions of cosmic time \( u \). At initial as well as at final stage, they attain constant values which suggest that the model has uniform expansion and uniform shear.

The spatial volume \( R \) is given by
\[
R^3 = e^{-u(n+1)} \left( \frac{P e^{-2u} + \frac{32 \pi K}{n} e^{-nu} \left( \frac{e^u}{(n-1)} + e^{-u} \right)}{2} \right)^{1/3}
\]
The spatial volume \( R \) of the model is an increasing function of cosmic time \( u \), in the presence of magnetic field \( K \) and
bulk viscosity $\beta$. It is seen that in the beginning of the model, its volume attains the constant value $R = P + \frac{32\pi K}{n(n-1)} - 2 \beta$, for $n \neq 1$ and it becomes infinite at later stage. This suggests that the model starts with constant volume (for $n \neq 1$) increasing continuously and diverges to infinity, at final stage.

IV. SOME SPECIAL CASES

A. In the absence of magnetic field $K$
In the absence of magnetic field $K$, we write $\epsilon (= \lambda) = (2n-1) \beta/64 \pi$

$$\Lambda = \frac{3}{8} n \beta e^{-(n+1)}$$

$$\theta = (n+1)\left[P - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-(n+1)}\right]^\frac{1}{2}$$

$$\sigma_1^1 = \frac{(2n-1)}{3} \left[P - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-(n+1)}\right]^\frac{1}{2}$$

$$\sigma_2^2 = -\frac{1}{2} \sigma_1^1 + \frac{L}{2}$$

$$\sigma_3^3 = -\frac{1}{2} \sigma_1^1 - \frac{L}{2}$$

$$\sigma_4^4 = 0$$

$$\sigma^2 = \frac{(2n-1)^2}{12} \left[P - \frac{3(n+1)u + 4}{2(n+1)} \beta e^{-(n+1)}\right] + \frac{L^2}{4}$$

$$R^3 = e^{-(n+1)}\left[P e^{-2u} - \frac{(3u + 4)}{2} \beta e^{-(n+3)u}\right]^\frac{1}{2}$$

It is observed that $\epsilon$, $\lambda$ and $\Lambda$ are positive for $n \neq 1/2$ and $\epsilon(= \lambda) = 0$, for $n = 1/2$. This suggests that the model exist with positive values of cosmological constant $\Lambda$ for $n \neq 1/2$ and for $n = 1/2$, the model exist but it is dusty universe with positive values of $\Lambda$. Thus the model always exist and cosmological constant $\Lambda$ is always positive, in the absence of magnetic field $K$. The scalar expansion $\theta$, the components of shear tensor and spatial volume $R$, as they diverge to infinity, for $n = 1$.

C. In the absence of magnetic field $K$ and bulk viscosity $\beta$
For $K = \beta = 0$, the physical quantities have been calculated as

$$\epsilon = \lambda = \Lambda = 0$$

$$\theta = 0$$

$$\sigma_1^1 = \sigma_4^4 = 0, \ - \sigma_2^2 = \sigma_3^3 = \frac{L}{2}, \ \sigma^2 = \frac{L^2}{4}, \ R^3 = 0$$

All the physical quantities $\epsilon$, $\lambda$, $\Lambda$, $\theta$, $\sigma_1^1$, $\sigma_4^4$ and $R$ attain zero values which shows that it is a vacuum model without expansion having zero volume and constant shear, in the absence of both magnetic field and bulk viscosity.

V. CONCLUSION

We have investigated Bianchi Type I bulk viscous fluid string dust, cosmological model with cosmological constant $\Lambda$ by solving Rosen’s field equations of bimetric theory of gravitation. The cosmological constant $\Lambda$ is playing the important role and it affects the behavior of the model in presence as well as in absence of the magnetic field $K$ and the bulk viscosity $\beta$ in the model. The model exists for $n > (64\pi K e^{2u})/3\beta$, for positive $\Lambda$. The model is never dusty with zero value of $\Lambda$ and model describes dusty
universe for \( n = ((64 \pi K e^{2n})/\beta) + (1/2) \) with \( \Lambda \neq 0 \). Also if \( \Lambda \) is negative then the model exist in presence of both magnetic field and bulk viscosity. The model has uniform expansion and uniform shear. The model starts with nonzero spatial volume and the volume diverges to infinity at final stage.

In the absence of magnetic field \( K \), the model exists for \( n \neq 1/2 \) and the model is dusty for \( n = 1/2 \) and in this case the cosmological constant \( \Lambda \) is always positive. The nature of scalar expansion, shear and spatial volume is same as that of the nature appeared in the presence of magnetic field. Thus the magnetic field does not affect the physical behavior of the model.

The bulk viscosity \( \beta \) makes a role and affects the physical behavior of the model. In the absence of it, the model does not exist, since \( \epsilon, \lambda \) and \( \Lambda \) are negative. Further in the absence of both magnetic field \( K \) and bulk viscosity \( \beta \), the model represents vacuum universe with zero cosmological constant \( \Lambda \) which has a constant shear.

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