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Research Article

Intervention Model Based on Exponential Smoothing Methods and ARIMA Modelling of the Nigerian Naira Exchange Rates

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Abstract— Nigeria's poor handling of the currency crisis has made exchange rates the country's main issue. This is made worse by her near total reliance on imports for consumption and her need for foreign cash from the export of crude oil. Consequently, Nigeria's economic success is heavily reliant on the fluctuations of other global currencies. The study is aimed at modelling exchange rates between the Nigerian naira and the Pakistani rupee during a financial slump using an intervention modelling approach constructed on exponential smoothing methods and the ARIMA model with a view to making comparisons. The dataset used in this study is the daily exchange rates of Pakistani rupees to Nigerian naira spanning from January to December 2016. Results revealed that the 2016 economic downturn had a negative impact on the naira, with a percentage change of 47.51. This implies that rupees appreciated over the naira such that Rs1 PKR = 1.9524 NGN compared to the periods before and after the intervention occurred. The economic recession was considered a step function with a delay of 1 period and a gradual-permanent effect with a decay rate of 0.60. Comparatively speaking, nonetheless, the ARIMA-intervention model outperformed the ETS-intervention model.

Keywords— Exponential smoothing methods, ARIMA, Intervention model, Exchange rates.

1. Introduction

Numerous predictive and seasonally adjustment mechanisms can be attributed directly to the concept of [1], thereby possibly becoming an exceptionally widely recognized technique in usage. The late 1950s saw the development of the exponential smoothing methods (ETS), which served as the foundation for some of the most effective forecasting techniques [2 - 4]. However, the ETS framework has been perceived as a technique for filtering and forecasting rather than a model built around statistical distributions. ETS has recently been designated as a statistical framework centered around statistical theories and rebranded as an inventive state-space algorithm [5]. A structure for evaluating the impacts of an event on a time series of interest is provided by intervention model building, which was first developed by [6]. The process is thought to be affected by the intervention by changing its mean level. As an illustration, extreme climatic shifts have led to low animal population levels. Therefore, it may be anticipated that the post-change population level will differ from the pre-change level. Moreover, a highway's 65 mph speed limit was raised to 70 mph. This could increase the risk of travelling on the main-road and frequency of automobile collisions. As a result, by examining the mean functions of the relevant stochastic process, we may examine these consequences. Another illustration that is relevant to this study is the effect of the economic downturn on the comparative value of the naira with respect to the Pakistani rupee.

Exchange rate is one of the tools used to assess a nation's economic standing. It gives insight into the strength of a nation's economy, which explains why it is closely examined and kept track of. As one of the main variables impacting growth in the economy, especially in emerging countries like Nigeria, the management of exchange rate policy is a top responsibility for the governing bodies. The rationale behind this is given that the international sector's sustainability is contingent upon the value of the currency at home vis-à-vis other currencies globally [7]. Whatever its conception, a currency's exchange rate measures a country's competitiveness in a clean market as well as serving as a vital value link connecting domestic and global markets for capital and services. Because of its impact on trade and the economic system as a whole, financiers and businesspeople are more inclined to support an established exchange rate instead of a fluctuating alternative [8]. Turbulent exchange rates refer to those that persist over time and often cause the native currency to continuously depreciate. Foreign exchange has lately become Nigeria's vital nutrients because of her dependency on oil for foreign trade in spite of falling crude oil prices on the global oil market and her utter reliance on imports for a majority of consumerism. In consequence of this, her wealth is strongly influenced by changes in various currencies across the globe.

ARIMA models have been the cornerstone of intervention modeling for a long time. Similar to how Etuk and Eleki [9] used ARIMA intervention analysis to predict the daily Yuan-Naira Exchange Rates. Girard [10] employed the same approach in investigating the prevalence of coughing pneumonia in the United Kingdom and Wales during 1940 and 1990. Yang [11] used an ARIMA with intervention model to investigate the influence of the introduction of novel products on income. Nelson [12] estimates the effect of the Bankruptcy Act of 1978 using ARIMA intervention analysis. With the intention of comparing the outcome with that of the intervention model utilizing a lag operator, Shittu and Inyang [13] used the Intervention design conducted on ARIMA to model the monthly crude oil prices in Nigeria. Lam et al. [14] used the concept of [6] to measure the impacts of the external factor and the convergent shift in computational findings of the company's process overhaul, which relies on the task model evaluation. Etuk et al. [15] use the intervention analysis method to determine the Yen/Naira daily exchange rate. Using an intervention analysis model, Sharma and Khare [16] investigated the effects of the Indian government's action to reduce pollution brought on by vehicle exhaust ejections. The intervention model, as proposed by Etuk and Udoudo [17], explains how the economic slump affects the daily rates of exchange between the Indian Rupee and the Nigerian Naira. Deutsch and Alt [18], analyzed to see how Boston's gun-related crimes were affected by Massachusetts' gun control law.

Etuk and Ntagu [19] looked at the modeling of the daily intervention of the comparative values of the Nigerian Naira (NGN) to the Swiss Franc (CHF) between May 18, 2016, and November 16, 2016. Min [20] employed intervention analysis to study both the immediate and long-term impacts of both the 1999 a natural disaster and the 2003 serious acute lung disease spread on the market for arriving tourists from Japan.

An intervention model for the conversion value between the Malaysian Ringitt (MYR) and the Liberian Dollar (LRD) was proposed by Etuk and George [21]. The effect of the Bt cotton variety on cotton yield in India was evaluated by Mrinmoy et al. [22]. The Bt cotton variety was introduced in 2002 as a step intervention. Etuk and Chukwukelo [23] used intervention analysis to model the daily Moroccan Dirham (MAD) and Nigerian Naira (NGN). Intervention time series analysis is used by Darkwah et al. [24] to evaluate the nature and consequences of community policing's establishment and implementation in Ghanaian communities. An autoregressive integrated moving average intervention model was fitted by Etuk et al. [25] to the daily Brazilian Real and the Nigerian Naira exchange rates.

A study by Jarrett and Kyper [26] investigated the detrimental effects of the worldwide monetary meltdown on the price of Chinese securities. Etuk and Amadi [27] examined how the GBP/USD exchange rate was affected by Great Britain's decision to leave the European Union. Using an intervention model, Lai and Lu [28] examine how the September 11, 2001 terrorist assault affected American demand for air travel. Moffat and Inyang [29] studied and analyzed how the government amnesty program (GAP) in Nigeria affected the country's production of crude oil. The impact of the declaration of cooperation on Nigeria's crude oil production was examined by Etuk et al. [30]. Using the Box-Tiao approach, Inyang et al. [31] studied how international oil politics affected Nigeria's crude oil price. The December 2016 intervention by the union had an abrupt and significant influence on the price of Nigerian oil upon its introduction, as seen by the corresponding 33.72% increase in price. Adeleye et al. [32] simulated a seasonal model following [1] methodology to the crude oil dataset. Results proved that a seasonal ARIMA model was sufficient.

Parvez and Azim [33] analyzed the Bangladesh population growth dataset using two approaches due to [1] and [4]. Their findings revealed that Bangladeshi population growth will be around 277 to 306 million people in year 2070, according to the prediction from the fitted model.

Among the authors that have enhanced the intervention model through its integration with exponential smoothing techniques are Inyang et al. [34] employed a time series intervention model constructed on the ESM and ARIMA models to model the daily Pakistan rupee to Nigerian naira exchange rates. Once more, Inyang et al [35] studied the response of the comparative value of the Bangladesh Taka to the naira as a direct consequence of the downturn in the economy in 2016 using an intervention model based on ETS and ARIMA models. Their results revealed that the intervention caused a 68.49% depreciation in the value of the Naira exchanged with the Bangladesh Taka in the exchange rate market, with a decay rate of 0.6. Seong and Lee [5] developed an intervention evaluation technique centered around exponential smoothing models by utilizing an entirely new state-space structure. In applications to the terrorist attacks of September on US airlines and the global pandemics of 2019, datasets. Results revealed that the intervention structure combined with the exponential smoothing methods outperformed that of the ARIMA framework. In order to explain the impact of the pandemic and develop precise predictive models, Jaganathan [36] employed ARIMA Linear Transfer Function and Exponential Smoothing (ES) with intervention models to model pandemic data. Results from the work revealed that the combined model was superior to the classical models. Trapero et al. [37] used a straightforward framework built on a transfer function combined with Single Exponential Smoothing (SES) to investigate the precision of predictive modeling in the event of career advancement. Therefore, this work seeks to explore the flexible and adaptable advantages inherent in exponential smoothing methods by broadening intervention modelling, which is primarily applied with ARIMA models to ETS models, with a view to making comparisons.

2. Data Presentation and Methodology

2.1. Data Description

The dataset analyzed is the daily Pakistani rupee swapped for a unit of the Nigerian naira spanning from January to December 2016, obtained from [38]. The dataset was split into datapoints prior to intervention (January 1–June 20, 2020) and after the intervention periods (June 21–December, 2020). The statistical package used for the analysis of this work is the R language [39].

2.2 Time series models

2.2.1 ARIMA process

Given an ARMA model with parameters ARMA(p,q) and the differencing operator $\nabla^d X_t$, the final structure is provided as ARIMA (p, d, q) [1, 40 - 41], written as

$$\phi(B)\nabla^d X_t = \theta(B)\varepsilon_t \tag{1}$$

With backward shift operator B defined as

$$B^0 \equiv 1 \Rightarrow X_t = B^0 X_{t-0}, BX_t = X_{t-1}, B^k X_t = X_{t-k}; k = 2,3,4,...$$

Where:

$\nabla^d X_t = X_t - X_{t-1}, d = 1$. Thus, $\nabla^d X_t = \nabla(\nabla^{d-1} X_t)$

$$X_t = \frac{\theta(B)}{\phi(B)}\varepsilon_t \tag{2}$$

2.2.2 Intervention Model

The impact of external variables I_t can be modeled dynamically using the form:

$$Y_t = \delta_1 Y_{t-1} + ... + \delta_r Y_{t-r} + \omega_0 I_{t-b} - \omega_1 I_{t-b-1} - ... - \omega_s I_{t-b-s} + X_t \tag{3}$$

It's can also be represented [6] as

$$f(\delta, \omega, I, t) = \sum_{j=1}^{k} R_{tj} = \sum_{j=1}^{k} \{\frac{\omega_j(B)}{\delta_j(B)}\} I_{tj} \tag{4}$$

2.2.2.1 ARIMA–Intervention Model

It is necessary to consider the impact caused by any external time series that may be present in each I_{tj}. The measurement variables are set to zero and one to represent the absence and presence of external event, respectively. Assume that the model becomes for one external variable with $k = 1$;

$$Y_t = R_{t_i} + X_t \tag{5}$$

Let's say that an intervention applied to Y_t occurs at time $t = T$. For $t < T$, R_t is presupposed to equals 0. According to type of intervention, R_{t_i} can take some common forms as follows:

$$R_{t_1} = \omega(B)B^b S_t^T \tag{6}$$

$$R_{t_2} = \omega(B)B^b P_t^T \tag{7}$$

$$R_{t_3} = \frac{\omega(B)B^b}{1-\delta(B)} S_t^T \tag{8}$$

$$R_{t_4} = \frac{\omega(B)B^b}{1-\delta(B)} P_t^T \tag{9}$$

Thus, $i = 1,2,3,4.$ represents a permeant shift in the mean function, a purely temporary change, a gradually increasing change, gradual decreasing change, respectively.

Where:

$\delta(B) = 1 - \delta_1 B - ... - \delta_r B^r$

$\omega(B) = \omega_0 - \omega_1 B - ... - \omega_s B^s$

b =delay parameter, ω =impact parameter,

δ =slope parameter $(0 < \delta < 1)$, Y_t is the response variable at t.

X_t is a Box – Jenkins ARIMA(p,d,q) model which represents the baseline daily Nigerian exchange rate with respect to the Pakistan Rupee in pre – intervention period.

$I_t = S_t^T, P_t^T$ indicator variable. Mathematically, they are written as

$$S_t^T = \begin{cases} 1, t \geq T \\ 0, t < T \end{cases} \tag{10}$$

$$P_t^T = \begin{cases} 1, t = T \\ 0, t \neq T \end{cases} \tag{11}$$

S_t^T and P_t^T are the step and pulse functions respectively.

Considering an intervention type in (8) and X_t in (2), the model in (5) becomes

$$Y_t = \frac{\omega(B)B^b}{1-\delta(B)} S_t^T + \frac{\theta(B)}{\phi(B)}\varepsilon_t \tag{12}$$

Similarly, for an intervention type in (7) and X_t in (2), the model becomes

$$Y_t = \omega(B)B^b P_t^T + \frac{\theta(B)}{\phi(B)}\varepsilon_t \tag{13}$$

2.2.2.2 Intervention Analysis with ETS

Taking into account the changes made to the aforementioned state-space simulations for the ETS in order to perfectly manage the impact of an external influence on the Y_t mean level. ETS with intervention can easily be expressed as:

$$Y_t = R_{t_i} + F^* X_t \tag{14}$$

Where Y_t and R_{t_i} remained as earlier defined, the process $F^* X_t$ denotes the fundamental time series without any external effect. The model structure in (14) is comparable to that of the intervention design with ARIMA models in (5), according to the intervention type. Suppose Y_t is subject to an intervention type with "gradually increasing effect" with ETS model with additive and multiplicative errors but with no trend and seasonality, (14) respectively becomes:

$$Y_t = \frac{\omega(B)B^b}{1-\delta(B)} S_t^T + ETS(A, N, N) \tag{15}$$

$$Y_t = \frac{\omega(B)B^b}{1-\delta(B)} S_t^T + ETS(M, N, N) \tag{16}$$

2.2.2.3 Intervention Effect

The impact of an intervention is numerically assessed by two values:

The initial impact given by ω_0 and then,

The long run term effect, given by

$$\frac{\omega(1)}{\delta(1)} = \frac{\omega}{1-\delta} \tag{17}$$

Then, the percentage change is given by

$$[1 - Exp(\omega)] \times 100\% \tag{18}$$

Where ω is the impact parameter and δ the growth rate.

2.3 Unit Root Test

When a first difference operation makes a nonstationary series with a trend in its mean over time trendless and hence stationary, the series is said to have only one or unit root. However, a system is characterized as having more than one unit root when the difference needed before stationarity is obtained is greater than one [42 – 43].

Hypothesis:

$H_0 : \beta = 0$ (series has unit root)

Against

$H_1 : \beta \neq 0$

Test statistics are:

$$T_\rho = \frac{\varphi - 1}{S.E(\varphi)} \sim t_\infty(n) \tag{19}$$

2.4 Model Validation

Diagnostic test is an important step in time series model building and this consist of scrutinizing a variety of diagnostics to determine whether the selected model is healthy and hence ready to forecast. We consider here;

2.4.1 Plot of the residual ACF

The autocorrelation functions of the residuals of the fitted model can be plotted to assess the quality of fit once a suitable ARIMA model has been fitted. A successful fit for the model is indicated by residuals that are white noise, if the majority of sample autocorrelation coefficients fall under the bound of $\pm\frac{2}{\sqrt{T}}$, where T is the series length.

2.4.2. Akaike Information Criterion (AIC)

The AIC [44], is formulated as

$$AIC = M_T \left[1 + \frac{2p}{T-p} \right] \tag{20}$$

Where:

M_T = Index related to production error (known as residual sum of squares)

p = No of parameters in the model, T= No. of data points.

2.4.3. Bayesian Information Criterion

The Bayesian Information Criterion (BIC) is a criterion that can be used to select a model from a subset of models [45 – 46]. A model with the least BIC value among two or more estimated models should be chosen. It is provided by:

$$BIC = n \ln \hat{\sigma}_e^2 + k \ln(n) \tag{21}$$

Where $\hat{\sigma}_e^2$ is the estimated error variance defined by

$$\hat{\sigma}_e^2 = \frac{1}{T} \sum_i^T (x_i - \bar{x})^2$$

x = Observed data, T = Series length, k = No. of free parameters to be estimated.

2.4.4 Ljung Box Test

Up to lag k, the Ljung Box Test can be used to determine if serial autocorrelation is present or not [47].

The statistic Q must be computed in order to perform the Ljung Box test. Given a series X_t of length ς:

$$Q(m) = \varsigma(\varsigma + 2) \sum_{j=1}^{N} \frac{r_j^2}{\varsigma - j} \tag{22}$$

Where: r_j = accumulated sample autocorrelations, N = the time lag.

Hypothesis:

H_0: (residuals do not show any autocorrelation)

Against

H_1: (null is false)

2.5 Measuring Forecast Error

As the performance metric, the predicting errors denoted by Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE), respectively, are selected.

$$MAE = \frac{1}{T} \sum_{t=1}^{T} |e_t| \tag{23}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e_t^{\,2}} \tag{24}$$

Where e_t is the residual at time t and T is the time series length. When a method fits historical time series data well, its mean absolute error (MAE) is close to zero; when a method fits historical series data badly, its MAE is high. As a result, the forecasting approach with the lowest MAE and RMSE is chosen as the most accurate when two or more are evaluated.

3. Results and Discussion

3.1 Time Plot

The time plot showing the daily Pakistan Rupee to Nigerian Naira exchange rates spanned from January to December 2016 is given in Figure 1.

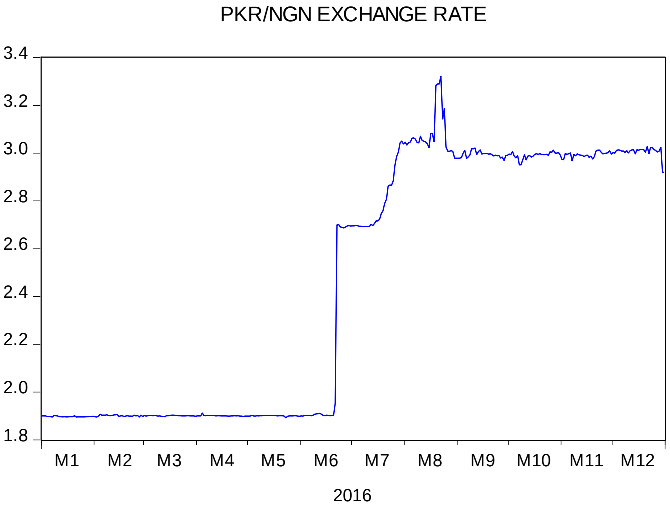

Figure 1: Time Series Plot of PKR/NGN2016 Exchange Rate (ER)

The graph of the series shows an upward trend which was marked in June 21 2016 as point of intervention with value 1.9542NGN and keep increasing. From the plot, we can observe the lowest exchange rate of 1.8923NGN on Monday 23, May 2016 while the highest exchange rate of 3.3217NGN was witnessed on Monday 22, August 2016. The average exchange rate in 2016 is 2.4607NGN.

3.2 Intervention Modelling

ARIMA-intervention model in equation (12) and ETS-intervention model in equation (15) described earlier are used to model the exchange rate dataset. The suspected point where the intervention took place on the series were labelled by indicator functions as

$$S_t^{\,T} = \begin{cases} 1, t \geq June21, 2016 \\ 0, t < June21, 2016 \end{cases} \tag{25}$$

Where: T= June 21, 2016 and $S_t^{\,T}$ is the Step function type.

3.3 Pre–Intervention Modelling

Dataset on the daily Pakistani rupee to Nigerian naira exchange rates from January 1, 2016 to June 20, 2016 (in Figure 2) is used to calculate parameters of the error part of the intervention model, and dataset from June 21, 2016 to December 31, 2016 (in Figure 3) has been used to compute parameters of the transfer function of the intervention model.

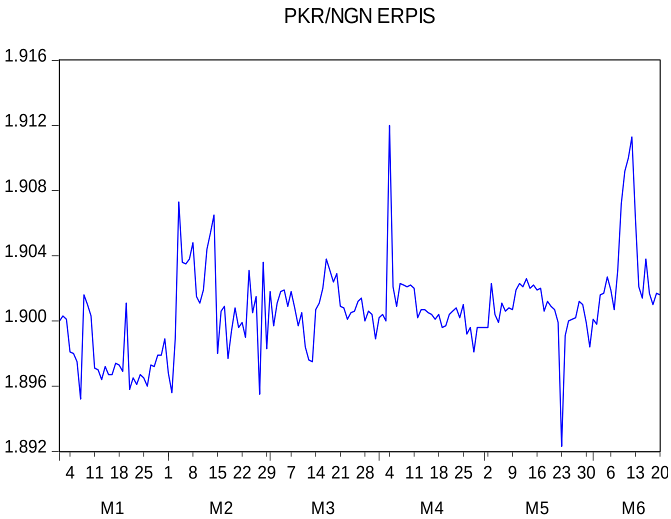

Figure 2. Time Series Plot of PKR/NGN2016 ER (Pre-series)

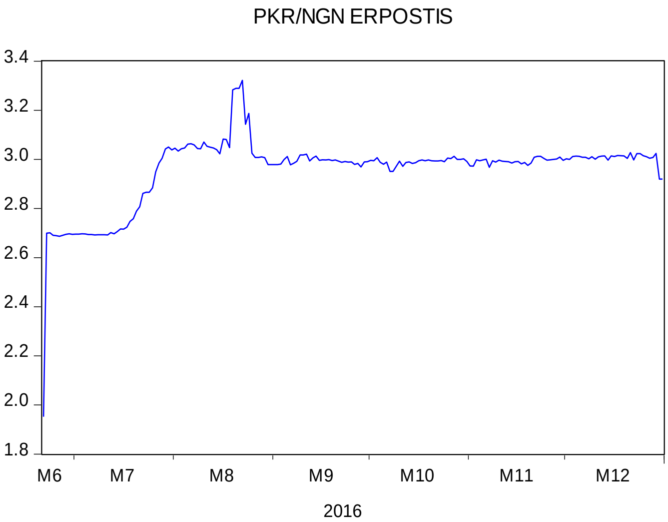

Figure 3. Time Series Plot of PKR/NGN2016 ER (Post-series)

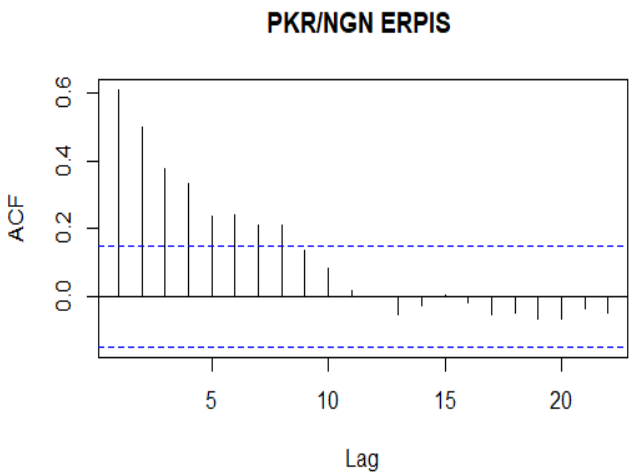

Figure 4. ACF of PKR/NGN2016 ER (Pre-series)

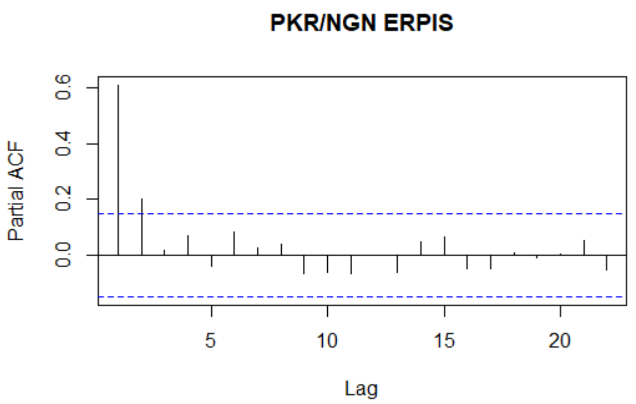

Figure 5. PACF of PKR/NGN2016 ER (Pre-series)

Pre-intervention plot against time shown in Figure 2, the graph of the sequence exhibits the characteristics of a non-stationary (also, see Figures 4 and 5). With peaks at interval of unequal length indicating the absence of seasonality.

The series was differenced once to attain stationarity. The graph of the differenced series in figures 6, 7 and 8 indicates that the series is stationary. The series' stationarity was further verified by a unit root test at the first difference because the Augmented Dickey-Fuller Test's p-value (p-value=0.01 < 0.05) is less than the alpha level. The series is therefore determined to be stationary, as shown in Table 1, rejecting the null hypothesis that the series contains a unit root.

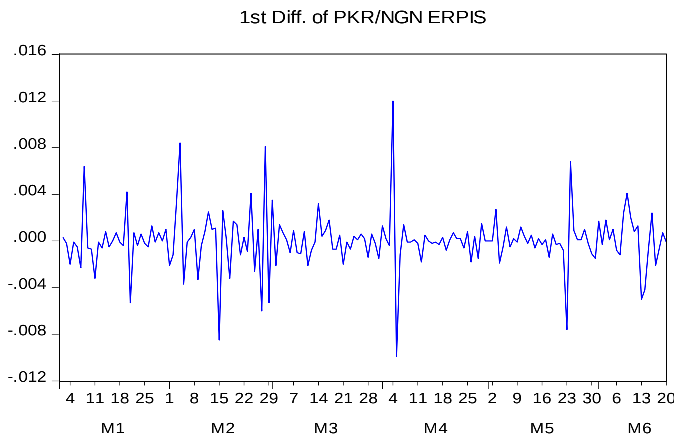

Figure 6. First Difference of PKR/NGN2016 ER (Pre-series)

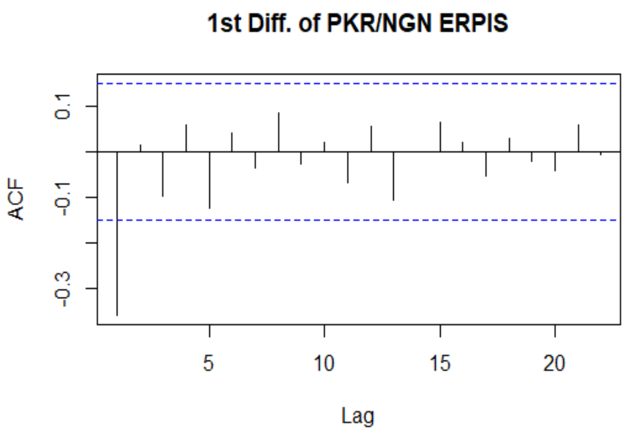

Figure 7. ACF of First Difference of PKR/NGN2016 ER (Pre-series)

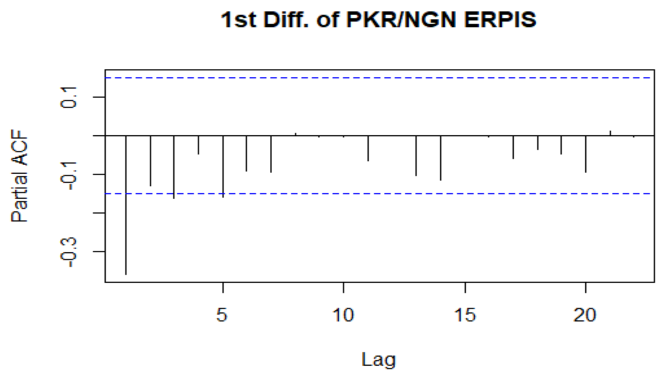

Figure 8. PACF of First Difference of PKR/NGN2016 ER (Pre-series)

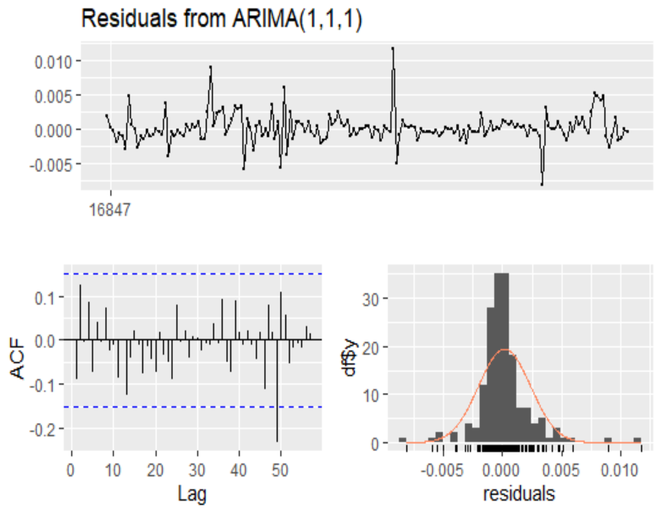

Figure 9. Residuals of the Fitted ARIMA(1,1,1) Model

Table 1. Unit Root Test

Test	Augmented Dickey-Fuller
Data	PKR/NG2016
Dickey-Fuller	-7.6185
Lag order	5
P-value	0.01
Alternative hypothesis	Stationary

Table 2. ARIMA Model Parameters

ARIMA(p,d,q) Model		Estimate	Std. Error	z value	Prob. Value
(0,1,1)	MA1	-0.499652	0.083992	-5.9488	2.701e-09 ***
(1,1,0)	AR1	-0.357033	0.071133	-5.0192	5.188e-07 ***
(1,1,1)	AR1	0.512404	0.100127	5.1175	3.095e-07 ***
	MA1	-0.938902	0.055411	-16.9444	2.2e-16 ***

Table 3. ARIMA Model Evaluation

Model	BIC	AIC
ARIMA (0,1,1)	-1591.991	-1598.274
ARIMA (1,1,0)	-1584.553	-1590.836
ARIMA (1,1,1)	-1592.941	-1602.366

Table 4. ARIMA (1,1,1) Model Ljung-Box Test

ARIMA(1,1,1)
Q* = 7.6953, df = 8, p-value = 0.4638
Model df:2 . Total lags used:10

Applying the concept of [1], three models were tentatively identified and the estimated ARIMA(p,d,q) models with their statistics are summarized in Table 2.

After the model has been determined and the parameters have been calculated, diagnostic checking is carried out to determine whether the fitted model is adequate. Utilizing the Akaike information criterion (AIC) and the Normalized Bayesian information criterion (BIC), as described in [1, 44–46]. When compared to other estimated models, Table 3 shows that the ARIMA (1,1,1) model has the lowest BIC and AIC values, at -1592.941 and -1602.366, respectively.

By exploring the autocorrelation function (ACF) and partial autocorrelation function (PACF) residuals of the fitted ARIMA (1,1,1) model, it's observed that all the coefficients of both the ACF and PACF residuals are within the significance bounds of plus and minus two standard errors $\left(\pm\frac{2}{\sqrt{172}}=\pm 0.1525\right)$ indicating the adequacy of the fitted model, as is reflected in Figure 9. Ljung-Box Q Statistics is also used to assess the adequacy of the fitted model. Table 4 shows that the model has no autocorrelation, with a p-value of 0.4638, which is greater than the alpha value of 0.05. As such, the model fits the dataset well and is statistically significant and sufficient.

3.4 ARIMA-Intervention Model

To calculate the impulse response function, Table 5, forecasting using the fitted model is applied to the initial few post-series data. Figure 10's impulse response function suggests that b=1, meaning that even though the intervention took place on June 21, 2016, its impact wasn't felt until June 22, 2016.

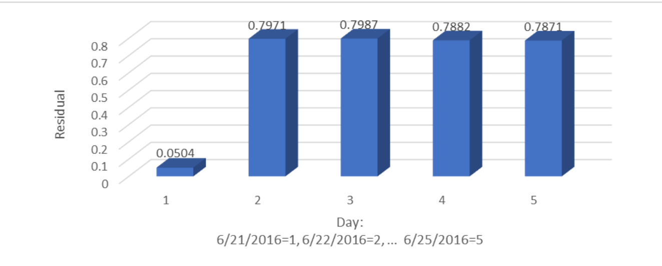

Figure 10. Impulse Response Function

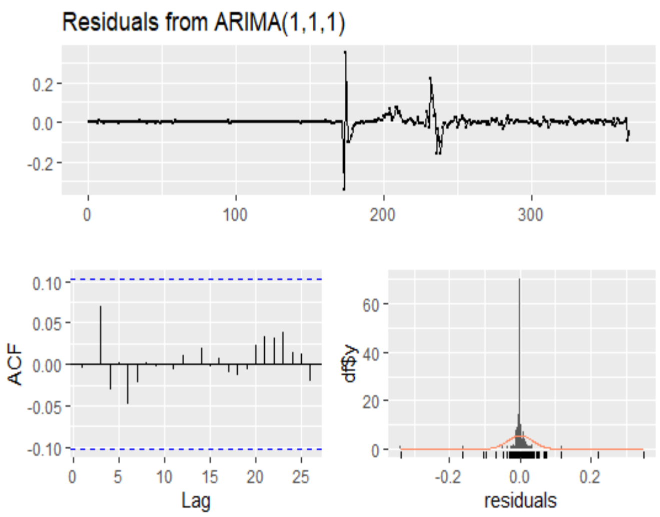

Figure 11. Residuals for ARIMA(1,1,1)-Intervention Model

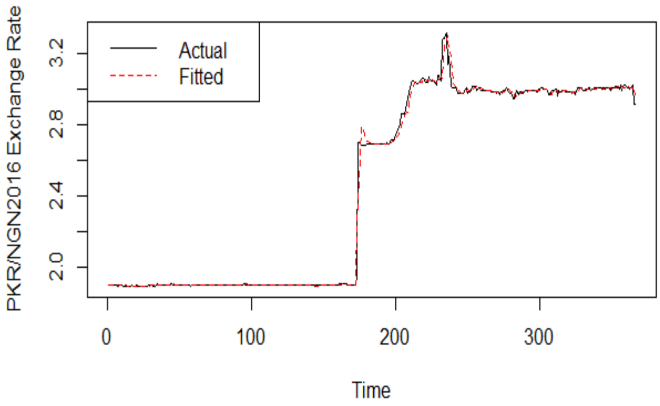

Figure 12. Fitted ARIMA(1,1,1)-Intervention Model Vs Actual Value

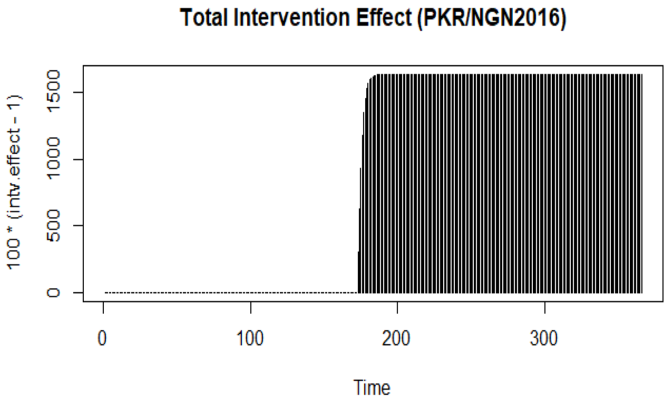

Figure 13. Total Intervention Effect for ARIMA(1,1,1)-Intervention Model

Table 5. ARIMA(1,1,1) Model Forecasts

Date	Actual value (POSTIS)	Forecast
2016/06/21	1.9524	1.902037
2016/06/22	2.6994	1.902261
2016/06/23	2.7011	1.902376
2016/06/24	2.6906	1.902435
2016/06/25	2.6896	1.902465

Table 6. ARIMA(1,1,1)-Intervention Model Parameters

Parameter	Estamte	Std.Error	Z-value	P-value
AR(1)	-0.227696	0.104826	-2.1721	0.029846 *
MA(1)	-0.262197	0.101784	-2.5760	0.009995 **
ω	0.388741	0.025084	15.4975	< 2.2e-16 ***
δ	0.594158	0.027752	21.4092	< 2.2e-16 ***
b	1			

Table 7. ARIMA(1,1,1)-Intervention Model Ljung-Box Test

ARIMA(1,1,1)-Intervention
Q* = 3.2953, df = 6, p-value = 0.771
Model df: 4., Total lags used: 10

Table 8. ARIMA(1,1,1)-Intervention Model Forecasts

S/N	Date	Actual Value	Forecast	95% Prediction Interval	
				Lower	Upper
336	1-Dec-2016	3.0022	2.995805	2.897960	3.094965
337	2-Dec-2016	3.0000	2.995634	2.854943	3.139059
338	3-Dec-2016	3.0115	2.995560	2.821580	3.173740
339	4-Dec-2016	3.0133	2.995527	2.793483	3.203260
340	5-Dec-2016	3.0128	2.995513	2.768844	3.229367
341	6-Dec-2016	3.0084	2.995507	2.746690	3.253008
342	7-Dec-2016	3.0088	2.995505	2.726425	3.274770

The impact parameter with value 0.3887 was significant with p-value of 0.0000, Table 6. The positive sign of the intervention parameter indicates that there was an increment in the exchange rate. This implies that, Pakistan Rupees appreciated over the Nigerian naira in the exchange rate market on Tuesday 21 June 2016 at Rs1 PKR=1.9524NGN, compared to the periods before and after the intervention occurred. With a percentage change of 47.51 computed from (18), the 2016 recession cause a 47.51% fall in the Nigerian Naira against the Pakistan Rupees in exchange rate market.

The ARIMA-Intervention model is represented mathematically as

$$Y_t = \frac{0.3887}{1-0.5942B} S_{t-1}{}^{T} + \frac{(1-.2622B)}{(1+.2277B)}\varepsilon_t \qquad (26)$$

The model in (26) was found to be statistically significant and adequate for the dataset when diagnosed, Table 7 and Figure 11. Confirmed by the plot of the ARIMA (1,1,1)-Intervention model and actual values, the fitted values mirror the actual values, Figure l2.

3.5 ETS-Intervention Model

A provisional specification of the Simple Exponential Smoothing model having additive error is presented in light of the series' lack of trend and seasonality. The model bears the initials ETS(A, N, N), which stands for additive error, no trend, and no seasonal components.

Table 9. ETS(A,N,N)+Intervention Parameters

Parameters	α	l	ω	δ
Estimates	0.7503	1.9000	-0.2373	0.60
Standard Errors	0.0749	0.0517	0.1080	

Table 10. Model Comparison

Model +Intervention	$\hat{\sigma}^2$	AIC	MAE	RMSE
$ARIMA(1,1,1)+\frac{\omega(B)B^b}{1-\delta(B)}S_t{}^T$	0.001262	-1390.222	0.011722	0.0354814
$ETS(A,N,N)+\frac{\omega(B)B^b}{1-\delta(B)}S_t{}^T$	0.002153	-1102.328	0.019	0.035

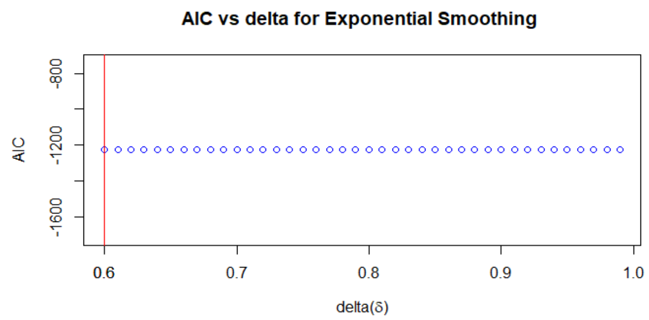

Figure 14. AIC vs Delta for ETS(A,N,N) Model

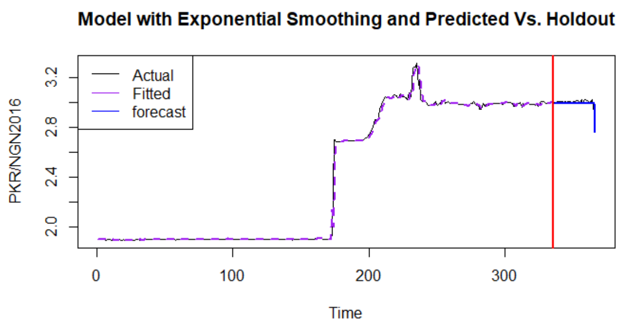

Figure 16. Fitted ETS(A,N,N)-Intervention Model Vs Actual Value

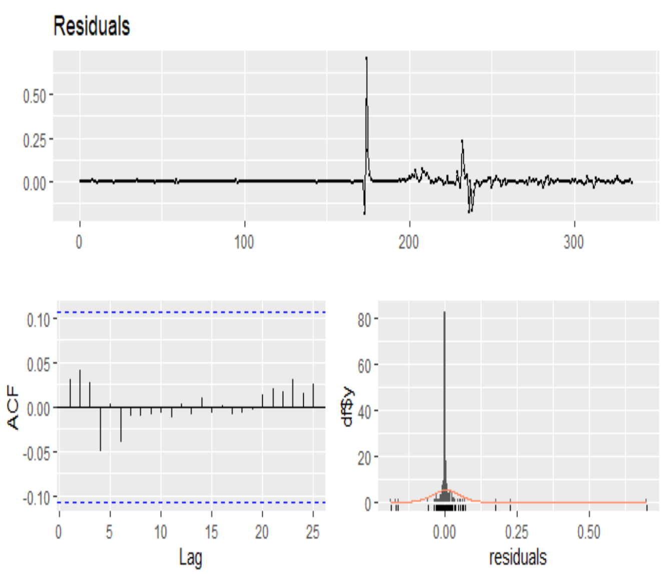

Figure 15. Residuals for ETS(A,N,N) Model

The smoothing constant α, which controls the flexibility of the level is estimated as $\alpha = 0.7503$. Corresponding intervention parameters are $\omega = -0.2373$ and $\delta = 0.60$ with a delay of one period after the intervention, Table 9. The estimates are all significant at 5% level. The impact parameter implies a negative change, that is, the 2016 global economic recession cause a fall in the value of the Nigerian Naira in exchange with the Pakistan Rupee.

The ETS-Intervention model is represented mathematically as

$$Y_t = \frac{-0.2373}{1-0.60B} S_{t-1}{}^{T} + ETS(A,N,N) \qquad (27)$$

The model in (27) was found to be statistically significant and adequate for the data, Figure 15. And this was confirmed by the plot of ETS-Intervention model and actual values, since the fitted and actual values mirrors one another (Figure 16).

4. Conclusion

The Pakistan Rupee to Nigeran Naira exchange rates for 2016 have been considered, with the step-intervention type being the advent of the economic recession in June 2016. To assess the impact of this intervention, two approaches to intervention modelling (ARIMA-Intervention and ETS-Intervention models) were adopted in studying the influence of this external event on the comparative value of the Naira with respect to the Pakistani rupee. Estimates from both models had significant parameters at the 5% level. In the error component of the intervention structure, ARIMA(1,1,1) and EST(A,N,N) models were superior to their counterparts in the nested family with the lowest BIC and AIC values when compared. Results from the ARIMA-Intervention model revealed an impact parameter with an initial value of 0.3887 and a long-run effect of 0.9579. The 2016 economic recession caused a 47.51% fall in the value of the naira against the rupee. With a growth rate of 0.60, it shows that the intervention effect is relatively persistent, and b = 1, which implies that the effect of the intervention was felt after a period of one delay. Interestingly, results from the ESM-Intervention model confirmed that the intervention depreciated the naira value in favor of the rupee and that 1 unit of the Pakistani rupee was exchanged for 1.9524 units of the naira. Models from the two methods passed diagnostic tests and were adjudged adequate for the series, with a perfect fit to actual values when plotted. Nevertheless, the ARIMA(1,1,1)-intervention model outperformed the ETS(A,N,N)-intervention model since it had smaller estimated error variance, AIC, and prediction accuracy metrics like MAE and RMSE than the ETS-intervention model, which contradicts [5]'s conclusion. The ARIMA(1,1,1)-intervention model produced in-sample forecasts that were found to be extremely close to the actual values, suggesting that the model is sound and that the methodology is suitable for the dataset being studied. This study backs up the argument made by [48] that researchers should avoid using a single methodology that works for all situations and instead should alternate between many methods as warranted. Consequently, the impact of the 2016 economic downturn on the Nigerian-naira/Pakistani-rupee exchange rate is characterized as having a gradual start but a long-term effect. Accurate forecasts from the models developed will help financial authorities and professionals improve their planning and decision-making processes as well as the value of the naira, since exchange rate projection is one of the most challenging uses of contemporary time series modeling.

Data Availability
The study's dataset is available in [38].

Conflict of Interest
The author has no competing interests to report.

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Not applicable.

Author Contribution
The author conducted the literature survey, data collection, formulated the methodology, processed and analyzed the dataset, and wrote the research paper.

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