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Anti-magic labeling for Boolean graph of path BG(P_n), (n⩾4)

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***Abstract*—** A graph G is anti-magic if there is a labelling of G is a one-to-one mapping taking the edges onto 1, 2, ..., |E| such that the sum of the labels assigned to edges incident to distinct vertices are different. A conjecture of Hartsfield and Ringel states that every connected graph different from K_2 is anti-magic. Our main result validates this conjecture for Boolean graph of path P_(n) (n⩾4).

Keywords— *Boolean graph BG(G) Anti-magic Labeling*

I. Introduction

Suppose $G(V,E)$ is a graph and let $E_G(v)$ be the set of edges of G incident to$v$, for each vertex v of G. We shall write $E(v)$ for $E_G(v)$. Let $f: E \to \{1,2,\ldots,|E|\}$ be a bijective mapping. The vertex-sum $\varphi_f(v)$ at v is defined as $\varphi_f(v) = \sum_{e\in E(v)} f(e)$. For any two distinct vertices u, v of G, $\varphi_f(v) \neq \varphi_f(u)$ gives an anti-magic labeling of G. A graph G is called anti-magic if G has an anti-magic labeling. The problem of anti-magic labeling of graphs was introduced by Hartsfield and Ringel [4]. They conjectured that all graphs with no single edge component are anti-magic. Graph Labeling has many applications in coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network addressing, and data base management.

II. Conjecture 1

[4] Every connected graph different from K_2 is anti-magic.
This conjecture is still open. Interestingly, the graph K_2 can be regarded as a tree on two vertices. Thus, if we restrict ourselves to trees, the above conjecture holds. Hartsfield and Ringel proved that paths, cycles and complete graph K_n, $(n\geq 3)$ are anti-magic. Recently, Alon et al. [1] have proved that the conjecture is true for some classes of dense graphs. They have shown that all dense graphs with $(n \geq 4)$ vertices and minimum degree $\Omega(\log n)$ are anti-magic. They also proved that if G is a graph with $(n \geq 4)$ vertices and the maximum degree $\Delta(G) \geq 4n-2$, then G is anti-magic and all complete bipartite graphs except K_2 are anti-magic. Anti-magic labeling of the Cartesian product of graphs was studied in [7]; if G is a regular anti-magic graph then for any graph H, the Cartesian product $H \times G$ is anti-magic. It was proved in [4] that 2-regular graphs are anti-magic and proved in [6] that 3-regular graphs are anti-magic. As a consequence, if G is 2-regular or 3-regular then for any graph H, $H \times G$ is anti-magic. In this paper, we extend anti-magic labeling to Boolean Graph of path.

III. Definition

Boolean graph $BG(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices in $BG(G)$ are adjacent if and only if they correspond to two adjacent vertices of G or to a vertex and non - incident edge of G.

IV. Theorem

The Boolean graph of path BG(P_n), (n≥4) is anti-magic

Proof: Let P_n be the path with vertices $v_1, v_2, v_3, \ldots, v_n$. By the definition of Boolean graph $BG(P_n)$ the vertex set is given by
$V\big(BG(P_n)\big) = \{v_i ; 1 \le i \le n\} \cup \{u_j ; 1 \le j \le n-1\}$
and the edge set is given by

$$E\big(BG(P_n)\big) = \{v_i v_{i+1} ; 1 \le i \le n-1\} \cup \{u_j u_{j+1} ; 1 \le j \le$$

We discuss Boolean graph of path in two cases.
Case (a): n ≡ 1 (mod 2)
Label the vertices of $BG(P_n)$ using the function $f: E \to N$ as follows:
$f(v_i v_{i+1}) = i$; $i = 1, 2, \ldots, n.$
$f(u_j u_{j+1}) = 2n-j$; $j = 1, 2, \ldots, n-1.$

$f(v_i u_j) = (n-1)(i+1) + j$ if $i < j$, where $1 \le i \le n + 1$ & $2 \le j \le n$.
$f(v_i u_j) = (n-1)(i-1) + n + j$ if $i > j$, where $j = 1, 2, \dots, n$ & $j < i \le n+1$
The induced function $f^* : V \to N$ such that
$f^*(v_i) = \sum_{u_j \in nbd(v_i)} f(v_i u_j)$
We consider the case when labels of vertices are distinct.
Subcase (i): when $i = 1$ and $j > i$.

$$f^*(v_i) = f(v_i v_{i+1}) + \sum_{\substack{j=2 \\ i<j}}^{n} f(v_i u_j)$$

$$= f(v_i v_{i+1}) + \sum_{j=2}^{n} [(n-1)(i+1) + j]$$

$$f^*(v_1) = f(v_1 v_2) + \sum_{j=2}^{n} [(n-1)(1+1) + j]$$

$$= 1 + \sum_{j=2}^{n} [(2n-2) + j]$$

$$= 1 + (n-1)(2n-2) + \left[\frac{n(n+1)}{2} - 1\right]$$

$$= 1 + (n-1)(2n-2) + \frac{n^2+n-2}{2}$$

$$= \frac{1}{2} [2 + 4n^2 - 4n - 4n + 4 + n^2 + n - 2]$$

$$f^*(v_1) = \frac{1}{2} [5n^2 - 7n + 4]$$

Sub case (ii): When $i = 2, 3, \dots, n$

$$f^*(v_i) = f(v_{i-1} v_i) + f(v_i v_{i+1}) + \sum_{\substack{j=1 \\ j \ne i-1, i}}^{n} f(v_i u_j)$$

$$= i + i - 1 + \sum_{\substack{j=1 \\ i>j}}^{i-2} f(v_i u_j) + \sum_{\substack{j=i+1 \\ i<j}}^{n} f(v_i u_j)$$

$$= 2i - 1 + \sum_{j=1}^{i-2} [(n-1)(i-1) + n + j] + \sum_{j=i+1}^{n} [(n-1)(i+1) + j]$$

$$= 2i - 1 + (i-2)[(n-1)(i-1) + n] + \frac{(i-2)(i-1)}{2} + (n-i)[(n-1)(i+1)] + \left[\frac{n(n+1)}{2} - \frac{i(i+1)}{2}\right]$$

$$= 2i - 1 + (i-2)[ni - n - i + 1 + n] + \frac{i^2 - 3i + 2}{2} + (n-i)(ni + n - i - 1) + \frac{n^2 + n - i^2 - i}{2}$$

$$= 2i - 1 + ni^2 - i^2 + i - 2ni + 2i - 2 + \frac{i^2 - 3i + 2}{2} + n^2 i + n^2 - ni - n - ni^2 - ni + i^2 + i + \frac{n^2 + n - i^2 - i}{2}$$

$$= \frac{1}{2} [8i - 4 - 8ni + 2n^2 i + 3n^2 - n]$$

$$f^*(v_i) = \frac{1}{2} [(2n^2 - 8n + 8)i + (3n^2 - n - 4)]$$

Subcase (iii): When $i = n + 1$ and $j > i$

$$f^*(v_i) = f(v_{i-1} v_i) + \sum_{\substack{j=1 \\ i>j}}^{n-1} f(v_i u_j)$$

$$= f(v_{i-1} v_i) + \sum_{j=1}^{n-1} [(n-1)(i-1) + n + j]$$

$$f^*(v_{n+1}) = f(v_n v_{n+1}) + \sum_{j=1}^{n-1} [(n-1)\ (n+1-1) + n + j]$$

$$= n + \sum_{j=1}^{n-1} [(n-1).n + n + j]$$

$$= n + \sum_{j=1}^{n-1} [n^2 - n + n + j]$$

$$= n + \sum_{j=1}^{n-1} [n^2 + j]$$

$$= n + (n-1) . n^2 + \frac{(n-1).n}{2}$$

$$= \frac{1}{2} [2n + 2n^3 - 2n^2 + n^2 - n]$$

$$f^*(v_{n+1}) = \frac{1}{2} [2n^3 - n^2 + n]$$

Consider the case when labels of edges are distinct.
Subcase (iv): When $j = 1$ and $i > j$

$$f^*(u_j) = f(u_j u_{j+1}) + \sum_{\substack{i=j+2 \\ i>j}}^{n+1} f(v_i u_j)$$

$$= 2n - j + \sum_{i=j+2}^{n+1} [(n-1)(i-1) + n + j]$$

f* (u_1) = (2n–1) + $\sum_{i=3}^{n+1}[(n-1)i-n+1+n+1]$

= (2n–1) + $\sum_{i=3}^{n+1}[(n-1)i+2]$

= (2n–1) + (n–1)2 + (n–1) $\left[\frac{(n+1)(n+2)}{2}-\frac{2.3}{2}\right]$

= 2n–1 + 2n–2 + $\frac{(n-1)(n^2+3n+2)}{2}-3(n-1)$

= $\frac{1}{2}$ [8n–6 + n^3 + $3n^2$ + 2n – n^2 – 3n – 2 – 6n + 6]

f* (u_1) = $\frac{1}{2}$ [n^3 + $2n^2$ + n – 2]

Subcase (v): When $j = 2,3,\dots,n-1$

f* (u_j) = f ($u_{j-1}u_j$) + f(u_j u_{j+1}) + $\sum_{\substack{i=1\\ i\neq j,j+1}}^{n+1} f(v_iu_j)$

= (2n – j + 1) + (2n – j) + $\sum_{\substack{i=1\\ i<j}}^{j-1} f(v_iu_j)+\sum_{\substack{i=j+2\\ i>j}}^{n+1} f(v_iu_j)$

= 4n – 2j + 1 + $\sum_{i=1}^{j-1}[(n-1)(i+1)+j]+\sum_{i=j+2}^{n+1}[(n-1)(i-1)+n+j]$

= 4n – 2j + 1 + $\sum_{i=1}^{j-1}[(n-1)i+(n-1)+j]+\sum_{i=j+2}^{n+1}[(n-1)i-(n-1)+n+j]$

= 4n – 2j + 1 + $\frac{(n-1)(j-1)j}{2}$ + (j–1) (n–1+j) + $\left[\frac{(n+1)(n+2)}{2}-\frac{(j+1)(j+2)}{2}\right]$ + (n–j) (1+j)

f* (u_j) = $\frac{1}{2}$ [n^3 +$2n^2$ + 5n + 4 – 6j]

Sub case (vi): When $j = n$ and $i < j$

f* (u_j) = f ($u_{j-1}u_j$) + $\sum_{\substack{i=1\\ i<j}}^{j-1} f(v_i\ u_j)$

= 2n – j+1 + $\sum_{i=1}^{j-1}[(n-1)(i+1)+j]$

f* (u_n) = 2n – n + 1 + $\sum_{i=1}^{n-1}[(n-1)(i+1)+n]$

= n+1 + $\sum_{i=1}^{n-1}[(n-1)i+n-1+n]$

= n+1 + $\sum_{i=1}^{n-1}[(n-1)i+2n-1]$

= n+1 + $\frac{(n-1)(n-1).n}{2}$ (n–1) (2n–1)

= $\frac{1}{2}$ [2n + 2 + n^3 – $2n^2$ + n + $4n^2$ – 2n – 4n + 2]

= $\frac{1}{2}$ [n^3 + $2n^2$ – 3n + 4]

∴ $BG(P_n)$ is anti-magic.

Case (b): n ≡ 0 (mod 2)

Label the vertices of $BG(P_n)$ using the function

f : E → N as follows:

f(v_i v_{i+1}) = 2 (n–i) + 1; i = 1, 2, …, n

f(u_j u_{j+1}) = 2 (n–j) ; j = 1, 2, …, n-1

f (v_iu_j) = (n–1) (i+1)+j for i< j

= (n–1) (i–1) + n + j for i> j

The induced function $f^*: V \to N$ such that

$f^*(v_i) = \sum_{u_j \in nbd(v_i)} f(v_iu_j)$

Consider the case when the labels of vertices are distinct.

Subcase (vii): When $i = 1$ and $j > i$

f* (v_i) = f (v_i v_{i+1}) + $\sum_{\substack{j=2\\ i<j}}^{n} f(v_iu_j)$

= 2(n–i)+1 + $\sum_{j=2}^{n}[(n-1)(i+1)+j]$

f*(v_1) = 2(n–1) + 1 + $\sum_{j=2}^{n}[(n-1)2+j]$

= 2n – 1 + (n–1) (2n–2) + $\frac{n(n+1)}{2}$ –1

f* (v_1) = $\frac{1}{2}$ [$5n^2$ – 3n]

Subcase (Viii): When $i = 2,\dots,n-1$

f* (v_i) = f(v_{i-1} v_i) + f (v_i v_{i+1}) + $\sum_{\substack{j=1\\ j=i-1,i}}^{n} f(v_iu_j)$

$= 2 [n – (i–1)] + 1 + 2 (n–i)+1 + \sum_{\substack{j=1 \\ i>j}}^{i-2} f(v_i\ u_j) + \sum_{\substack{j=i+1 \\ i<j}}^{n} f(v_i\ u_j)$

$= 4n – 4i+4 + \sum_{j=1}^{i-2}[(n-1)(i-1)+n+j] + \sum_{j=i+1}^{n}[(n-1)(i+1)+j]$

$= 4n – 4i + 4 + \sum_{j=1}^{i-2}[(n-1)i-n+1+n+j] + \sum_{j=i+1}^{n}[(n-1)i+n-1+j]$

$= 4n – 4i + 4 + (i – 2) [(n–1)i + 1] + \frac{(i-2)(i-1)}{2} + (n–i) (ni – i + n–1) + \left[\frac{n(n+1)}{2} - \frac{i(i+1)}{2}\right]$

$= \frac{1}{2} [2n^2 i – 8ni – 4i + 3n^2 + 7n + 6]$

$f^*(v_i) = \frac{1}{2} [(2n^2 – 8n – 4)i + (3n^2 + 7n + 6)]$

Subcase (iX): when $i = n+1$ and $j < i$

$f^*(v_i) = f(v_{i-1}\ v_i) + \sum_{\substack{j=1 \\ i>j}}^{n-1} f(v_i u_j)$

$= 2 [n – (i–1)] + 1 + \sum_{j=1}^{n-1}[(n-1)(i-1)+n+j]$

$f^*(v_{n+1}) = 2 [n–(n+1–1)] + 1 + \sum_{j=1}^{n-1}[(n-1)(n+1-1)+n+j]$

$= 1 + \sum_{j=1}^{n-1}[(n-1)n+n+j]$

$= 1 + \sum_{j=1}^{n-1}[n^2+j]$

$= 1 + (n–1)\ n^2 + \frac{(n-1).n}{2}$

$f^*(v_{n+1}) = \frac{1}{2} [2n^3 – n^2 – n + 2]$

We consider the case when the labels of edges are distinct.

Subcase (x): when $j = 1$ and $i > j$

$f^*(u_j) = f(u_j\ u_{j+1}) + \sum_{\substack{i=j+2 \\ 1>j}}^{n+1} f(v_i u_j)$

$= 2(n – j) + \sum_{i=j+2}^{n+1}[(n-1)(i-1)+n+j]$

$f^*(u_1) = 2 (n – 1) + \sum_{i=3}^{n+1}[(n-1)(i-1)+n+1]$

$= 2(n – 1) + \sum_{i=3}^{n+1}[(n-1)i-n+1+n+1]$

$= 2 (n – 1) + (n – 1)\left[\frac{(n+1)(n+2)}{2} - \frac{2.3}{2}\right] + (n – 1)2$

$= \frac{1}{2} [n^3 + 2n^2 + n – 4]$

Subcase (xi): when $j = 2, 3, \ldots, n-1$

$f^*(u_j) = f(u_{j-1}u_j) + f(u_j\ u_{j+1}) + \sum_{\substack{i=1 \\ i\neq j, j+1}}^{n+1} f(v_i u_j)$

$= 2(n–j) + 2 + 2(n–j) + \sum_{\substack{i=1 \\ i<j}}^{j-1} f(v_i u_j) + \sum_{\substack{i=j+2 \\ i>j}}^{n+1} f(v_i u_j)$

$= 4n – 4j + 2 + \sum_{i=1}^{j-1}[(n-1)(i+1)+j] + \sum_{i=j+2}^{n+1}[(n-1)(i-1)+n+j]$

$= 4n – 4j + 2 + \sum_{i=1}^{j-1}[(n-1)i+n-1+j] + \sum_{i=j+2}^{n+1}[(n-1)i-(n-1)+n+j]$

$= 4n – 4j + 2 + (n–1)\ \frac{(j-1)j}{2} + (j-1)(n-1+j) + (n-1)\left[\frac{(n+1)(n+2)}{2} - \frac{(j+1)(j+2)}{2}\right] + (n – j) (1 + j)$

$f^*(u_j) = \frac{1}{2} [n^3 + 2n^2 + 5n + 6 – 10j]$

Subcase (xii): when $j = n$ and $i < j$

$f^*(u_j) = f(u_{j-1}u_j) + \sum_{\substack{i=1 \\ 1<j}}^{j-1} f(v_i\ u_j)$

$$= 2(n-j) + 2 + \sum_{i=1}^{j-1}[(n-1)(i+1)+j]$$

$$f^*(u_n) = 2(n-n) + 2 + \sum_{i=1}^{n-1}[(n-1)i+n-1+n]$$

$$= 2 + (n-1)\frac{(n-1)n}{2} + (n-1)(2n-1)$$

$$f^*(u_n) = \frac{1}{2}[n^3 + 2n^2 - 5n + 6]$$

Hence in all the above cases the labeling of all the vertices and the edges of the Boolean graph of path is anti-magic.

$\therefore BG(P_n)$ is anti-magic.

V. CONCLUSION

Finally we conclude that the anti-magic labeling to Boolean Graph of path is anti-magic.

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