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Research Article

Heat and Mass Transfer in Radiant-Magnetohydrodynamics Squeeze Flow of Vanadium Pentoxide (V_2O_5)-Based Jeffrey Hybrid Nanofluid with Heat Source/Sink

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Abstract— our work exposes the properties of the transmission of heat and mass in Jefferies nanofluids that are hybrid over the squeezing tunnel that travels across a porous material, as well as the nature of magnetohydrodynamics (MHD) fluid flow. The influence by thermal radiation parameters together with other pertinent parameters is studied. Suitable similarity transformation is applied to the system of dimensionless equations to discretize them. Vanadium Pentoxide (V_2O_5) dispersions are deliberated in the base fluid. Validation of this research is achieved by relating to published results. Graphs and tables are used to discuss the momentum, temperature, Nusselt number and concentration profiles. The graphical results show that the momentum profile accelerated with De and $S < 0$, while it reduced with Ha, λ as well as $S > 0$.

Keywords— Jeffrey hybrid nanofluid, MHD, Squeeze flow, Thermal radiation, Chemical reaction, squeezing flow, heat source/ sink

1. Introduction

Due to its wide range of physical applications, the investigation of the transfer of heat and flow for squeezing unstable kinematic flow of fluid via disparate plates has proved a fascinating field of study over millennia. For example, food processing, hydrodynamic machinery, lubrication setup, compression, and crop damage from freezing, emergence, along with variability, amongst other things. The volatile flow in two dimensions of a viscous magnetohydrodynamic MHD fluid across two parallel inconceivable plates was investigated and analyzed by Akbar et al. [1]. A review of the literature reveals that the squeeze Jeffrey flow across two parallel plates is the subject of the majority of studies. Squeezing flow is the term used to describe the constriction of fluid across two separate plates that results in the fluid beginning to flow. Stefan [2] proposes to explore the squeezing of viscosity over a horizontal tube. Cameron [3] subsequently investigated the squeezing lubricant fluid review beyond two unbounded sheets. For viscous fluid of squeezing flow, Wang [4] found a completely new similarity transformation that converts Navier-Stokes towards ordinary differential equations. Bujurke et al. [5] then looked at an analytical solution in accordance with [4] findings. In addition to Rashidi et al. [6], Khan et al. [7]. Scientists are quite interested in Jeffrey fluid because it exhibits both relaxation and retardation characteristics. How the fluid behaves in response to applied shear stress. Because it can change from a shear thinning flow to a Newtonian fluid when a high force is applied to the fluid, it is categorized as such. Noor et al. [8]. Matinfar et al. [9] went into great detail into heat radiation effects on Jeffery fluid flows during squeezing. The impact of thermal radiation and heat generation/absorption on the squeezing of unstable cu as well as TiO_2-nanofluid was investigated quantitatively as well as computationally by Madaki et al. [10]. The influence of electromagnetic radiation on unsteady free convection magnetohydrodynamic flows of Brinkman-type systems over a medium that is porous with Newtonian heat has been examined by Mamatha et al. [11]. Thermal radiation influence on squeezing flow fluid from Casson between parallel plates was reported by Khan et al. [12]. Makinde et al. [13] examined the effects of Brownian motion and thermophoresis on MHD bioconvection of nanofluid through quartic chemical process along with the nonlinear thermal radiation beyond an upper horizontal surface of a paraboloid of rotation. The influence of an integrated electromagnetic field upon radiant bioconvection flow via a plate that is vertical having thermophoresis along with Brownian motion was studied by Avinash et al. [14]. The impacts of suspended nanoparticles and non-linear thermal radiation on the convection and heat transfer boundary layer flow of nanofluids on a heated vertical sheet were examined by Mahanthesh et al. [15]. The purpose of a heat generation/absorption is to decrease and increase a fluid's thermal conductivity, correspondingly. The temperature of the high conductivity fluid rises, while the low conductivity fluid exhibits the reverse behavior. Noor et al. [16]. Using the power series technique, Qasim [17] examined the effects of the heat generation/absorption around temperature along mass movement over a vertical stretching plate.

Ahmed et al. [18] state that several studies have demonstrated the usage of composite nanofluid in technological advancement programs, including refrigerant for engines for vehicles, nuclear energy facilities, and electrical devices. Numerous physical models have demonstrated the flow of hybrid nanofluids. Alghamdi et al. [19] investigated the MHD flow and thermal transmission at a stagnation point over a horizontal stretching surface for the Casson hybrid nanofluid. Subsequently, the effects of MHD, thermal slip, viscous dissipation, and an exponential convective heat exchange on stationary point movement at vertical stretched surfaces were examined by Abbas et al. in [20]. Dual sheet as well as multi-wall nanotubes made of carbon with conventional fluid are considered. Mahabaleshwar et al. [21] investigated the MHD flow of a Cu–Al_2O_3/H_2O composite nanofluid on a permeable stretch/shrink medium with slip in velocity conditions, in addition to radiative heat transfer. The concept of MHD nanofluid flow has for long being the concern of several researches because it is applications in various fields of science and technology such as in the geometric model of lubricant flow in pneumatic elevators, moulding with injections, and bearings as well, it has been researched by a number of scientists, such as Muhammad et al. [22], Muhammad et al [23], Madaki et al. [24], [25], [26], Hussaini et al. [27] and [28].

There has not yet been any research done on the mass along with heat transmission of the MHD composite nanofluid through the squeezing of a pair of parallel plates acting as thermal radiation sources, with vanadium pentoxide (V_2O_5) acting as the solid material (nanoparticle), to the best of our knowledge. Squeezing Jeffrey's hybrid nanofluid flow due to its complexity in terms of governing equations has generally received little attention. Therefore, this research is aimed at obtaining the mathematical solutions of the Mass and heat transfer in radiant-MHD vanadium pentoxide (V_2O_5)-based squeezing flow Jeffrey nanofluid hybrids are equipped with a source or heat generation/ absorption (i.e., filling the void left by Noor and Shafie [29]), and hence compare the present result with the existing ones.

2. Description of the Problem

This research will contain a numerical solution describing the behaviour of the hybrid nanofluid Jeffrey squeezed between a pair of parallel plates. However, our study would focus on the effects of radiant heating, magnetic field on the fluid flow and the heat source/sink on the general flow pattern. Thus, the fluid flow would consider to be squeezing, Newtonian and incompressible.

We study the unstable squeezed Jeffrey hybrid nanofluid flow of across a pair of plates over a porous material with magnetohydrodynamic along heat generation/absorption together with chemical reaction. Vanadium peroxide hybrid nanoparticles are taken into consideration. There is an expanse of $y = \pm h(t) = l(1-\alpha t)^{1/2}$ between the two sheets. Both the top and bottom plates are subject to the external velocity $v_2(t) = \frac{\partial h(t)}{\partial t}$. Up to t=1/α, the two separate sheets shift away when α < 0 but nearer when α > 0. Just as shown on fig. 1 below. The bottom surface is exerted with magnetic field vertically Noor and Shafie [29]. Muhammad et al. ([22] and [23]) articles contains a more general formulation of the Jeffrey hybrid nanofluid's momentum, temperature, nanoparticle concentration, and continuity equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

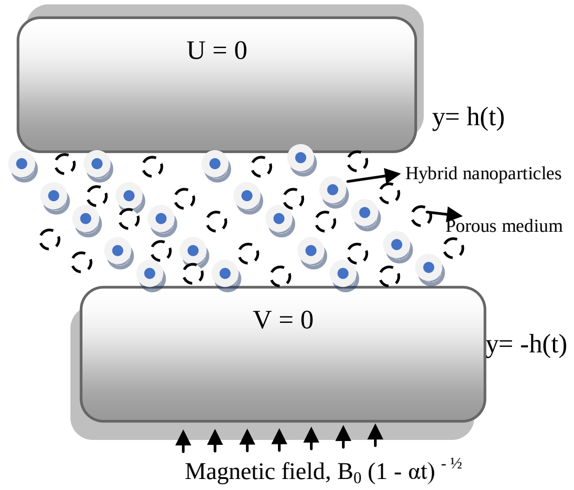

Fig. 1. Geometry of the problem.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = \frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{\partial^2 v}{\partial y^2} + \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\lambda_2}{1+\lambda_1}$$
$$\left(\frac{\partial^3 v}{\partial t \partial y^2} + u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right) \tag{2}$$
$$-\frac{\sigma_{hnf}B(t)}{\rho_{hnf}}u - \frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{\varphi}{k_1(t)}u = 0,$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_{hnf}}{(\rho Cp)_{hnf}}$$
$$\left(1+\frac{1}{\lambda_1}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] - \frac{1}{(\rho Cp)_{hnf}}\frac{\partial q_r}{\partial y} \tag{3}$$
$$+\frac{Q(T-T_\infty)}{\rho_{hnf}} = 0,$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-k_c\left(C-C_\infty\right)=0, \tag{4}$$

Subject to the boundary conditions

$$u=0, v=v_w=\frac{\partial h(t)}{\partial t}, T=T_w, C=C_\infty, \; at \; y=h(t), \tag{5}$$

$$\frac{\partial u}{\partial y}=\frac{\partial^3 u}{\partial y^3}=\frac{\partial T}{\partial y}=\frac{\partial C}{\partial y}=v0, at\, y=0, \tag{6}$$

Table 1. Thermophysical properties of the fluid and solid particles

Physical Properties	Fluid phase (water)	Vanadium Pentoxide (V_2O_5)
$C\rho$ (j/Kg k)	4179	127.7
ρ (Kg/m^3)	997.1	3.35
K (W/mK)	0.613	4.22
$\sigma(\mu S/cm)$	0.05	13.0

According to the Stefan-Boltzmann rule, the amount of radiation absorbed q_r over period of time departing an object corresponds with a fourth of a degree of the temperature in absolute terms, or $q_r=\sigma T^4 A$. This represents the radiation heat fluctuation. Where T is the absolute temperature, A is the emitting body's area, σ is the Stefan Boltzmann constant, and q is the heat transfer per unit time (ω). Nevertheless, Roseland (1931) also clearly expressed the radiative heat flux factor that appears in equation (4) as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

In reference to certain research conducted by Kothandapani and Prakash (2015) as well as Akbar et al. (2013), it is believed for the temperature difference inside the flow is significantly limited, and the term T^4 can be expressed as a function of temperature. Because of this, T^4 is expanded using the Taylor series expansion around T_∞, ignoring the higher-order constants. As a result, we devise $T^4=T_\infty^4+4T_\infty^3-4T_\infty^4$ (8)

Substituting eqs. (7) and (8) into (3), gives

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{hnf}}{(\rho Cp)_{hnf}}$$

$$\left(1+\frac{1}{\lambda_1}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]+\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\frac{Q(T-T_\infty)}{\rho_{hnf}}=0, \tag{9}$$

3. Experimental Method (Method of solution with Runge-Kutta fourth order along with shooting technique)

We used the same non-dimensional variables as Shafie and Noor (2023) to transform the mechanism of partial differential system (PDEs) into ordinary differential system (ODEs).

$$u=\frac{ax}{2(1-at)}f'(\eta), v=-\frac{al}{2\sqrt{(1-at)}}f(\eta),$$

$$\eta=\frac{y}{l\sqrt{(1-at)}}f'(\eta), \theta=\frac{T}{T_w}, \varphi=\frac{C}{C_w} \tag{10}$$

When the similarity variables (8) are substituted into equations (1)–(4), the resulting non-dimensional ODE is

$$\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)f^{iv}-S\left(\eta f'''+3f''+f'f''-ff'''\right)$$

$$+\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{De}{2}\left(\eta f^{v}+5f^{iv}+2f''f'''-ff^{iv}-ff^{v}\right)$$

$$-\frac{\sigma_{hnf}}{\sigma_f}\frac{\rho_f}{\rho_{hnf}}Ha^2 f''-\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\left(1+\frac{1}{\lambda_1}\right)\frac{1}{Da}f''=0, \tag{11}$$

$$\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}\frac{k_{hnf}}{k_f}\frac{1}{\Pr}\left(1+\frac{4}{3}Rd\right)\theta''+S\left(f\theta'-\eta\theta'\right)+\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}\gamma\theta$$

$$+\frac{\mu_{hnf}}{\mu_f}\frac{(\rho Cp)_f}{(\rho Cp)_{hnf}}Ec\left[\left(1+\frac{1}{\lambda_1}\right)\left[\left(f''\right)^2+4\delta^2(f')^2\right]\right]=0, \tag{12}$$

$$\frac{1}{Sc}\varphi''+Sc\left(f\varphi'-\eta\varphi'\right)-R\varphi=0, \tag{13}$$

These are the associated boundary conditions

$$f(\eta)=0, f''(\eta)=0, f^{iv}(\eta)=0, \theta(\eta)=0, \varphi'(\eta)=0 \; at \; \eta=0,$$

$$f(\eta)=1, f'(\eta)=0, \theta(\eta)=1, \varphi(\eta)=1 \; at \; \eta=1, \tag{14}$$

The following lists are the correlations between the thermophysical characteristics of the hybrid nanofluid:

$$\frac{\mu_{hnf}}{\mu_f}=\frac{1}{\left(1-\phi_{hnf}\right)^{2.5}} \quad \text{Dynamic viscosity } (\mu) \tag{15}$$

$$\rho_{hnf}=\left(1-\phi_{hnf}\right)_{\rho f}+\phi_{V_2O_5}$$

Density of hybrid nanofluid (ρ) (16)

$$(\rho Cp)_{hnf} = (1-\phi_{hnf})(\rho Cp)_f + \phi_{v_2O_5}(\rho Cp)_{v_2O_5}$$

Heat capacity (ρCp) (17)

$$\frac{\sigma_{hnf}}{\sigma_f} = \left[1 + \frac{3\phi_{hnf}\left(\phi_{v_2O_5}\sigma_{v_2O_5}\right)}{\phi_{v_2O_5}\sigma_{v_2O_5} + 2\phi_{hnf}\sigma_{hnf} - \phi_{hnf}\sigma_f \left(\phi_{v_2O_5}\sigma_{v_2O_5} - \sigma_f(\phi_{v_2O_5}\sigma_{v_2O_5})\right)} \right]$$

Electrical conductivity (σ) (18)

$$\frac{k_{hnf}}{k_f} = \frac{\dfrac{\left(\rho_{V_2O_5}k_{V_2O_5} + \phi_{hnf}k_{V_2O_5}\right)}{\phi_{hnf}} + 2k_f + \left(\phi_{V_2O_5}k_{V_2O_5} - 2\phi_{V_2O_5}k_f\right)}{\left(\dfrac{\phi_{V_2O_5}k_{V_2O_5}}{\phi_{V_2O_5}}\right) + 2k_f - \left(\phi_{V_2O_5}k_{V_2O_5}\right) + \phi_{V_2O_5}k_f}$$

Thermal conductivity (k) (19)

The substantial expressions from the governing equations are defined as

$$Rd = \frac{4\sigma^* T_\infty^3}{k^* k_f}, S = \frac{\alpha l^2}{2v_f}, Ha = lB_0\sqrt{\frac{\sigma}{\rho_f v_f}}, R = \frac{ak_2 l^2}{v_f}, Sc = \frac{v_f}{D_m},$$

$$\Pr = \frac{v_f}{\alpha_f}, Da = \frac{k_0}{\varphi l^2}, \gamma = \frac{Q_0 l^2}{v_f(\rho Cp)_f}, Ec = \frac{\alpha^2 x^2}{4C_p T_w (1-at)^2},$$

$$De = \frac{\alpha\lambda_2}{1-at}, \delta = \frac{1}{x}(1-at)^{\frac{1}{2}},$$

4. Results and Discussion

This portion includes a brief explanation for better comprehension as well as graphical representations of the characteristics of fluid profiles for many relevant parameters. For hybrid nanofluid containing vanadium pentoxide, the important features of the heat radiation parameter, squeezing number, Eckert number, chemical reaction, ratio of retardation time parameter, and some other relevant factors on fluid profiles are described in depth. Additionally, table 1 above discusses the thermophysical characteristics of hybrid nanoparticles. By applying appropriate similarity transformations, the flow's governing equations are converted into ODEs. Using a shooting technique, a transformed collection of ODEs is solved using the Runge–Kutta–Fehlberg fourth–fifth-order (RKF 45) method. In order to validate the current finding for −f (1), Table 2 has been produced using published results from Jyothi et al. (2021) and Shafie and Noor (2023). Whereas, table 3, shows the computed values for the reduced Nusselt number in relation to the thermal radiation parameter and the squeeze number. It is clear that irrespective of S < 0, S = 0 or S > 0, the fact remained the same that for any increase in the radiation parameter, the Nusselt number is decreased. The momentum profile f (η) for a range of Deborah number De values is shown in Fig. 2. It shows that when the Deborah number increases, the momentum profile is greatly improved.

Table 2. Numerical results of - $f''(1)$ for Squeeze number (S) With $\lambda_1 \to \infty$, $Da \to \infty$, $De \to \infty$, $Ha = Ec = \delta = \gamma = R = \varphi_2 = \mathbf{0}$ and $Sc = Pr = 1$.

Squeeze number **(s)**	Jyothi et al. (2021) $-f''(1)$	Shafie and Noor (2023) $-f''(1)$	Present result $-f''(1)$
-1.0	2.170090	2.170255	2.170223
-0.5	2.617403	2.617512	2.617391
0.01	3.007133	3.007208	3.007001
0.5	3.336449	3.336504	3.336620
2.0	4.167389	4.167411	4.167389

Table 3. Computations showing the reduced Nusselt number ($-\theta'(0)$) when $\Pr = 10, \gamma = 0.4, Ec = 0.8, \delta = 7$

Therm **(Rd)**	Nur $S=-5$	Nur $S=-1$	Nur $S=0$	Nur $S=1$	Nur $S=5$
0	8.8305	13.8697	10.3759	10.2712	10.6023
1	4.4842	6.6242	5.0088	4.8989	4.9278
3	2.6512	3.6454	2.8690	2.8060	2.8001
5	2.0820	2.7294	2.2186	2.1752	2.1676
7	1.8046	2.2846	1.9039	1.8709	1.8640

The consequences of Hartmann number parameter Ha on the momentum profile $f'(\eta)$ is displayed via Figure 3. It indicates that, as the Ha values enhance, the fluid's momentum slows down. When Ha is elevated, the Lorentz force in fluid flow is induced, which weakens the adhesion force at the outermost layer boundary. Conversely, the fluid near the exterior surface moves more slowly as the permeability of the porous material increases. This means that the fluid is decelerating at the boundary region because the friction force within the fluid is weakened.

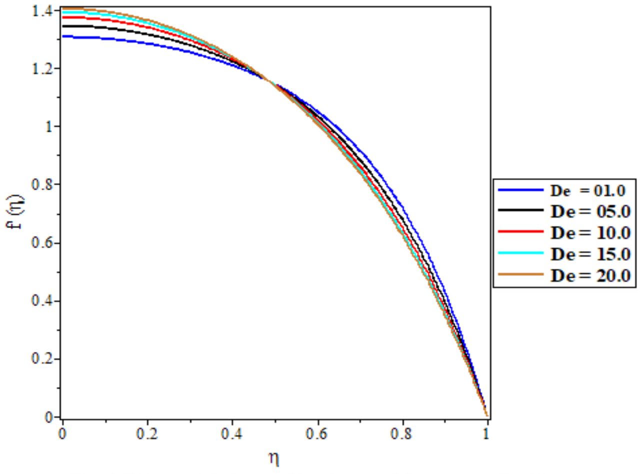

Fig. 2 Effects of Deborah Number on Momentum Profile

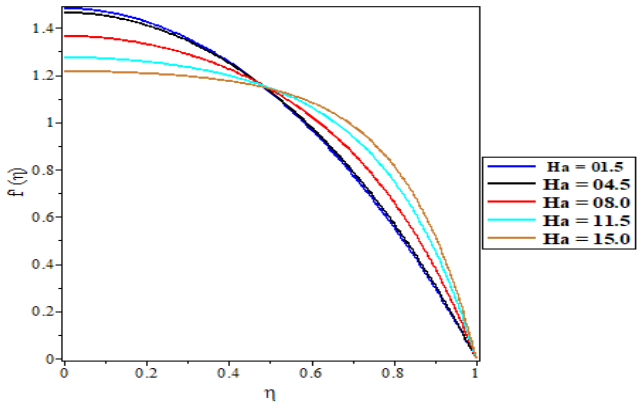

Fig. 3 Effects of Hartmann number on Momentum Profile

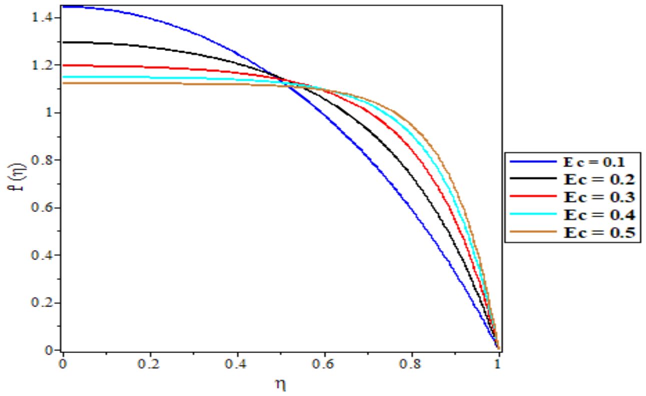

Fig. 4 Effects of Eckert number on Momentum Profile

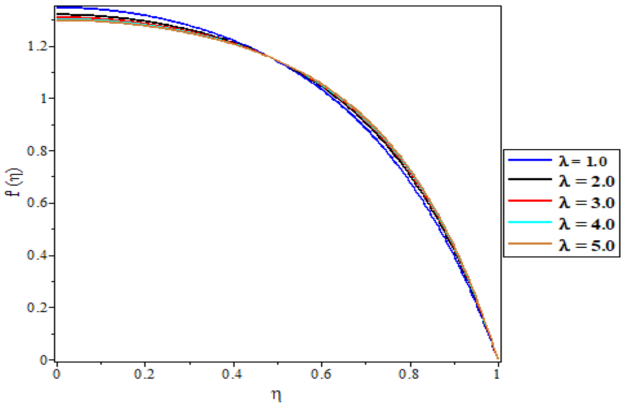

Fig. 5 Effects of Ratio of Retardation Time on Momentum Profile

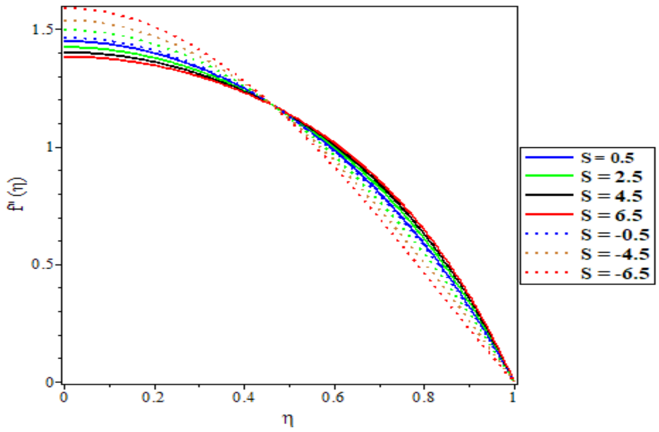

Fig. 6 Effects of Squeeze Number on Momentum Profile

The effects of Eckert number Ec on momentum profile $f'(\eta)$ is illustrated on fig. 4, in which it demonstrates that, the momentum profile becomes less as the Eckert number rises. This is because heat energy is released into the fluid when frictional forces are present, which lowers the momentum in the flow zone. Furthermore, viscous dissipation reduces momentum in the presence of viscous dissipation. Subsequently we observed that the momentum equation expressly does not contain the Eckert number, but rather in the temperature equation but yet has a significant effect on the momentum and hence momentum can be easily regulated by controlling the values of the Eckert number. Furthermore, Fig. 5 illustrates how the ratio retardation time parameter affects the momentum profile, showing that as λ increases, the axial momentum decreases. Additionally, it is seen that the addition of λ increases the particle's intermolecular forces, causing a drop in momentum at the bottom plate and an increase in flow viscosity. The squeezing parameter, S, is an indicator of the motility between two plates. Surfaces that are traveling apart when $S < 0$ and closer together when $S > 0$. Whether $S > 0$ or $S < 0$, Figure 6 depicts the fluid's momentum as it slows down. The physical interpretation states that whereas surface compression causes the fluid to travel fast in the thin channel, substantial resistance causes the fluid to flow more slowly in the larger channel.

6. Conclusion and Future Scope

For this study, we investigated the Casson hybrid nanofluid flow behaviour between two parallel plates while taking heat radiation and particle deposition of vanadium pentoxide (V2O5) into account. The governing equations that illustrate the fluid flow are then converted into non-linear ODEs with the support of appropriate similarity variables. The resulting equations are then numerically solved using the shooting technique and the RKF 45 scheme. Subsequently, Graphs are used to show how some relevant parameters affect various fluid profiles. The following is a summary of the findings above:

1. The momentum profile accelerates with an increase in Deborah's number.
2. The momentum profile is decreased with an increase in the Hartmann number.
3. The momentum profile is observed to decrease in the Eckert number.
4. An increase in ratio retardation time parameter, results in a decrease in the momentum
5. The momentum profile is said to decrease as soon as the planes remain squeezed ($S > 0$) while it slows down when the planes are disjointed ($S < 0$) in the medium of the channel.

Furthermore, this research can be extended by considering the effects of impermeability on the momentum profile, thermophoresis or Brownian diffusion on the temperature profile or suction/ injection to be added to the boundary conditions.

Data Availability

None.

Conflict of Interest

The authors declare that they have no competing interests.

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Authors' Contributions

The authors contributed correspondingly towards the research. They unanimously reviewed the literature and conceived the study. They harmoniously contributed in writing the first draft of the manuscript, reviewed and edited the manuscript hence approved the final camera-ready manuscript.

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