Measuring the Technical Efficiency of Decision Making Units by CCR Model in Data Envelopment Analysis

Raju Nellutla [1*], M. Goverdhan [2], V.V. Haragopal [3]

[1]Department of Mathematics, Guru Nanak Institutions Technical Campus, Hyderabad, Telangana ,India.
[2] Department of Mathematics, C V R College of Engineering , Hyderabad, Telangana ,India.
[3] Department of Mathematics, Birla Institute of Technology and Science , Pilani- Hyderabad Campus , Telangana ,India .

[*]*Corresponding Author: rajunellutla@gmail.com, Tel.: +91-98495-86081*

Abstract— In this paper we analyse the Board of Intermediate Public Examination (IPE) data by CCR model in Data Envelopment Analysis for the Andhra Pradesh state district wise. To see the Technical efficiency Performance of the districts in Andhra Pradesh state prior to the division of state. The Performance of the districts is presented along with the Peer , Reference set , potential improvements and Technical efficiency Performance of the state by CCR Model in Data Envelopment Analysis and also based on the reference set and peer weight, we can express the input and output values of in efficient districts needed to bring in to efficient status.

Keywords— CCR Model, Data Envelopment Analysis , Performance, Peers, Potential Improvement, Technical Efficiency.

I. INTRODUCTION

For decades, many applied fields share common concern over design and action on how to improve the data analytics. The concept of best practices is to deliberate action in Private and Government sectors for improvement. There are three important characteristics associated with a ''best practice'': a comparative process, an action and link between an action and some out comes or goals. From 1980's on wards, researchers started to direct their efforts towards extending ''best practice'' to Education sector. This led to school improvement initiatives and studies on the characteristics of School Environment conductive to learning [1]. In various parts of the world researchers like Rhodes, Cooper and Thanassoulis started seeking appropriate measurement methodologies for school efficiency. Rodhes and Southwick studied about the efficiency in U.S.A. Private Universities in comparison to the Public Universities, by applying Data Envelopment Analysis model and they regarded as decision making units on the university as whole and analysed the data.

Kwimbere [2] applied DEA model to assess the performance of Decision Making Units viz., Engineering, Mathematics and Physics departments of a set of universities in U.K and [3] assessed the Performance of Management Schools in Secondary School Examinations of Andhra Pradesh State for academic years 2009-10 and 2010-11 by Data Envelopment Analysis.In this study we analyse the

Board of Secondary Education (BSE) in Andhra Pradesh to assess which district fares well for the data collected for the academic year 2012-14 by CCR in Data Envelopment Analysis.

II. DATA ENVELPMENT ANALYSIS

Data Envelopment Analysis is relatively "data oriented" approach for evaluating the performance of a set of peer entities called decision making entities, which convert multiple inputs in to multiple outputs .In the recent years, the DEA has emerged in to a greater variety of application for using evaluating the performance of many different kinds of entities engaged in many different activities in many different contexts in many different countries world over.DEA represents a great progression of continuous advancing for the data analysis , which find extensive use in industry , society and even in Education.[4] performed a critical Data Envelopment Analysis of hospital efficiency in India and found interesting results in health care issues.

In this study we consider Data Envelopment analysis (DEA) is used in an attempt to deal with the issue of measuring the Technical efficiency of the districts in Andhra Pradesh. The technique DEA was employed for the multiple inputs and outputs for evaluating the summary measure of efficiency of the data. This evaluation can be conducted not only at the organization level but also in sub units such as

number of boys and girls appeared in examination in district wise and their results. [5] assessed the Technical efficiency of Management Schools in Secondary School Examinations of Andhra Pradesh by CCR model.

2.1 Educational inputs: The resources or input indicators are units of measurement, which represent the factors used to carry out the delivery of services. Identification and measurement of these factors is crucial in a fair evaluation of the economy and efficiency in the programs and services management. Previous studies on other performance models [6] have shown that inputs of universities can be categorized in various ways. Here Educational Inputs are district wise number of boys and girls appeared in Public Examination.

2.2 Educational Outputs: Output indicators measure the level of activity of programs and services. Furthermore, it is always useful to disclose indicators that provide information about the quantity and the quality of the activity [7]. The quality, as an attribute that affects the user's perception, can also modify the productive process input/output relation. For this reason, it must be considered to access the efficiency of the process. Authors [8] Constructed DEA methodology to measure risk of commercial Banks. Raju Nellutla [9] Constructed Data Envelopment Analysis to know the Performance of the districts in SSC Public Examinations for the years 2009 - 2011 of Andhra Pradesh. In our case we give the Educational outputs of the Public Examination is number boy and girls passed out in public Examination in district wise.

III. EFFICIENCY

The efficiency analysis is always computed to assess the performance of an organization. The Efficiency is an important factor in economic analysis, where the process has a single input and single output , then efficiency is defined as :

$$\text{Efficency} = \frac{\text{Output}}{\text{Input}} \qquad (1)$$

the theory of production from the economic point of view then can be considered as a formal model to link inputs and outputs, This theory has several strengths. First , some formal relationship between inputs and outputs exists and a '' best practice '' can be identified by comparing different units transforming in to inputs to output where all units are assessed relative to that of optimum. The production process that occurs in educational institutions seem to have the same characteristics of the above economic model in the business sector- utilization of physical and human resources as inputs to compute outputs as shown in below.
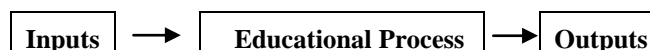
| Inputs | → | Educational Process | → | Outputs |

Figure 1 Transformation Input/ Output   Process

3.1 The CCR Model: In Data Envelopment Analysis (DEA) the most widely used model is CCR Model [10] and [11] . A Constant Return to Scale relationship is assumed between Inputs and Outputs. It was the first Data Envelopment Analysis model to be developed CCR after Charnes, Cooper and Rhodes who introduced this.This model calculates the Overall Efficiency (OE) for each unit, Where the both Technical Efficiency and Scale Efficiency are aggregated in to one value. The Primal CCR model is explained as follows:

Decision Making Units DMU_j : The jth Decision Making Unit   j= 1,2,3,…,n

x_{ij}: The amount of the ith input of the jth Decision making Units $x_{1j}, x_{2j}, x_{3j}, …, x_{nj}$

y_{ij}: The amount of the jth output of the jth Decision making Units $y_{1j}, y_{2j}, y_{3j}, …, y_{sj}$

v_i: The weight assigned to the ith input   , i =1,2 3,…,n.

u_r: The weight assigned to the rth output , r =1,2 3,…,s.

The Fractional Programming Problem (FPP) is :

$$\textbf{Maximize } \ R = \frac{u_1 y_{1k} + u_2 y_{2k} + \cdots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \cdots + v_m x_{mk}} \qquad (2)$$

$$\textbf{Subject to constraints}: \frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \leq \textbf{1}, \qquad \textbf{j} = \textbf{1, 2, ..., n.} \qquad (3)$$

$$\textbf{and non negativity} \qquad u_1, u_2, u_3, …, u_s \geq 0, v_1, v_2, v_3, … v_m \geq 0 \qquad (4)$$

The ratio of input and output should not exceed 1 for every decision making unit. The objective is to Maximize the decision making units. The optimal value of R^* is at most one. Mathematically, non negativity constraints (4) is not sufficient for the fractional terms in (3) to have a positive value. Now we replace the Factional Program (FP) by the following Linear Programming Problem (LPP),

$$\text{Maximize } R(u,v) = u_1 y_{1k} + u_2 y_{2k} + - - - + u_s y_{sk} \qquad (5)$$

Subject to $\quad v_1 x_{1j} + v_2 x_{2j} + --- + v_m x_{mj} = 1$ (6)

$u_1 y_{1j} + u_2 y_{2j} + --- + u_s y_{sj} \leq v_1 x_{1j} + v_2 x_{2j} + --- + v_m x_{mj}$ (7)

$u_1, u_2, u_3, \dots, u_s \geq 0, v_1, v_2, v_3, \dots v_m \geq 0$ (8)

Optimal Solution (v^*, u^*, R^*)

The reference set

$$R_s = \left\{ j: \sum_{r=1}^{s} u_r^* y_{rj} = \sum_{i=1}^{m} v_i^* x_{ij} \qquad j = 1,2,3, \dots, n \right\}$$ (9)

The Reference set R_s is the Primal Problem . The Primal Problem becomes

$$Maximize \; R^*(v^*, u^*) = \sum_{r=1}^{s} u_r y_{rk}$$ (10)

$$Subject \; to \; constraints: \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \; \leq 0 \qquad j = 1,2,3, \dots, n$$ (11)

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$ (12)

$Non \; negativity \; u_r \geq 0, v_i \geq 0$

The above linear programming problem yield the Optimal Solution R^*, where efficiency score is called Technical Efficiency or CCR Efficiency for the particular DMU$_j$ and Efficiency scores for all of them are obtained by repeating them for each DMU$_j$, j= 1,2,…n. The value of R^* is always less than or equal to unity. DMUs for which $R^* < 1$ are relatively inefficient and those for which $R^* = 1$ are relatively efficient, having their virtual input-output combination points on the frontier. The frontier itself consists of linear facts spanned by efficient units of the data, and the resulting frontier production function has no unknown parameters. As per the above model we implement empirical model evaluation for the data considered in the next section by explaining the Potential Improvement (P.I) and Reference Comparison (R.C) .

3.2 Potential Improvement: An efficient study not only provides an efficiency score per each unit but also indicates by how much and in which areas an inefficient unit need to improve in order to efficient. This information can enable the targets to be set which could help inefficient units to be improved in their Performance.

3.3 Reference comparison: If the assessment of units was found as inefficient then it is felt to be justified then the information provided can be used as a basis for setting targets for the units .As a first step in setting targets, the inefficient unit should be compared with the units in its reference set.

3.4 Peer Group: Data Envelopment Analysis identifies for each inefficient unit a set of excellent units, called Peer Group, which includes those units that are efficient if evaluated with the optimal weights of inefficient unit. The Peer Group , made up of Decision Making Units which are characterized by operating methods similar to the inefficient unit being examined , which is a realistic term of comparison which unit aim to imitate in order to improve its performance. Another name for the peer group is reference set.
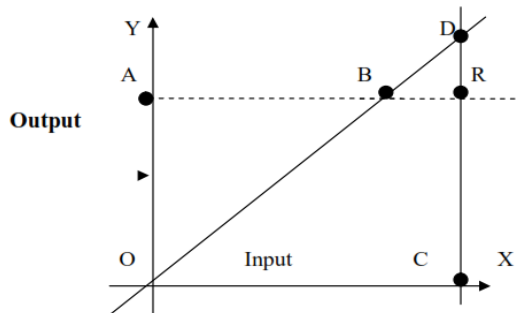

Fig 2 Constant Returns to Scale

From the above figure we understand that, a production of a single output is illustrated graphically. In fig (2) it can be seen that the function *f(x),* where *f(x)* is a

straight line and has a single slope.　　Hence, for every unit increase in the input that goes into the process, the output produced increases by a constant proportional quantity; hence it represents Constant Returns to Scale (CRS). In this case, R could be projected onto the frontier either under an input- reducing consideration or an output – increasing consideration. B and D are projected points on the frontier obtained for comparison.

IV.　EMPIRICAL STUDY ON DATA SET

It is well known that every state in India holds a Public Examination at 12th grade. This data was chosen to see the Technical Efficiency/ Peer Performance of the 23 districts of Andhra Pradesh. The Inter Public Examinations data for the academic years 2012-13 and 2013-14 of Andhra Pradesh in the 23 districts were considered the data for Measuring the Technical Efficiency of the Board of Intermediate Education in Andhra Pradesh district wise. In Inter Public Examinations (IPE), March, 2013, 801419 Regular Candidates have appeared for Examinations. Out of 767208 candidates, 401098 boys and 366110   girls candidates have appeared for Inter Public Examinations and Inter Public Examinations (IPE), March, 2014, 801419 Regular Candidates have appeared for Examinations. Out of 801419 candidates, 413098 boys and 388321 girls candidates have appeared for Inter Public Examinations. In every district , number of boys and girls have appeared for examination and number of boys and girls have passed out in examination date were consider for this analysis. The CCR Model district wise Technical Efficiency (T.E) is presented below.

Table 1   District wise Technical Efficiency for the academic year 2012-13

S. No	District(DMU)	CCR Technical Efficiency R^*	References	Peers	Name of the Peers
1	Adilabad	0.767	0	2	West Godavari, Nellore
2	Anathapur	0.837	0	2	West Godavari, Nellore
3	Chitoor	0.963	0	2	West Godavari, Nellore
4	East Godavari	0.874	0	1	West Godavari
5	Guntur	0.906	0	2	West Godavari, Nellore
6	Hyderabad	0.874	0	2	West Godavari, Nellore
7	Kadapa	0.821	0	2	West Godavari, Nellore
8	Karim Nagar	0.766	0	1	West Godavari
9	Khammam	0.798	0	2	West Godavari, Nellore
10	Krishna	0.998	0	2	West Godavari, Nellore
11	Kurnool	0.899	0	2	West Godavari, Nellore
12	Mahaboob Nagar	0.660	0	2	West Godavari, Nellore
13	Medak	0.727	0	2	West Godavari, Nellore
14	Nalgonda	0.709	0	2	West Godavari, Nellore
15	Nellore	1.000	18	0	Nellore
16	Nizamabad	0.688	0	2	West Godavari, Nellore
17	Prakasam	0.935	0	1	Ranga Reddy
18	Ranga Reddy	1.000	2	0	Ranga Reddy
19	Srikakulam	0.857	0	2	West Godavari, Nellore
20	Vishakhapatnam	0.950	0	2	West Godavari, Nellore
21	Vizia Nagaram	0.668	0	2	West Godavari, Nellore
22	Warangal	0.704	0	2	West Godavari, Nellore
23	West Godavari	1.000	20	0	West Godavari

From the above Table 1 The Technical Efficiency variation for the 23 districts has the following bound $0.660 \le R^* \le 1.000$. Also three districts have been emerged as efficient namely Nellore, Ranga Reddy, West Godavari and the remaining 20 districts input loses due to CCR Technical efficiency. Potential improvement is required in order to improve in their Performance with regards to results .

From the Table 1 it is clear that Nellore, Ranga Reddy, West Godavari are Technically (CCR) Efficient when compared to the 23 districts. It is observed that the Peers to the all other districts seem to be Nellore, Ranga Reddy, West Godavari.   West Godavari district is having highest

references. Peer contribution of this district is more comparable to other districts.

In DEA every technically efficient DMU by its self role model. For Example, Nellore, Ranga Reddy and West Godavari districts are Technically (CCR) Efficient by its self is a role model district. From the above table 1 is noticed that the Technical Efficiency (T.E) of Adilabad district is 0.767. Hence, Adilabad district is technically inefficient. If returns to scale is constant it could have produced its current outputs 0.767 or 76.70 percent of inputs. Thus , removal of all inefficiencies is achieved by reducing all inputs by 0.233 or

Approximately,23% of their observed values .In fact , based on the reference set and Peer Weight, we can express the input and output values needed to bring Adilabad district in to efficient status. Similarly we analyse the remaining inefficient district in order to improve their performance to words improve the results.

The distribution of score graph for the academic year 2012-13 is presented below:
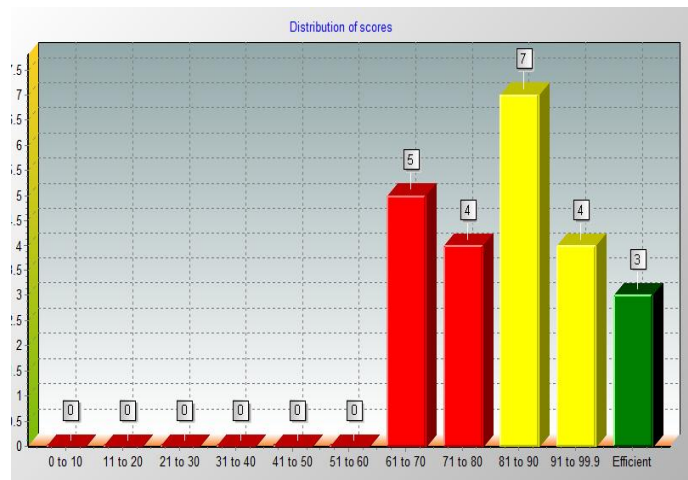

Figure 3   The   Distribution of scores graph for the academic year 2012-13

From the above distribution score graph we observed that 61-70 score limit 5 districts, 71-80 score limit 4 districts ,81-90 score limit 7 districts ,91-99.9 score limit 4 are technically inefficient Frontier and 3 districts is in efficient status.

The results of Data Envelopment Analysis (DEA) for district wise Technical Efficiency for the academic year 2013-14 is presented below:

Table 2   District wise Technical Efficiency for the academic year 2013-14

S. No	District(DMU)	CCR Technical Efficiency R^*	References	Peers	Name of the Peers
1	Adilabad	0.718	0	2	Krishna , West Godavari
2	Anathapur	0.852	0	2	Krishna , West Godavari
3	Chitoor	0.913	0	2	Krishna , West Godavari
4	East Godavari	0.956	0	1	West Godavari
5	Guntur	0.912	0	1	West Godavari
6	Hyderabad	0.923	0	2	Krishna , West Godavari
7	Kadapa	0.874	0	2	Krishna , West Godavari
8	Karim Nagar	0.910	0	1	West Godavari
9	Khammam	0.876	0	2	Krishna , West Godavari
10	Krishna	1.000	22	0	Krishna
11	Kurnool	0.861	0	1	Krishna
12	Mahaboob Nagar	0.686	0	1	Krishna
13	Medak	0.668	0	2	Krishna , West Godavari
14	Nalgonda	0.733	0	2	Krishna , West Godavari
15	Nellore	0.943	0	1	Krishna
16	Nizamabad	0.760	0	2	Krishna , West Godavari
17	Prakasam	0.859	0	1	Krishna
18	Ranga Reddy	0.925	0	1	Krishna
19	Srikakulam	0.848	0	2	Krishna , West Godavari

20	Vishakhapatnam	0.964	0	1	Krishna
21	Vizia Nagaram	0.873	0	2	Krishna , West Godavari
22	Warangal	0.759	0	2	Krishna , West Godavari
23	West Godavari	1.000	16	0	West Godavari

From the above Table 2 The Technical Efficiency variation for the 23 districts has the following bound $0.668 \le R^* \le 1.000$. Also two districts have been emerged as efficient namely Krishna, West Godavari and the remaining 21 districts input loses due to CCR Technical efficiency. Potential improvement is required in order to improve in their Performance with regards to results in this state.

From the Table 2 it is clear that Krishna and West Godavari are Technically (CCR) Efficient when compared to the 23 districts. It is observed that the Peers to the all other districts seem to be Krishna and West Godavari. Krishna district is having highest references. Peer contribution of this district is more comparable to other districts.

In DEA every efficient DMU by its self role model. For Example, Krishna and West Godavari are Technically (CCR) Efficient by its self is a role model. From the above table is noticed that the Technical Efficiency (T.E) of Mahaboob Nagar district is 0.686. Hence, Mahaboob Nagar district is technically inefficient. If returns to scale is constant it could have produced its current outputs 0.686 or 68.60 percent of inputs. Thus , removal of all inefficiencies is achieved by reducing all inputs by 0.3140 or Approximately,31% of their observed values .In fact , based on the reference set and Peer Weight, we can express the input and output values needed to bring Mahaboob Nagar district in to efficient status.   Similarly we analyse the remaining inefficient district in order to improve their performance to words improve the results.

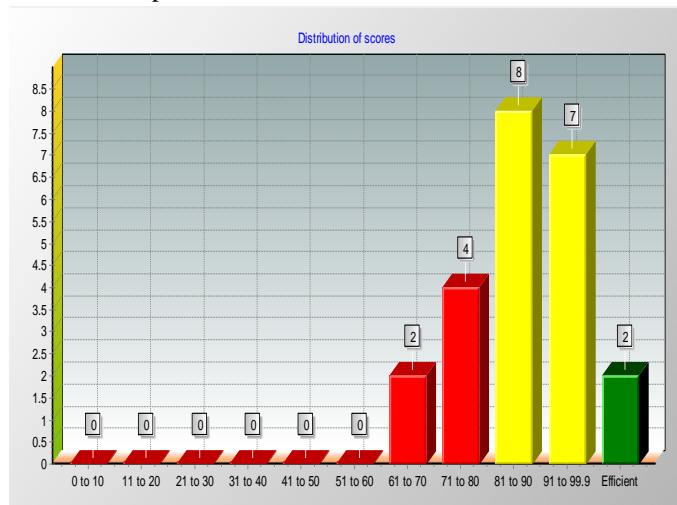The Distribution of scores graph for the academic year 2013-2014 is presented below:

Figure 4 The Distribution of scores graph for the academic year 2013-14

From the above distribution score graph we observed that 61-70 score limit 2 districts, 71-80 score limit 4 districts ,81-90 score limit 8 districts ,91-99.9 score limit 7 are technically inefficient Frontier and 2 districts is in efficient status.

The Summary of Technical Efficiency for the academic year 2012-13 and 2013-14 is presented below for clarity.

Table 3 Summary of CCR Technical Efficiency of Andhra Pradesh

	2012-13	2013-14
T.E Mean	0.844	0.861
T.E S.D	0.116	0.097
T.E C.V	13.74%	11.27%
Minimum Technical Efficiency	0.660	0.668
Maximum Technical Efficiency	1	1
No. of Efficient districts	3 (Nellore, West Godavari, Ranga Reddy)	2 (Krishna, WestGodavari)
Total number of districts	23	23

V.   CONCLUSION

From this analysis we found that for the academic year 2012-13 Nellore, West Godavari and Ranga Reddy district performs well as per CCR Technical Efficiency. For the academic year 2013-14 Krishna, West Godavari performs well as per CCR Technical Efficiency. Remaining districts Potential improvement is required for improving in their performance with regards to results. West Godavari district performs well for the both academic years. For the academic year 2012-13 average technical efficiency is 0.844 and 2013-14 is 0.861.While compare coefficient of variation c.v of technical efficiency for the academic year 2012-13 and 2013-14, it shows greater consistency between these two years in Andhra Pradesh . If returns to scale is constant in CCR model it could have produced its current outputs percent of inputs. Thus , removal of all inefficiencies is achieved by reducing all inputs of their observed values .In fact , based on the reference set and peer weight, we can express the input and output values needed to bring every DMUs (districts) in to efficient status. For this every district in this state, the educational institutions should focus more on improvement of overall quality of education through continuous improvement programs. If continuous improvement is required any educational institutes. Then only their performance will be in optimal level.

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AUTHORS PROFILE

Dr. Raju Nellutla, working as Associate Professor of Mathematics/ Statistics and Department Coordinator in Department of Humanities and Sciences, Guru Nanak Institutions Technical Campus, Hyderabad, Telangana, India, Obtained his Master degree in M. Sc (Applied Statistics) from University College of Science, Osmania University, Hyderabad. He did his Ph. D in Statistics from Kakatiya University, Warangal, Telangana. His area of research includes Statistics, Advanced Operations Research and Data Envelopment Analysis . He has published 8 research papers in reputed international journals. Dr. Raju Nellutla is sessional member of   ISCA. He has 15 years of teaching experience.

Dr. M. Goverdhan , working as Assistant Professor of Statistics in Department of Humanities and Sciences, CVR College of Engineering, Hyderabad, Telangana, India, Obtained his Master degree in M. Sc (Applied Statistics) , M.Phil.(Statistics) , Osmania University, Hyderabad. He did his Ph. D in Statistics from Kakatiya University, Warangal, Telangana. He has 15 years teaching experience.

Dr. V. V Haragopal was a former Professor of Statistics , Head and Chainman Board of Statistics and Director CQM at Osmania University, Hyderabad. He has guided around 20 Ph. D and 7   M. Phil students and published more than 100 papers in reputed international and national journals. He has delivered many invited talk across the India. Presently   working as Professor of Mathematics in Birla Institute of Technology and Science , Pilani, Hyderabad Campus, Hyderabad.