Forecasting and Modeling Monthly Rainfall in Bengaluru, India: An Application of Time Series Models

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Abstract - Rainfall is the most essential stochastic phenomenon which plays an important role in Indian agriculture sector and is necessary for economic growth of the country. These days, the prediction of rainfall has become a most challenging task as it is drastically affected by climate changes due to the worsen effects global warming. For the accurate and timely rainfall predictions, in this article seasonal Naive, triple exponential smoothing and seasonal ARIMA time series models have been applied and the comparison of accuracy of forecasts of these time series models has been checked through various scale dependent error forecast methods and the residual analysis. Further, the best fitted time series model for monthly prediction of rainfall of Bengaluru, Karnataka is suggested through the empirical analysis by utilizing the monthly rainfall data recorded from the years 2009 to 2018 of the Bengaluru city of Karnataka State of India and the results show that the seasonal autoregressive moving average model i.e. ARIMA(0,0,2)(1,1,1)₁₂ provides the more accurate results to forecast the rainfall pattern in Bangalore City of India than the other time series models.

Keywords- Rainfall Forecasting, Time Series Models, SARIMA, Seasonal Naive, Exponential Smoothing.

I. INTRODUCTION

As a result of global warming, the climate is changing drastically and the effects of abnormal variations in the climate can directly be observed by the rising temperature and the changing pattern of rainfall, which are induced by the increased anthropogenic activities such as industrialization, urbanization, deforestation, agriculture, change in land use pattern etc. lead to emission of greenhouse gases and the rate of climate change is increasing much faster (Arnell [5], IPCC [14]). Significantly increased Carbon dioxide in the atmospheric concentration over the last century raised the global temperature by 0.74 centigrade as compared with the pre-industrial era (UNFCCC 2007). Scientists have portended that long-term changes in the climate will admit to increase in the sea level, an increase in heat waves and heavy precipitation, and decreased water resources in semi-arid regions.

Potential change in climate and its impact on rainfall is imposing a threat to the resources of water worldwide. The climate crisis is impacting weather patterns and the water cycle in dangerous ways. The Intergovernmental panel on climate change (IPCC) concluded that the impacts of climate change on fresh water resources as the drought affected areas will likely increase in extent; and heavy precipitation events are very likely to increase in frequency and intensity (Kundzewicz et. al [18]). Also IPCC has accounted that the Indian sub-continent will adversely be affected by enhanced variation in climate, rise in temperature and substantial reduction of summer rainfall in some parts and water stress by 2020 Cruz et al. [8].

Since, the rainfall is a natural and random phenomenon, to choose the best appropriate forcasting procedure which can provide the better estimates for future is very crucial. Hence, In this article, the rainfall forecasting is done by the comparing the various time series models

Organization of the paper: Section II deals with related work done by several researchers in the field of prediction of rainfall. Section III discusses about the data used for empirical analysis and the various time series models with forecast accuracy criteria. In section IV, empirical analysis has been done using various time series forecasting models, namely Seasonal Naive model, Holt-Winter’s seasonal model and Seasonal ARIMA model for the given time series data set and results are explained using graphical representations and tabulation. Further, the results so obtained are analyzed by comparing the models to forcaste the rainfall pattern for Bengaluru city, India through forecast accuracy error criteria and residual analysis. Section V presents the concluding remarks in context of the problem discussed in this article and the empirical findings show that seasonal ARIMA performed better as compared to the other time series models in prediction of monthly rainfall in Bengaluru, Karnataka. The section ends by suggesting possibility for development.

II. RELATED WORK

In last decades, various researchers showed the decreasing pattern in the rainfall in some regions of the country
(Mooley et al. [22] and Kulkarni et al. [19]). A significant decrease in winter rainfall in all sub division except Jharkhand is showed by Rajeevan et al. [26] and Guhathakurta [12]. Due to global warming, the rainfall pattern has been found to be somewhat subdued â€“ monsoon rainfall reduced by 2.4% during 1979-2009 as compared to the period 1949-1978 (Ranade et al. [27]). Annamalai et al. [3] and Dash et al. [9] have shown the increase in the number of monsoon break days, while Ajayamohan et al. [2] depicted the decline in the number of monsoon depressions over India.

The agriculture sector is the most sensitive sector which is affected by the changes in the rainfall because the rainfall pattern of a region of any country determines the nature and characteristics of yield and food production. Thus, Smallholder subsistence farmers are among the worst hit by the variability in the climate due to their low adaptive capacity and their dependence on rain-fed agriculture (Ifejika Speranza [13] and Easterling [10]). It is said that agriculture is the backbone for Indian economy and is the 2nd largest economy based on agriculture, that accounts for 18% of gross domestic product (GDP) and provides employment to 50% of the countries workforce. In the recent decades, the contribution of Indian agriculture to the total national output is between 48% and 60% (Report on policies and action plan for a secure and sustainable agriculture, India).

Since, climate change is an emerging environmental and developmental challenge these days. Changes in the climate are inflaming unfavorable conditions and causing the problem to shift the rainfall pattern. In most of the sub-regions of India, the agriculture production is predominantly rain-fed, intra annual rainfall variability is high and droughts are recurrent. The state Karnataka of India is likely to be more vulnerable to climate change than other states as it has a bittersweet relationship with rainfall pattern. The economy of Karnataka is agrarian and its population is engaged in agriculture sector at larger scale. Only 26.5% of the sown land is subjected to irrigation and rest of the farmland fully depends on rainfall. Rainfall also influences the quantity of ground water, which in turns affects the amount of drinking water and some of the regions are facing severe and perennial water shortage problem in the Karnataka state. According to the United States Geological Survey, rain water seeps into the ground as infiltration. Some of the water seeps deep beneath the top layers of soil where it fills up the space between subsurface rocks and it becomes ground water, less than 2 percent of the earth’s water is ground water, but it provides 30 percent of our fresh water. Without rain water’s continued replenishment of the ground water, potable water would become scarcer than it already is.

Apart from this, the changing pattern of rainfall is also strongly influencing the crop engineering, tourism perspectives, flood protection work scheduling, rain water harvesting strategies, urban water manipulation strategies, and other pluralistic activities related to water resources. Though, Karnataka has come up with innovative methods like rainwater harvesting in order to solve the drinking water scarcity in the state but still, it is very important and essential to know the future rainfall pattern to recharge the depleting ground water and for the planning of water harvesting to cope up with the severe problems of shortage of water by the government of the state.

In literature, researchers have employed a number of statistical models to predict the rainfall amount. Some of the researchers have used various mathematical models which are based on the probability that help us in knowing the probable weekly, monthly or annually rainfall. Fundamentally, two approaches are used to predict the rainfall, first an empirical approach which is based on the past data and its relationship with the number of atmospheric factors. Second approach is the dynamic approach in which physical model based on the system of equations are used for predictions. The meteorologist have developed atmospheric models that estimate the changes in weather conditions such as temperature, pressure etc. by using mathematical equations. The most widely used models to predict the weather are regression models, artificial neural network, fuzzy logic and group methods of data handling and time series models. In past decades, Time series modeling has become a widely used tool in hydrology. The time series model was used by Nail & Momani [23] to detect the changes in rainfall and runoff patterns to search for significant changes in the components of a number of rainfall time series. The most popular models in time series models to predict the weather conditions which are Naive, Exponential smoothing, Gray model, auto regressive moving average model (ARMA) and autoregressive integrated moving average (ARIMA) and seasonal autoregressive integrated moving average (SARIMA). The Box-Jenkins ARIMA model has been successfully used for many environmental management applications and water resource related predictions by Nail & Momani [23]. ARMA (1, 1) was used by Attah et al. [7] to analyze the annual rainfall in lower kaduna catchment Nigeria. In 2013, Abdul et al. [1] forecasted the rainfall pattern in Ghana by using SARIMA(0,0,0)(2,1,0)12 and the model SARIMA (0,0,0)(1,1,1)12 was applied by Papalaskaris, et al.[25] to perform short term forecast of monthly rainfall in Kavala city, Region of Eastern Macedonia- Thrace, North-Eastern Greece. Uba et al. [29] showed the application of time series analysis in modeling monthly rainfall data for Maiduguri, North-Eastern Nigeria by using ARIMA(1,1,0) model. Recently, Asfaw et al. [6] used time series trend analysis to rainfall and temperature in north-central Ethiopia. ARIMA model was compared with markov chain model to predict the meteorological parameters by Khadar Babu, et al. [15] and found that ARIMA model performed better for prediction of rainfall flow in vellore, TamilNadu than the markov chain model. These studies motivates us to forecasting the rainfall situation by using various forecasting based timeseries model for a data set.
III. METHODOLOGY

In this study the Seasonal Naive, Exponential Smoothing and Seasonal ARIMA models considered and the accuracy of the forecast is analyzed by the Standard Error of Residuals, Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Scaled Error (MASE) and Akaike's Information Criterion (AIC). On the basis of accuracy, best fitted model is chosen and the rainfall pattern is predicted for the Bengaluru city of Karnataka, India.

Data Used:
Here, the forecast of rainfall pattern for the Bangalore City of Karnataka, India is done. For empirical analysis of the rainfall prediction, the monthly time series data on rainfall (in mm) is taken from the Open Government Data (OGD) Platform India (https://data.gov.in) from January 2009 to December 2018 for Bangalore city of Karnataka, India. The RStudio 1.2.5033 software is used for the analysis.

Time Series Models:

Naive Model: There are two different Naive models to forecast the time series data. First one is Naive I, in which each forecast is generated by using the previous value of the time series and second one is Naive II that includes the forecast of the current observation is considered as same as the previous observation and multiplied by the growth rate of the current observation over previous observation. The seasonal naive method is same as the naive method in which last observed value of the previous season is considered as predicted value of the same season. For Example, the predicted value of all subsequent months of July will be equal to the previous value observed for July. This method works for the data having high seasonality. The Naive models for the given time series, say \( X_t \) and the predicted time series \( \hat{X}_t \) at the time \( t \) are as follows:

\[
\text{Naive I: } \hat{X}_t = X_t \\
\text{Naive II: } \hat{X}_t = X_{t-1} + \frac{X_{t-1} - X_{t-2}}{X_{t-2}} \\
\text{Seasonal Naive: } \hat{X}_{t+m} = X_{t+m \mod k} 
\]

where \( m \) is seasonal period and \( k = [(h-1)/m] + 1 \).

Triple Exponential Smoothing Procedure: The triple exponential smoothing method is an extension of exponential smoothing that explicitly adds the support for the seasonality component to the univariate time series. This method some times called Holt-Winter's seasonal method. Holt (1957) and Wintr (1960) generalized the Holt’s method to capture the seasonality. The Holt-Winter's seasonal method constitutes the forecast equation and the three smoothing equations: one for the level \( l_t \), one for the trend \( b_t \) and one for the seasonal component \( s_t \), with the corresponding smoothing parameters \( \alpha, \beta, \gamma \), \( m \) is used to denote the frequency of the seasonality i.e. the number of seasons in a year.

Two variations are used in this method which are based on the nature of the seasonality. If the seasonal fluctuations are roughly constant throughout the time series then additive method is preferred while the multiplicative method is preferred if the seasonal fluctuations are proportional to the level of time series.

In this article, multiplicative method is used because the seasonality is not constant throughout the time. With the multiplicative method, seasonal component is expressed in relative terms, and the series is seasonally adjusted by diving the seasonal component and within each year the seasonal component is sum up to approximately \( m \).

The Holt-Winter's seasonal model for the given time series, say \( X_t \) in the component form for the multiplicative method is:

\[
\text{Forecast: } \hat{X}_{t+h|m} = (l_t + hb_t)s_{t+h-m(k+1)} \\
\text{Level: } l_t = \alpha \frac{X_t}{s_{t-m}} + (1 - \alpha)(l_{t-1} + b_{t-1}) \\
\text{Trend: } b_t = \beta^*(l_t - l_{t-1}) + (1 - \beta^*)b_{t-1} \\
\text{Seasonal: } s_t = \gamma \frac{X_t}{(l_{t-1} + b_{t-1})} + (1 - \gamma)s_{t-m}
\]

where \( k \) is the integer part of \((h-1)/m\), which ensures that the estimates of the seasonal indices used for forecasting come from the final year of the sample. The level equation shows a weighted average between the seasonally adjusted observation and the non-seasonal component \( X_t - s_{t-m} \) and the seasonal component \( s_t \), and the seasonal index of the same season last year (i.e. \( m \) time period ago).

The seasonal component is often expressed as:

\[
s_t = \gamma^*(X_t - l_t) + (1 - \gamma^*)s_{t-m}
\]

after putting \( l_t \) from the smoothing equation for the level of component from the above, we get

\[
s_t = \gamma^*(1 - \alpha)(X_t - l_{t-1} - b_{t-1}) + [1 - \gamma^*(1 - \alpha)]s_{t-m}
\]

which is identical to the smoothing equation for the seasonal component we specify here, with \( \gamma = \gamma^*(1 - \alpha) \) and the parameter restriction are \( 0 \leq \gamma^* \leq 1 \) and \( 0 \leq \gamma \leq 1 - \alpha \).

SARIMA Model: Box-Jenkins (1976) introduced the SARIMA model for modeling a time series which incorporates both non-seasonal and seasonal factors in the multiplicative model and the usual notation for the SARIMA model is:
ARIMA\( (p, d, q) \times (P, D, Q)_s \)

with \( p = \) non-seasonal AR order, \( d = \) non-seasonal differencing, \( q = \) non-seasonal MA order, \( P = \) seasonal AR order, \( D = \) seasonal differencing, \( Q = \) seasonal MA order, and \( s = \) time span of repeating seasonal pattern.

For the given time series is \( X_t (t = 1, 2, ..., T) \), the SARIMA model can be given as:

\[
\varphi_p(B) \Phi_P(B^s) \Delta^d \Delta^D_s X_t = \theta_q(B) \Theta_Q(B^s) a_t, \quad t = 1, 2, ..., T
\]

Where, Non - seasonal AR component:

\[
\varphi_p(B) = 1 - \varphi_1 B - \cdots - \varphi_p B^p
\]

Seasonal AR component:

\[
\Phi_P(B^s) = 1 - \Phi_1 B^s - \cdots - \Phi_P B^{ps}
\]

Non - seasonal MA component:

\[
\theta_q(B) = 1 + \theta_1 B + \cdots + \theta_q B^q
\]

Seasonal MA component:

\[
\Theta_Q(B^s) = 1 + \theta_1 B^s + \cdots + \theta_Q B^{qs}
\]

\[
\Delta^d \Delta^D_s X_t = (1 - B)(1 - B^s)X_t
\]

Here, \( B \) is the backshift operator, \( \varphi_p < 1, \Phi_P < 1, \theta_q < 1, \Theta_Q < 1 \) and \( a_t \) follows white noise \((0, \sigma^2)\).

The model SARIMA is used to fit time series after transforming the series into the stationary and eliminating the seasonal component for the series. Differencing is one of the method which is frequently used to obtain the stationary series and for the constant variance, logarithm is generally taken to the series. Here, the deseasonalised rainfall time series is transformed to stationary series using first difference \((d = 1)\) and the orders of seasonal and non-seasonal autoregressive (AR) and moving average (MA) models are identified by autocorrelation and partial autocorrelation functions. Further, The forecast accuracy of the models is checked by various forecast error methods.

**Forecast Accuracy of the Models:** In this article, to check the forecast accuracy, various forecast error measures have been used. In which two types of forecast error procedures are most frequently follows namely as

1. Measures based on scale-dependent errors
2. Measures based on Percentage errors

In the measures based on the scale dependent errors procedure, forecast errors are on the same scale as the data and the accuracy measures based only on the \( e_t \) and therefore can not be used for comparison between the time series having different units. The most commonly used measures based on the scale-dependent errors are Mean absolute error (MAE), Root mean squared error (RMSE) and Mean absolute scaled error (MASE), which are defined as:

Mean absolute error:

\[
MAE = \frac{\sum_{t=1}^n e_t}{n}
\]

And Root mean squared error:

\[
RMSE = \sqrt{\frac{\sum_{t=1}^n e_t^2}{n}} = \sqrt{\frac{\sum_{t=1}^n (X_t - \hat{X}_t)^2}{n}}
\]

Mean absolute scaled error:

\[
MASE = \frac{\sum_{t=1}^n |X_t - \hat{X}_t|/Q}{n}
\]

where \( Q \) is a stable measure of the scale of time series \( X_t \) and for seasonal time series the value of \( Q \) is:

\[
Q = \frac{\sum_{t=m+1}^n |X_t - \hat{X}_t|/Q}{n - m}
\]

Another, in the measures based on the percentage errors procedure measures are unit free and frequently use for comparison of forecast performance of different data series. The most commonly used measure based on percentage errors is Mean absolute percentage error i.e.

Mean absolute percentage error:

\[
MAPE = \frac{\sum_{t=1}^n |p_t|}{n} = \frac{\sum_{t=1}^n (|X_t - \hat{X}_t|/X_t) \times 100}{n}
\]

where \( n \) is the sample size, \( X_t \) is the value of the actual time series and \( \hat{X}_t \) is the forecast in the \( t^{th} \) month. The major disadvantage of the measures based on percentage errors is being infinite and undefined if \( X_t = 0 \) for any period of interest \( t \) and having extreme values if any \( X_t \) is close to zero. Another disadvantage of percentage errors is that it can not be used if the units of measurement has meaningful zero. For example, a percentage error make no sense when measuring the accuracy of temperature forecast as temperature has an arbitrary zero values either in Fahrenheit or Celsius scales.
Thus, in this article, the accuracy of the time series models used for forecasting of the rainfall data is checked by the measures based on the scale-dependent errors (i.e. MAE, RMSE and MASE) because the rainfall data have meaningful units (in mm) as zero (rainfall units are recorded as zero or close to zero in some of the months) in few of the months in the data series.

IV. EMPIRICAL ANALYSIS

Here, the monthly rainfall prediction using various time series model such as seasonal Naive, Holt-Winter’s seasonal method and seasonal ARIMA is carried out. For empirical analysis, the data has been recorded from the year 2009 to 2018 for the Bangalore city of India. The results obtained from above three models are compared using the measures based on the scale-dependent errors, residual plots and the AIC criterion. The model with the least MAE, RMSE, MASE and AIC is chosen as the best model fitted to the rainfall data of Bangaluru city of India.

For the time series analysis, first the actual time series data on rainfall (in mm) for Bengaluru city of India from the year 2009 to 2018 is plotted to know the pattern of rainfall. Since, the time series data is having heavy seasonal component. So, in this article, we have used three seasonal models (Seasonal Naive, Holt-Winter’s seasonal model and Seasonal ARIMA model) to obtain the more accurate forecasts. For the model adequacy, various forecast error measurements have been applied and then the best fitted model is suggested for the prediction of rainfall pattern in Bengaluru city of Karnataka, India.

In this case, the series have too much fluctuation over the time, so to gain the stability in the series, log transformation and second differencing \( (d = 2) \) is used. Logarithms transformation stabilized the variation in the series and the differencing \( (d = 2) \) stabilized the mean of the time series by removing the changes in the level of a series and therefore reduced the trend-cycle and seasonality from the series. To test the stationarity of the series “Augmented Dickey-Fuller Test” is also used and found the p-value is 0.01 with the lag order 4, which shows that the series is stationary at 95% confidence level.

For the seasonal ARIMA model, the the values of the orders of non-seasonal and seasonal autoregressive (AR) and moving average (MA) components has been identified using autocorrelation function (ACF) and partial autocorrelation function (PACF). The best fitted model to the given time series data of rainfall is chosen on the basis of minimum value of Akaike’s information criterion (AIC) i.e. \( ARIMA(0,0,2)(1,1,1)_{12} \) as represented in the Table 2 and the accuracy of the forecast by the \( ARIMA(0,0,2)(1,1,1)_{12} \)
is checked under various criterion shown in the table 1 and the Figure 7. Further, the forecast error values are compared with other time series models and found that only seasonal ARIMA$(0,0,2)(1,1,1)_{12}$ model is satisfying all the criterion of providing the accurate forecast for the given time series data set of rainfall. Hence, the forecasting for the rainfall (in mm) is done for the years 2019 to 2021 for Bengaluru city of Karnataka state of India, which is shown in the Table 3 and in Figure 8.

![Figure 3. Average Monthly Rainfall (mm) in Bengaluru (From 2009 to 2018).](image)

![Figure 4. Monthly Rainfall (in mm) in Bengaluru (From 2009 to 2018).](image)

![Figure 5. Residual plots of monthly rainfall (Using seasonal Naive model).](image)

![Figure 6. Residual plots of monthly rainfall (Using seasonal Holt-Winter's model).](image)

![Figure 7. Residual plots of monthly rainfall (Using seasonal ARIMA model).](image)

<table>
<thead>
<tr>
<th>SARIMA Model</th>
<th>AIC Values</th>
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<tr>
<td>ARIMA$(0,0,0)(1,1,1)_{12}$           :</td>
<td>393.34</td>
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</table>
the rainfall of Bengaluru city of Karnataka, India by utilizing the monthly data form the January 2009 to December 2018. The comparison of forecast accuracy of these models is done under various measures based on forecast errors and the residual plots. The time series model is suggested to forecast the rainfall in Bengaluru district of Karnataka by comparing the various time series models on the basis of forecast accuracy. The Empirical findings show that seasonal ARIMA performed better as compared to the seasonal naive and seasonal Holt-Winter's model and the best fitted SARIMA model to the rainfall data of Bengaluru city of India is $ARIMA(0,0,2)(1,1,1)_{12}$ . Further, the forecasting for three years (2019, 2020 and 2021) is also done for the Bengaluru city by the best suited SARIMA model which indicates that in most of the months, the rainfall is decreasing except May and June months in the coming three years which is not a good indication for the government of Karnataka as Bengaluru is already suffering with the severe problems of shortage of water. This study can be helpful in making rain water harvesting strategies, urban water manipulation strategies, and other pluralistic activities related to water resources.

Conflict of Interest: The authors declare that they have no conflict of interest.

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