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Generalized Ratio-Cum-Product Estimator for Estimating Population Mean in Systematic Sampling.

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Abstract -This manuscript suggests a generalized class of ratio-cum-product estimator in systematic sampling. The Mean Square Error (MSE) of the suggested class and mathematical conditions have been also derived to prove the efficiency of proposed estimators. The results are justified numerically.

Keywords *Auxiliary variate; Bias; Efficiency; Mean Squared Error.*

I. INTRODUCTION

Systematic sampling was studied by [1]. In literature, many authors have discussed the estimation procedures in systematic sampling for instance [2], [3], [4], [5], [6], [7], [8] and [9].

II. NOTATIONS AND SOME EXISTING ESTIMATORS

Let y_{ij} and x_{ij} denote the variate y and the variate x respectively bearing the serial number $i+(j-1)k$ in the population $(i=1,2,....,k;\ j=1,2,.....,n)$. If the i^{th} sampling unit is randomly taken from the first k units, then $\bar{y}_{sys}$ and $\bar{x}_{sys}$ are defined as

$$\bar{y}_{sys} = \bar{y}_{i.} = \frac{1}{n}\sum_{j=1}^{n} y_{ij}\,, \bar{x}_{sys} = \bar{x}_{i.} = \frac{1}{n}\sum_{j=1}^{n} x_{ij}\ .$$

[10] proposed the ratio estimator in systematic sampling given by

$$\bar{y}_R^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}}{\bar{x}_{sys}}\right), \qquad (1)$$

And MSE of the estimator is given as

$$MSE\left(\bar{y}_R^{sys}\right) = \lambda \bar{Y}^2\left(\rho_y^* C_y^2 + \rho_x^* C_x^2\left(1-2K\sqrt{\rho^{**}}\right)\right), \qquad (2)$$

where

$$K = \rho_{xy}\frac{C_y}{C_x}, \rho_y^* = \{1+\rho_y(n-1)\}, \rho_x^* = \{1+\rho_x(n-1)\},$$

$$\rho^{**} = \left(\frac{\rho_y^*}{\rho_x^*}\right), \rho_{yx} = \frac{S_{yx}}{S_y S_x}, \lambda = \frac{N-1}{nN}$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij}-\bar{X}\right)\left(y_{ij}-\bar{Y}\right),$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(y_{ij}-\bar{Y}\right)^2, S_x^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij}-\bar{X}\right)^2.$$

The product estimator in systematic sampling given by [11] is

$$\bar{y}_P^{sys} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys}}{\bar{X}}\right), \qquad (3)$$

And the expression (MSE) is given as

$$MSE\left(\bar{y}_P^{sys}\right) = \lambda \bar{Y}^2\left(\rho_y^* C_y^2 + \rho_x^* C_x^2\left(1+2K\sqrt{\rho^{**}}\right)\right). \qquad (4)$$

[12] suggested the exponential ratio and exponential product estimator in systematic sampling given as

$$\bar{y}_{ER}^{sys} = \bar{y}_{sys}\exp\left(\frac{\bar{X}-\bar{x}_{sys}}{\bar{X}+\bar{x}_{sys}}\right), \qquad (5)$$

$$\bar{y}_{EP}^{sys} = \bar{y}_{sys} \exp\left(\frac{\bar{x}_{sys} - \bar{X}}{\bar{x}_{sys} + \bar{X}}\right). \qquad (6)$$

The MSE of the exponential ratio and exponential product estimator are given as

$$MSE\left(\bar{y}_{ER}^{sys}\right) = \lambda \bar{Y}^2 \left(\rho_y^* C_y^2 + \rho_x^* \left(\frac{C_x^2}{4}\right)\left(1 - 4K\sqrt{\rho^{**}}\right)\right), \qquad (7)$$

$$MSE\left(\bar{y}_{EP}^{sys}\right) = \lambda \bar{Y}^2 \left(\rho_y^* C_y^2 + \rho_x^* \left(\frac{C_x^2}{4}\right)\left(1 + 4K\sqrt{\rho^{**}}\right)\right). \qquad (8)$$

III. PROPOSED ESTIMATOR

Following [13] we define a class of the ratio-cum-product estimators in systematic sampling given as

$$T = \bar{y}_{sys}\left[\frac{a\bar{X}+b}{a\bar{x}_{sys}+b}\right]^{g}\left[\frac{a\bar{x}_{sys}+b}{a\bar{X}+b}\right]^{1-g} = \bar{y}_{sys}\left[\frac{a\bar{X}+b}{a\bar{x}_{sys}+b}\right]^{2g-1}, \qquad (9)$$

where g is a suitable chosen scaler.

For obtaining the bias and MSE, we write

$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$,

such that

$$E(e_0) = E(e_1) = 0,$$

and

$E\left(e_0^{\,2}\right) = \lambda . C_y^{\,2}\rho_y^*$, $E\left(e_1^{\,2}\right) = \lambda . C_x^{\,2}\rho_x^*$,

$E(e_0 e_1) = \lambda . K C_x^2 \sqrt{\rho_y^* \rho_x^*}$.

Expressing equation (9) in terms of e's, we get

$$T = \bar{Y}(1+e_0)(1+\theta e_1)^{-(2g-1)}, \qquad (10)$$

where $\theta = \dfrac{a\bar{X}}{\left(a\bar{X}+b\right)}$.

Solving equation (10) and retaining the terms up to second order, we have

$$\left(T - \bar{Y}\right) = \bar{Y}\begin{pmatrix} e_0 - (2g-1)\theta e_0 \\ -(2g-1)\theta e_0 \theta e_1 + g(2g-1)\theta^2 e_1^2 \end{pmatrix}. \qquad (11)$$

Squaring above equation both sides, we have

$$\left(T - \bar{Y}\right)^2 = \bar{Y}^2\left[e_0^2 - 2(2g-1)\theta e_0 e_1 + (2g-1)^2\theta^2 e_1^2\right]. \qquad (12)$$

The MSE is given as

$$MSE(T) = \lambda \bar{Y}^2 \begin{bmatrix} C_y^2\rho_y^* + \theta^2 C_x^2 \rho_x^* (2g-1)^2 \\ -2\theta K C_x^2 \sqrt{\rho_y^*\rho_x^*}(2g-1) \end{bmatrix}. \qquad (13)$$

We differentiate the $MSE(T)$ with respect to g and equating the derivative to zero, g_{opt} thus obtained is given by,

$$g_{opt} = \frac{1}{2}\left[1 + \frac{K\sqrt{\rho_y^*\rho_x^*}}{\theta \rho_x^*}\right] = \frac{1}{2}\left[1 + \frac{K\sqrt{\rho^{**}}}{\theta}\right],$$

using the value of g_{opt}, we get the $MSE_{\min}(T)$ given as

$$MSE_{\min}(T) = \gamma \bar{Y}^2 \rho_y^* C_y^{\,2}\left(1 - \rho_{xy}^2\right).$$

IV. EFFICIENCY COMPARISONS

We have derived the conditions under which the proposed estimator is better than the usual unbiased estimator, ratio estimator, product estimator is given as under

Table 1: Conditions for Comparison

Existing estimator	**Conditions**
$\bar{y}_{sys}$	$\lambda \bar{Y}^2 \rho_y^* C_y^2 \rho_{xy}^2 > 0$
$\bar{y}_R^{sys}$	$\lambda \bar{Y}^2 \rho_x^* C_x^2 \left(1 - K\sqrt{\rho^{**}}\right) > 0$
$\bar{y}_P^{sys}$	$\lambda \bar{Y}^2 \rho_x^* C_x^2 \left(1 + K\sqrt{\rho^{**}}\right) > 0$

V. Numerical Illustrations

In order to show that our estimator is more efficient numerically, we have used the data given by [14].

Table 2: Some Special Cases of the Proposed Class.

Constants			**Estimator**
a	**b**	**g**	
a	b	$\frac{1}{2}$	$T \to \bar{y}_{sys}$ [The usual unbiased estimator]
1	0	1	$T \to \bar{y}_R^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}}{\bar{x}_{sys}}\right)$ [Ratio estimator]
1	0	0	$T \to \bar{y}_P^{sys} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys}}{\bar{X}}\right)$ [product estimator]
1	C_x	1	$T \to \bar{y}_{SD}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}+C_x}{\bar{x}_{sys}+C_x}\right)$
1	ρ	1	$T \to \bar{y}_{ST}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}+\rho}{\bar{x}_{sys}+\rho}\right)$
1	$\beta_2(x)$	1	$T \to \bar{y}_{STT}^{sys} = \bar{y}_{sys}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}_{sys}+\beta_2(x)}\right)$
a	b	1	$T \to \bar{y}_1^{sys} = \bar{y}_{sys}\left(\frac{a\bar{X}+b}{a\bar{x}_{sys}+b}\right)$ [Generalised ratio type estimator]
a	b	0	$T \to \bar{y}_2^{sys} = \bar{y}_{sys}\left(\frac{a\bar{x}_{sys}+b}{a\bar{X}+b}\right)$ [Generalised product type estimator]

Table 3: Data Statistics

N	15	C_y	0.56
n	3	S_x^2	149.55
$\bar{Y}$	80	C_x	0.28
$\bar{X}$	44.47	S_{xy}	538.57
ρ_{xy}	0.9848	ρ_x	0.707
S_y^2	2000	ρ_y	0.6652

Table 4 shows the (MSE) and Percent Relative Efficiency (PRE) of various estimators given as,

Estimators	**MSE**	**PRE**
$\bar{y}_{sys}$	1455.08	100.00
$\bar{y}_R^{sys}$	373.32	398.62
$\bar{y}_P^{sys}$	786.06	189.45
$\bar{y}_{ER}^{sys}$	820.09	177.43
$\bar{y}_{EP}^{sys}$	1044.42	139.42
T	45.74	3196.57

VI. CONCLUSION

We suggested a generalised ratio-cum-product estimator in systematic sampling and its properties are derived. We have also evaluated the performance of the estimator by using the real life data set. From table 3 it is observed that the estimator is better than the existing estimators, thus it is preferable to use in practical surveys.

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