On Contra Delta Generalized Pre-Continuous Functions

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Available online at: www.isroset.org

Accepted 27/Jun/2018, Online 30/Aug/2018

Abstract- In this paper, the notion of contra \( \delta gp \)-continuous functions is introduced by utilizing \( \delta gp \)-closed sets in topological spaces. Some of their fundamental properties are studied and the relationships of contra \( \delta gp \)-continuous functions with other related functions are discussed.

Keywords- \( \delta gp \)-open set, contra continuous function, contra pre-continuous function, \( \delta gp \)-continuous function.

I. INTRODUCTION

In 1996, Dontchev [8] initiated the study of contra continuous functions. Subsequently, Jafari and Noiri [15, 16] exhibited contra \( \alpha \)-continuous and contra pre-continuous functions in topological spaces. In this paper, a new class of generalized contra continuous functions by using \( \delta gp \)-closed sets, called contra \( \delta gp \)-continuous functions is introduced and study some of their basic properties. Relationships between contra \( \delta gp \)-continuous functions and other related functions are investigated.

II. PRELIMINARIES

Definition 2.1 A subset A of a topological space X is called pre-closed [19] (resp, b-closed [1], regular closed [26], semi-closed [18] and \( \alpha \)-closed [21]) if \( cl(int(A)) \subseteq A \) (resp,\( cl(int(A)) \cap int(cl(A)) \subseteq A \), \( A = cl(int(A)), int(cl(A)) \subseteq A \) and \( int(cl(int(A))) \subseteq X \).

Definition 2.2 A subset A of a topological space X is called \( \delta \)-closed [28] if \( A = cl_{\delta}(A) \) where \( cl_{\delta}(A) = \{ x \in X; int(cl(U)) \cap A = \emptyset, U \in \tau \text{ and } x \in U \} \)

Definition 2.3 A subset A of a topological space X is called (i) \( \delta gp \)-closed [5] (resp, \( gp \)-closed [17]) if \( pcl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \delta \)-open (resp, regular open and open) in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.4 A function \( f : X \rightarrow Y \) from a topological space X into a topological space Y is called (i) contra continuous [8] (resp, contra pre-continuous[15], contra \( \alpha \)-continuous[16], contra \( gp \)-continuous[7] and contra \( gp \)-continuous) if \( f^{-1}(G) \) is closed (resp, pre-closed, \( \alpha \)-closed, \( gp \)-closed and \( gp \)-closed) in X for every open set G of Y.

(ii) perfectly-continuous [23] if \( f^{-1}(G) \) is clopen in X for every open set G of Y.

(iii) pre-closed [10] if for every closed subset A of X, \( f(A) \) is pre-closed in Y.

(iv) \( \delta gp \)-continuous [27] (resp, completely-continuous [2] and super continuous [20]) if \( f^{-1}(G) \) is \( \delta gp \)-open (resp, regular-open and \( \delta \)-open) in X for every open set G of Y.

Definition 2.5 A space X is called (a) extremely disconnected [12] if the closure of every open subset of X is open.

(b) strongly irresolvable [11] if every open subspace of X is irresolvable.

(c) semi-regular [6] if every open set is \( \delta \)-open in X.

(d) Urysohn [29] if for each pair of distinct points x and y of X, there exist open sets U and V containing x and y respectively such that \( cl(U) \cap cl(V) = \emptyset \).

(e) regular [29] if U is open in X and x \( \in \) U, then there is an open set V containing x such that \( cl(V) \subseteq U \).

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3. Contra $\delta gp$-Continuous Functions.

**Definition 3.1** A function $f:X\to Y$ is called contra delta generalized pre-continuous (briefly, contra $\delta gp$-continuous) if the inverse image of every open set of $Y$ is $\delta gp$-closed in $X$.

**Theorem 3.2** A function $f:X\to Y$ is contra $\delta gp$-continuous if and only if $f^{-1}(U)$ is $\delta gp$-open in $X$ for every closed set $U$ of $Y$.

**Remark 3.3** From Definitions 2.4 and 3.1, we have the following diagram of implications for a function $f:X\to Y$

\[
\begin{array}{c}
\text{Perfectly continuity} \\
\downarrow \\
\text{contra pre-continuity} \iff \text{contra continuity} \\
\downarrow \\
\text{contra gp-continuity} \rightarrow \text{contra $\delta gp$-continuity} \\
\downarrow \\
\text{contra gpr-continuity}
\end{array}
\]

None of the implications in above diagram is reversible.

**Example 3.4** Consider $X=\{a,b,c,d\}$ with the topologies $T = \{X,\emptyset,\{a\},\{b\},\{a\},\{a,b,c\}\}$ and $\sigma = \{X,\emptyset,\{a\},\{b\},\{a,b\},\{a,c\},\{a,b,c\}\}$. Define $f:(X,\tau)\to(X,\sigma)$ by $f(a)=f(b)=a$, $f(c)=b$ and $f(d)=c$. Then $f$ is contra gpr-continuous but not contra $\delta gp$-continuous, since $\{a\}$ is open in $Y$ but $f^{-1}(\{a\})=\{a,b\}$ is not $\delta gp$-closed in $X$.

**Remark 3.5** (a) Contra $\delta gp$-continuity and $\delta gp$-continuity are independent each other.
(b) Contra $\delta gp$-continuity and contra $\delta$-continuity are independent each other.

**Example 3.6** In Example 3.4, $f$ is $\delta gp$-continuous but not contra $\delta gp$-continuous.

**Example 3.7** Consider $X$, $\tau$ and $\sigma$ as in Example 3.4. Define $h:(X,\tau)\to(X,\sigma)$ by $h(a)=d$, $h(b)=c$, $h(c)=a$ and $h(d)=b$. Then $h$ is contra $\delta gp$-continuous but not $\delta gp$-continuous, since $\{a,b\}$ is open in $Y$ but $h^{-1}(\{a,b\})=\{c,d\}$ is not $\delta gp$-open in $X$.

**Definition 3.8** A space $X$ is called locally $\delta gp$-indiscrete if every $\delta gp$-open set is $\delta gp$-closed in $X$.

**Theorem 3.9** If $f:X\to Y$ is a contra $\delta gp$-continuous and $X$ is locally $\delta gp$-indiscrete space, then $f$ is $\delta gp$-continuous.

**Proof:** Let $V$ be a closed set in $Y$. Since $f$ is contra $\delta gp$-continuous and $X$ is locally $\delta gp$-indiscrete space, then $f^{-1}(V)$ is $\delta gp$-closed in $X$. Hence $f$ is $\delta gp$-continuous.

**Definition 3.10** [22] A space $X$ is called locally indiscrete if every open set is closed in $X$.

**Theorem 3.11** If $f:X\to Y$ is a $\delta gp$-continuous and $Y$ is locally indiscrete space, then $f$ is contra $\delta gp$-continuous.

**Proof:** Let $G$ be any open set of $Y$. Since $Y$ is locally indiscrete space and $f$ is $\delta gp$-continuous, then $f^{-1}(G)$ is $\delta gp$-closed in $X$. Hence $f$ is contra $\delta gp$-continuous.

**Theorem 3.12** [27] (a) In extremely disconnected space $X$, every $g\delta s$-closed set is $\delta gp$-closed.
(b) In strongly irresolvable space $X$, every $\delta gp$-closed set is $g\delta s$-closed.

As a consequence of Theorem 3.12, we have the following Theorem 3.13 and Theorem 3.14.

**Theorem 3.13** If $f:X\to Y$ is a contra $g\delta s$-continuous and $X$ is extremely disconnected space, then $f$ is contra $\delta gp$-continuous.

**Theorem 3.14** If $f:X\to Y$ is a contra $\delta gp$-continuous and $X$ is strongly irresolvable space, then $f$ is contra $g\delta s$-continuous.

**Theorem 3.15** If $f:X\to Y$ is contra $\delta gp$-continuous and $X$ is $T_{\delta gp}$-space, then $f$ is contra continuous.

**Proof:** Suppose $X$ is $T_{\delta gp}$-space and $f$ is contra $\delta gp$-continuous. Let $G$ be an open set in $Y$ by hypothesis. $f^{-1}(G)$ is $\delta gp$-closed in $X$ and hence $f^{-1}(G)$ is closed in $X$. Therefore $f$ is contra continuous.

**Theorem 3.16** If $f:X\to Y$ is contra $\delta gp$-continuous and $X$ is $\delta gpT_{1/2}$-space, then $f$ is contra pre-continuous.

**Proof:** Suppose $X$ is $\delta gpT_{1/2}$-space and $f$ is contra $\delta gp$-continuous. Let $G$ be an open set in $Y$ by hypothesis. $f^{-1}(G)$ is $\delta gp$-closed in $X$ and hence $f^{-1}(G)$ is pre-closed in $X$. Therefore $f$ is contra pre-continuous.

**Theorem 3.17** If $f:X\to Y$ is contra $\delta gp$-continuous and $X$ is semi regular, then $f$ is contra gp-continuous.

**Proof:** Follows from the fact that every open set is $\delta$-open in semi-regular space.

**Lemma 3.18** [27] For a subset $A$ of a space $X$, the following are equivalent:
(a) $A$ is clopen;
(b) $A$ is open and pre-closed;
(c) $A$ is open and gp-closed;
(d) $A$ is $\delta$-open and $\delta gp$-closed;
(e) $A$ is regular-open and $gp$-closed.

**Lemma 3.19** For a subset $A$ of a space $X$, the following are equivalent:
(a) $A$ is clopen.
(b) $A$ is regular-open and pre-closed.
(c) $A$ is $\delta$-open and pre-closed.

Following Theorem is immediate from Lemma 3.18 and Lemma 3.19:

**Theorem 3.20** The following statements are equivalent for a function $f:X \rightarrow Y$:
(a) $f$ is perfectly continuous.
(b) $f$ is continuous and contra $\delta$-pre-continuous.
(c) $f$ is continuous and contra $gp$-continuous.
(d) $f$ is $\delta$-continuous and contra $\delta gp$-continuous.
(e) $f$ is $\delta$-continuous and contra $\delta$-pre-continuous.
(f) $f$ is $\delta$-continuous and contra $\delta gp$-continuous.

**Theorem 3.21** If $f:X \rightarrow Y$ is contra $\delta gp$-continuous, then the following equivalent statements hold:
(i) For each $x \in X$ and each closed set $B$ of $Y$ containing $f(x)$, there exists an $\delta gp$-open set $A$ in $X$ containing $x$ such that $f(A) \subset B$.
(ii) For each $x \in X$ and each open set $G$ of $Y$ not containing $f(x)$, there exists a $\delta$-open set $H$ in $X$ not containing $x$ such that $f^{-1}(G) \subset H$.

**Proof:** Let $B$ be a closed set in $Y$ such that $f^{-1}(B) \subset B$, then $x \in f^{-1}(B)$. By hypothesis, $f^{-1}(B)$ is $\delta$-open set in $X$ containing $x$. Let $A = f^{-1}(F)$, then $f(A) = f(f^{-1}(B)) \subset B$.

**Theorem 3.22** [5] Let $A \subset X$. Then $x \in \delta gp cl(A)$ if and only if $U \cap A = \emptyset$, for every $\delta gp$-open set $U$ containing $x$.

Recall that for a subset $A$ of a space $(X,\tau)$, the set $\bigcap \{U \in \tau \mid A \subseteq U \}$ is called the kernel of $A$ and is denoted by $ker(A)$.

**Lemma 3.23** [14] The following properties hold for subsets $A$ and $B$ of a space $X$:
(i) $x \in ker(A)$ if and only if $A \cap F = \emptyset$ for any closed set $F$ of $X$ containing $x$.
(ii) $A \subset ker(A)$ and $A = ker(A)$ if $A$ is open in $X$.
(iii) If $A \subset B$, then $ker(A) \subset ker(B)$.

**Definition 3.24** A space $X$ is said to be $\delta gp$-additive if $\delta GPC(X)$ is closed under arbitrary intersections.

**Theorem 3.25** Let $X$ be $\delta gp$-additive, then the following are equivalent for a function $f:X \rightarrow Y$.
(i) $f$ is contra $\delta gp$-continuous.
(ii) For each $x \in X$ and each closed set $D$ of $Y$ containing $f(x)$, there exists an $\delta gp$-open set $C$ in $X$ containing $x$ such that $f(C) \subset D$.
(iii) $f(\delta gp cl(C)) \subset ker(f(C))$ for every subset $C$ of $X$.
(iv) $\delta gp cl(f^{-1}(D)) \subset f^{-1}(ker(D))$ for every subset $D$ of $Y$.

**Proof:** (i) $\rightarrow$ (ii) It follows from Theorem 3.21 (ii) $\rightarrow$ (i). Let $C$ be a closed set in $Y$ containing $f(x)$, then $x \in f^{-1}(G)$. From (ii), there exists $\delta gp$-open set $U_x$ in $X$ containing $x$ such that $f(U_x) \subset D$, $U_x \subset f^{-1}(G)$.

Thus $f^{-1}(G) = \cup \{U_x : x \in f^{-1}(G)\}$ is $\delta gp$-open in $X$.

(i) $\rightarrow$ (iii) Let $C$ be any subset of $X$. Suppose $y \notin ker(f(C))$, then by Lemma 3.23, there exists a closed set $D$ in $Y$ containing $y$ such that $f(C) \cap D = \emptyset$. Hence we have, $C \cap f^{-1}(D) = \emptyset$ and $\delta gp cl(C) \cap f^{-1}(D) = \emptyset$, which implies $f(\delta gp cl(C)) \cap D = \emptyset$ and hence $y \notin f(\delta gp cl(C))$. Therefore $f(\delta gp cl(C)) \subset ker(f(C))$.

(iii) $\rightarrow$ (iv) Let $D \subset Y$, then $f^{-1}(D) \subset X$. By (ii) and Lemma 3.23, $f(\delta gp cl(f^{-1}(D))) \subset ker(f(f^{-1}(D))) \subset ker(D)$. Thus $\delta gp cl(f^{-1}(D)) \subset f^{-1}(ker(D))$.

(iv) $\rightarrow$ (i) Let $U$ be any open subset of $Y$. Then by (iv) and Lemma 3.23, $\delta gp cl(f^{-1}(U)) \subset f^{-1}(ker(U)) = f^{-1}(U)$ and $\delta gp cl(f^{-1}(U)) = f^{-1}(U)$. Therefore $f^{-1}(U)$ is $\delta gp$-closed set in $X$.

**Theorem 3.26** If a surjective function $f(X,\tau) \rightarrow (Y,\sigma)$ is contra $\delta gp$-continuous and preclosed with $X$ as a $T_{\delta gp}$-space, then $Y$ is locally indiscrete.

**Proof:** Let $H$ be any open set in $Y$. Since $f$ is contra $\delta gp$-continuous and $X$ is $T_{\delta gp}$-space, then $f^{-1}(H)$ is closed in $X$. Since $f$ is preclosed, then $H$ is preclosed in $Y$. Thus we have $cl(H) = cl(int(H)) \subset H$ and hence $H$ is closed in $Y$.

**Theorem 3.27** If $f(X,\tau) \rightarrow (Y,\sigma)$ is contra $\delta gp$-continuous, $X$ is $\delta gp$-additive and $Y$ is regular, then $f$ is $\delta gp$-continuous.

**Proof:** Let $x \in X$ and $N$ be any open set of $Y$ containing $f(x)$. As $Y$ is regular, there exists an open set $M$ in $Y$ containing $f(x)$ such that $cl(M) \subset N$. If $f$ is contra $\delta gp$-continuous, there exists an $\delta gp$-open set $U$ in $X$ containing $x$ such that $f(U) \subset cl(M)$. Then $f(U) \subset cl(M) \subset N$. Hence by Theorem 3.25, $f$ is $\delta gp$-continuous.

Recall that, for a function $f:X \rightarrow Y$, the subset $\{ (x,f(x)) : x \in X \} \subset X \times Y$ is called the graph of $f$ and is denoted by $G(f)$.
Definition 3.28 The graph \(G(f)\) of a function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is said to be contra \(\delta\text{-}gp\)-closed if for each \((x,y)\in(X\times Y)\) there exist \(\delta\text{-}gp\)-open set \(U\) and \(V\) containing \(x\) and \(y\) respectively such that \((U\times V)\cap G(f)=\varphi\).

Theorem 3.29 The graph \(G(f)\) of a function \(f:(X,\tau)\rightarrow(Y,\sigma)\) is contra \(\delta\text{-}gp\)-closed in \(X\times Y\) if and only for each \((x,y)\in(X\times Y)\) there exist \(\delta\text{-}gp\)-open set \(U\) and \(V\) containing \(x\) and \(y\) respectively such that \((U\times V)\cap G(f)=\varphi\).

Theorem 3.30 If \(f:(X,\tau)\rightarrow(Y,\sigma)\) is contra \(\delta\text{-}gp\)-continuous and \(Y\) is Urysohn, then \(G(f)\) is contra \(\delta\text{-}gp\)-closed in the product space \(X\times Y\).

Proof: Let \((x,y)\in(X\times Y)\) then \(y=f(x)\) and there exist open sets \(U\) and \(V\) such that \((x,y)\in(U\times V)\) and \((U\times V)\cap G(f)=\varphi\). Since \(f\) is contra \(\delta\text{-}gp\)-continuous, there exists \(\delta\text{-}gp\)-open set \(G\) such that \(x\notin G\) and \((U\times V)\cap G=\varphi\). Hence \(G(f)\) is contra \(\delta\text{-}gp\)-closed in \(X\times Y\).

Theorem 3.31 Let \(g:X\rightarrow X\times Y\) be the graph function of \(g:X\rightarrow Y\), then \(f(x)=(x,f(x))\) for each \(x\in X\). Then \(f\) is contra \(\delta\text{-}gp\)-continuous if \(g\) is contra \(\delta\text{-}gp\)-continuous.

Proof: Let \(V\) be any open set in \(Y\), then \(X\times V\) is an open set in \(X\times Y\). It follows that \(f^{-1}(U)=g^{-1}(X\times V)\) is \(\delta\text{-}gp\)-closed in \(X\) since \(g\) is contra \(\delta\text{-}gp\)-continuous. Hence \(f\) is contra \(\delta\text{-}gp\)-continuous.

Definition 3.32 [24] A space \(X\) is submaximal if every pre-open set is open in \(X\).

Theorem 3.33 If \(M\) and \(N\) are \(\delta\text{-}gp\)-closed sets in a submaximal space \(X\), then \(M\cup N\) is \(\delta\text{-}gp\)-closed in \(X\).

Proof: Let \(U\) be \(\delta\)-open set in \(X\) such that \(M\cup N\subseteq U\). Then \(pcl(M)\subseteq U\) and \(pcl(N)\subseteq U\). Hence \(M\cup N\) is \(\delta\text{-}gp\)-closed.

Corollary 3.34 If \(A\) and \(B\) are \(\delta\text{-}gp\)-open sets in submaximal space \(X\), then \(A\cap B\) is \(\delta\text{-}gp\)-open in \(X\).

Theorem 3.35 [5] If \(A\subseteq X\) is \(\delta\text{-}gp\)-closed, then \(A=gpcl(A)\).

Remark 3.36 Converse of above theorem is true if \(X\) is \(\delta\text{-}gp\)-additive.

Theorem 3.37 Assume that \(X\) is \(\delta\text{-}gp\)-additive. If \(f:X\rightarrow Y\) and \(g:X\rightarrow Y\) are contra \(\delta\text{-}gp\)-continuous, \(X\) is submaximal and \(Y\) is Urysohn. Then \(F=\{x\in X:x=g(x)\}\) is \(\delta\text{-}gp\)-closed in \(X\).

Proof: Let \(x\in F\), then \(f(x)=g(x)\). Therefore, there exist open sets \(U\) and \(V\) such that \(f(x)\in U\) and \(g(x)\in V\). Then \(U\cap V\neq\varphi\) and \(x\in X\). Since \(f\) and \(g\) are contra \(\delta\text{-}gp\)-continuous, \(f^{-1}(U)\) and \(g^{-1}(V)\) are \(\delta\text{-}gp\)-open sets in \(X\). Let \(M=f^{-1}(U)\) and \(N=g^{-1}(V)\). Hence \(f(x)\in M\cap N\). Then \(M\cap N\) are \(\delta\text{-}gp\)-open sets containing \(x\). Since \(O=M\cap N\), then \(O\) is \(\delta\text{-}gp\)-open set in \(X\). Hence \((O\cap N)\cap g(O)=f(M\cap N)\cap g(M\cap N)\subseteq f(M)\cap g(N)=\varphi\). Therefore \(F\) is \(\delta\text{-}gp\)-closed in \(X\).

Definition 3.38 A space \(X\) is called \(\delta\text{-}gp\)-connected if \(X\) is not the union of two disjoint nonempty \(\delta\text{-}gp\)-open sets.

Theorem 3.39 For a space \(X\) the following are equivalent: (a) \(X\) is \(\delta\text{-}gp\)-connected, (b) \(\delta\)-open and \(\delta\text{-}gp\)-open subset of \(X\). If \(X\) is \(\delta\text{-}gp\)-closed and \(\delta\text{-}gp\)-open in \(X\), then \(X=A\cup(X-A)\). Hence \(A=\varphi\) and \(X\) is \(\delta\text{-}gp\)-connected.

Proof: (a)\(\rightarrow\)(b): Suppose \(A\) is any proper \(\delta\text{-}gp\)-open and \(\delta\text{-}gp\)-closed subset of \(X\). Then \(A\cap (X-A)=\varphi\) which contradicts the fact that \(X\) is \(\delta\text{-}gp\)-connected.

(c) Assume that \(A\neq\varphi\) and \(X\) is \(\delta\text{-}gp\)-connected. Then \(X=A\cup(X-A)\). Hence \(A=\varphi\) and \(X\) is \(\delta\text{-}gp\)-connected.

(b)\(\rightarrow\)(c): Let \(f:X\rightarrow Y\) be a contra \(\delta\text{-}gp\)-continuous function where \(Y\) is a discrete space with at least two points. Then \(f^{-1}(\{y\})=\varphi\) and \(\delta\text{-}gp\)-open for each \(y\in Y\).

Theorem 3.40 If \(f:X\rightarrow Y\) is a contra \(\delta\text{-}gp\)-continuous function and \(X\) is \(\delta\text{-}gp\)-connected space, then \(Y\) is not a discrete space.

Proof: If \(\delta\text{-}gp\)-open and \(\delta\text{-}gp\)-closed subset of \(X\). Let \(f:X\rightarrow Y\) be a contra \(\delta\text{-}gp\)-continuous function defined by \(f(N)=\{y\}\) and \(f(X-N)=\{z\}\) for some distinct points in \(Y\). Hence \(f\) is constant if follows that \(N=X\).
contra δgp-continuous, then $f^{-1}(A)$ is proper nonempty δgp-open and δgp-closed subset of $X$ which contradicts the fact that $X$ is δgp-connected space. Hence $Y$ is not discrete.

**Theorem 3.41** If a surjective function $f:X\to Y$ is contra δgp-continuous and $X$ is δgp-connected space, then $Y$ is connected.

**Proof:** Suppose that $Y$ is not a connected space. Then there exist disjoint open sets $U$ and $V$ in $Y$ such that $Y=U\cup V$. Therefore $U$ and $V$ are closed sets in $Y$. Since $f$ is contra δgp-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are δgp-open sets in $X$. Also $f$ is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and $X=f^{-1}(U)\cup f^{-1}(V)$ which contradicts the fact that $X$ is δgp-connected space. Hence $Y$ is connected.

**Theorem 3.42** Let $X$ be a δgp-connected and $Y$ be $T_1$-space. If $f:X\to Y$ is contra δgp-continuous, then $f$ is constant.

**Proof:** By hypothesis $Y$ is $T_1$-space, $K=\{ y \in Y \}$ is a disjoint δgp-open partition of $X$. If $|K|\geq 2$, then $X$ is the union of two nonempty δgp-open sets. This is contradiction to the fact that $X$ is δgp-connected. Therefore $|K|=1$ and hence $f$ is constant.

**Definition 3.43** A topological space $X$ is said to be δgp-Hausdorff space if for any pair of distinct points $x$ and $y$, there exist disjoint δgp-open sets $G$ and $H$ such that $x \in G$ and $y \in H$.

**Theorem 3.44** If an injective function $f:X\to Y$ is contra δgp-continuous and $Y$ is an Urysohn space. Then $X$ is δgp-Hausdorff.

**Proof:** Let $x$ and $y$ be any two distinct points in $X$ and $f$ is injective, then $f(x)=f(y)$. Since $Y$ is an Urysohn space, there exist open sets $A$ and $B$ in $Y$ containing $f(x)$ and $f(y)$ respectively, such that $\text{cl}(A)\cap\text{cl}(B)=\emptyset$. Then $f(x) \in \text{cl}(A)$ and $f(y) \in \text{cl}(B)$. Since $f$ is contra δgp-continuous, then by Theorem 3.8, there exist δgp-open sets $C$ and $D$ in $X$ containing $x$ and $y$, respectively, such that $f(C) \subseteq \text{cl}(A)$ and $f(D) \subseteq \text{cl}(B)$. We have $C \cap D \subseteq f^{-1}(\text{cl}(A)) \cap f^{-1}(\text{cl}(B)) = f^{-1}(\emptyset) = \emptyset$. Hence $X$ is δgp-Hausdorff.

**Definition 3.45** [25] A space $X$ is called Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.

**Definition 3.46** A topological space $X$ is said to be δgp-normal if each pair of disjoint closed sets can be separated by disjoint δgp-open sets.

**Theorem 3.47** If $f:X\to Y$ be contra δgp-continuous closed injection and $Y$ is ultra normal, then $X$ is δgp-normal.

**Proof:** Let $E$ and $F$ be disjoint closed subsets of $X$. Since $f$ is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in $Y$. Since $Y$ is ultra normal there exist disjoint clopen sets $U$ and $V$ in $Y$ such that $f(E)\subseteq U$ and $f(F)\subseteq V$. This implies $E \subseteq f^{-1}(U)$ and $F \subseteq f^{-1}(V)$. Since $f$ is contra δgp-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint δgp-open sets in $X$. This shows $X$ is δgp-normal.

**Remark 3.48** The composition of two contra δgp-continuous functions need not be contra δgp-continuous as seen from the following examples.

**Example 3.49** Let $X=Y=Z=\{ a,b,c \}$, $	au = \{ X,\emptyset,\{ a \},\{ b \},\{ a,b \} \}$, and $\tau = \{ Y,\emptyset,\{ a \} \}$ and $\eta = \{ Z,\emptyset,\{ b,c \} \}$ be topologies on $X$, $Y$, and $Z$ respectively. Define a function $f:X\to Y$ as $f(a)=a$, $f(b)=b$ and $f(c)=c$ and a function $g:Y\to Z$ as $g(a)=b$, $g(b)=c$, and $g(c)=a$. Then $f$ and $g$ are contra δgp-continuous but $gf:X\to Z$ is not contra δgp-continuous, since there exists an open set $\{ b,c \}$ in $Z$ such that $(gf)^{-1}(\{ b,c \})=\{ a,b \}$ is not δgp-closed in $X$.

**Theorem 3.50** For any two functions $f:X\to Y$ and $g:Y\to Z$, the following hold:

(i) $g\ast f$ is contra δgp-continuous if $f$ is contra δgp-continuous and $g$ is contra continuous.

(ii) $g\ast f$ is contra δgp-continuous if $f$ is δgp-continuous and $g$ is contra continuous.

(iii) $g\ast f$ is contra δgp-continuous if $f$ is δgp-irresolute and $g$ is contra δgp-continuous.

**Proof:** (i) Let $U$ be an open set in $Z$. Then $g^{-1}(V)$ is open in $Y$ since $g$ is continuous. Therefore $f^{-1}(g^{-1}(U))=(g\ast f)^{-1}(U)$ is δgp-closed in $X$ because $f$ is contra δgp-continuous. Hence $g\ast f$ is contra δgp-continuous. The proofs of (ii) and (iii) are analogous to (i) with the obvious changes.

**Theorem 3.51** Let $f:X\to Y$ be contra δgp-continuous and $g:Y\to Z$ be δgp-continuous with $Y$ is $T_{\delta}gp$-space, then $gf:X\to Z$ is contra $\delta gp$-continuous.

**Proof:** Let $V$ be any open set in $Z$. Since $g$ is $\delta gp$-continuous, $g^{-1}(V)$ is $\delta gp$-open in $Y$ and since $Y$ is $T_{\delta}gp$-space, $g^{-1}(V)$ open in $Y$. Since $f$ is contra $\delta gp$-continuous, $f^{-1}(g^{-1}(V))=(gf)^{-1}(V)$ is $\delta gp$-closed set in $X$. Therefore $gf$ is contra $\delta gp$-continuous.
**Definition 3.52** A function \( f: X \to Y \) is called pre \( \delta g \)-closed if the image of every \( \delta g \)-closed set of \( X \) is \( \delta g \)-closed in \( Y \).

**Theorem 3.53** Let \( f: X \to Y \) be pre \( \delta g \)-closed surjection and \( g: Y \to Z \) be a function such that \( g \circ f: X \to Z \) is contra \( \delta g \)-continuous, then \( g \) is contra \( \delta g \)-continuous.

**Proof:** Let \( U \) be any open set in \( Z \). Then \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) is \( \delta g \)-closed in \( X \). Since \( f \) is a pre \( \delta g \)-closed surjection, \( f^{-1}(g^{-1}(U)) = g^{-1}(U) \) is \( \delta g \)-closed set in \( Y \). Therefore, \( g \) is contra \( \delta g \)-continuous.

**REFERENCES**


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