Fixed point Theorems of Multivalued Mappings in Cone Metric Spaces via Cone C-Class function

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Available online at: www.isroset.org
Accepted 18/Aug/2018, Online 30/Aug/2018

Abstract — Let $P$ be a subset of a Banach space $E$ and $P$ is normal and regular cone on $E$, we prove the existence of the fixed point for multi valued maps and $\varphi$-$\psi$-contractive mappings in cone metric spaces via cone C class functions.

Keywords — Cone metric space, Multivalued mappings, Fixed point, Cone C class function

I. INTRODUCTION

In recent years, several authors (see [1-5]) have studied the strong convergence to a fixed point with contractive constant in cone metric spaces. Seong Hoon Cho and Misen Kim [5] have proved certain fixed point theorems by using Multivalued mapping in the setting of contractive constant in metric spaces. Note on $\varphi - \psi$ -contractive type mappings and related fixed point are proved by Arslan Hojat Ansari [8]. Fixed point theorems of Multivalued mappings in Cone metric spaces proved by Dr.M. Marudai and Dr.R. Krishnakumar [1].

II. PRELIMINARIES

Definition 1.1: Let $E$ be a Banach space and a subset, $P$ of $E$ is said to be a cone if it satisfies the following conditions
(i) $P \neq \emptyset$ and $P$ is closed;
(ii) $ax + by \in P \forall x, y \in P$ and $a, b$ are non-negative real numbers
(iii) $P \cap (-P) = \emptyset$

Given a cone $P \subseteq E$, we define a partial ordering $\leq$ with respect to the cone $P$ by $x \leq y$ if and only if $y - x \in P$. If $y - x \in interior \ of \ P$, then it is denoted by $x \ll y$. The cone $P$ is said to be Normal if a number $K > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$. The cone $P$ is called regular if every increasing sequence which is bounded above is convergent and every decreasing sequence which is bounded below convergent.

Definition 1.2: Let $X$ be a non-empty set, and suppose the mapping $d: X \times X \rightarrow E$ is said to be a cone metric space if it satisfies
(i) $0 \leq d(x, y) \forall x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$
(iii) $d(x, y) = d(x, z) + d(z, y)$ for all $x, y, z \in X$

Example 1.3: Let $E = R^2, P = \{(x, y) \in E; x, y \geq 0\}, X = R$ and $d: X \times X \rightarrow E$ defined by
$d(x, y) = (|x - y|, \alpha |x - y|)$

Where $\alpha \geq 0$ is a constant. Then $(X, d)$ is a cone metric space.

Definition 1.4: Let $(X, d)$ be cone metric space, $x \in X$ and $\{x_n\}$ a sequence in $X$. Then
(i) $\{x_n\}$ converges to $x$ whenever for every $c \in E$ with $0 \ll c$ there is a natural number $N$ such that $d(x_n, x) \ll c$ for all $n \geq N$
Definition 1.5: Let \((X,d)\) is said to be a complete cone metric space, if every Cauchy sequence is convergent in \(X\).

Definition 1.6: Let \((X,d)\) be a metric space. We denote \(CB(X)\) the family of nonempty closed bounded subset of \(X\). Let \(H(\cdot,\cdot)\) be the Hausdorff distance on \(CB(X)\). That is, for \(A,B \in CB(X)\)
\[
H(A,B) = \max\{\sup_{a \in A} d(a,B), \sup_{b \in B} d(A,b)\}
\]
Where \(d(a,B) = \inf_{b \in B} d(a,b)\) is the distance from the point \(a\) to the subset \(B\). An element \(x \in X\) is said to be a fixed point of a multi-valued mapping \(T:X \rightarrow 2^X\) if \(x \in T(x)\)

Definition 1.7: A function \(\psi: P \rightarrow P\) is called an altering distance function if the following properties are satisfied:
(i) \(\psi\) is non-decreasing and continuous
(ii) \(\psi(t) = 0\) if and only if \(t = 0\)

Definition 1.8: An ultra altering distance function is a continuous, non decreasing mapping \(\varphi:P \rightarrow P\) such that \(\varphi(t) > 0\), \(t > 0\) and \(\varphi(0) \geq 0\)

We denote this set with \(\Phi_u\)

Definition 1.9: A mapping \(f:P^2 \rightarrow P\) is called cone \(C\) -class function if it is continuous and satisfies following axioms:
1) \(F(s,t) < s\)
2) \(F(s,t) = s\) implies that either \(s = 0\) or \(t = 0\); for all \(s,t \in P\)

We denote cone \(C\) -class functions as \(\mathcal{C}\)

Example 2.9: The following functions \(F:P^2 \rightarrow P\) are elements of \(\mathcal{C}\), for all \(s,t \in [0,\infty)\):
(i) \(F(s,t) = s - t\)
(ii) \(F(s,t) = ks\), where \(0 < k \leq 1\).
(iii) \(F(s,t) = s\beta(s), \) where \(\beta: [0,\infty) \rightarrow [0,1]\).
(iv) \(F(s,t) = \Psi(s), \) where \(\Psi:P \rightarrow P, \Psi(0) = 0, \Psi(s) > 0\) for all \(s \in P\) with \(s \neq 0\) and \(\Psi(s) \leq s\) for all \(s \in P\).
(v) \(F(s,t) = s - \varphi(s), \) where \(\varphi: [0,\infty) \rightarrow [0,\infty]\) is a continuous function such that \(\varphi(t) = 0 \Leftrightarrow t = 0\).
(vi) \(F(s,t) = s - h(s,t), \) where \(h: [0,\infty) \times [0,\infty) \rightarrow [0,\infty]\) is a continuous function such that \(h(s,t) = 0 \Leftrightarrow t = 0\) for all \(s,t > 0\).
(vii) \(F(s,t) = \varphi(s), F(s,t) = s \Rightarrow s = 0, \) here \(\varphi: [0,\infty) \rightarrow [0,\infty]\) is an upper semi continuous function such that \(\varphi(0) = 0\) and \(\varphi(t) < t\) for \(t > 0\).

Lemma 1.10: Let \(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively, \(F \in \mathcal{C}\) and \(\{s_n\}\) a decreasing sequence in \(P\) such that
\[
\psi(s_{n+1}) \leq F(\psi(s_n), \varphi(s_n))
\]
For all \(n \geq 1\). Then \(\lim_{n \to \infty} s_n = 0\)

III. MAIN RESULTS

Theorem 2.1: Let \((X,d)\) be a complete cone metric space and the mapping \(T:X \rightarrow CB(X)\) be multivalued map satisfying for each \(x,y \in X\)
\[
\psi(H(Tx,Ty)) \leq F(\psi(a[d(x,Tx) + d(y,Ty)] + b[d(x,Ty) + d(Tx,y)])
\]
\[
\phi(a[d(x,Tx) + d(y,Ty)] + b[d(x,Ty) + d(Tx,y)])\text{ for all }x,y \in X \text{ and } a + b < \frac{1}{2}, a,b \in \left[0,\frac{1}{2}\right].
\]
\(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively. \(F \in \mathcal{C}\) such that \(\psi(t+s) \leq \psi(t) + \psi(s)\). Then \(T\)
has a fixed point in \(X\)

Proof: for every \(x_0 \in X\) and \(n \geq 1, x_1 \in Tx_0 \text{ and } x_{n+1} \in Tx_n\)
\[
\psi(d(x_{n+1},x_n)) \leq \psi(H(Tx_n,Tx_{n-1})) \leq F(\psi(a[d(x_n,Tx_n) + d(x_{n-1},Tx_{n-1})] + b[d(x_n,Tx_{n-1}) + d(Tx_n,x_{n-1})]),
\]

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\[ \phi(a[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] + b[d(x_n, T_{x_{n+1}}) + d(T_{x_{n+1}}, x_m)]) \]
\[ \leq F(\phi(a[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] + b[d(x_n, x_{n+1}) + b[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)]) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] + b[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)]) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] + b[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)]) \]
\[ \leq F(\psi((a + b)[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)]) \]
\[ \leq \psi((a + b)[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)]) \]
\[ d(x_{n+1}, x_m) \leq L(d(x_n, x_{n+1}) + d(x_{n+1}, x_m)) \]

For \( n \geq 0 \) to \( n \geq m \), we have
\[ d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_m) \leq [L^{n+1} + L^{n+1} + \ldots + L^m]d(x_1, x_0) \]
\[ \leq \frac{L^m}{1-L}d(x_1, x_0) \]

Let \( 0 < c \) be given, choose a natural number \( N_1 \) such that \( \frac{L^m}{1-L}d(x_1, x_0) < c \) for all \( m \geq N_1 \). This implies \( d(x_n, x_m) \leq c \). For \( n \geq m \), \( \{x_n\} \) is a Cauchy sequence in \((X, d)\) is a complete cone metric space, there exists \( p \in X \) such that \( x_n \rightarrow p \). Choose a natural number \( N_2 \) such that \( d(x_n, p) < \frac{L^m}{1-L}d(x_1, x_m) \leq N_2 \). Hence for \( n \geq N_2 \) we have \( d(x_n, p) < \frac{c(1-L)}{3} \) where \( k = a + b \)
\[ \psi(d(Tp, p)) \leq \psi(H(Tx_n, Tp) + d(Tx_n, p)) \]
\[ \leq F(\psi(a[d(x_n, Tx_n) + d(p, Tp)] + b[d(x_n, Tp) + d(Tx_n, p)] + d(x_{n+1}, p)) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(p, Tp)] + b[d(x_n, Tp) + d(x_{n+1}, p)] + d(x_{n+1}, p)) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(p, Tp)] + b(d(x_n, Tp) + d(x_{n+1}, p)) + d(x_{n+1}, p)) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(p, Tp)] + b(d(x_n, Tp) + d(x_{n+1}, p)) + d(x_{n+1}, p)) \]
\[ \leq F(\psi(a[d(x_n, x_{n+1}) + d(p, Tp)] + b(d(x_n, Tp) + d(x_{n+1}, p)) + d(x_{n+1}, p)) \]
\[ \leq (1 - k)d(Tp, p) \leq kd(Tx_n, Tp) + kd(Tx_n, p) + d(x_{n+1}, p) \]
\[ \leq \frac{L^m}{1-L}d(Tx_n, Tp) + d(x_{n+1}, p) \]
\[ \leq \frac{L^m}{1-L}d(Tx_n, Tp) + d(x_{n+1}, p) \]
\[ \leq \frac{L^m}{1-L}d(Tx_n, Tp) + d(x_{n+1}, p) \]
\[ \leq \frac{L^m}{1-L}d(Tx_n, Tp) + d(x_{n+1}, p) \]
\[ d(Tp, p) \leq \left( \frac{L^m}{1-L}d(Tx_n, Tp) + d(x_{n+1}, p) \right) \]
For all \(x, y \in X\) and \(r \in [0, 1]\). \(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively, \(F \in C\) such that 
\[
\psi(t + s) \leq \psi(t) + \psi(s).
\]
Then \(T\) has a fixed point in \(X\).

**Proof:** for every \(x_0 \in X\) and \(n \geq 1\), \(x_1 \in Tx_0\) and \(x_{n+1} \in Tx_n\)
\[
\psi(d(x_{n+1}, x_n)) \leq \psi(H(Tx_n, Tx_{n-1})
\]
\[
\leq F(\psi(r \max\{d(x_n, x_{n-1}), d(x_n, Tx_n), d(x_{n-1}, Tx_{n-1})\}), \varphi(r \max\{d(x_n, x_{n-1}), d(x_n, Tx_n), d(x_{n-1}, Tx_{n-1})\}))
\]
\[
\leq F(\psi(r \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_{n+1})\}), \varphi(r \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_{n+1})\}))
\]
\[
\leq F(\psi(rd(x_n, x_{n-1})), \varphi(rd(x_n, x_{n-1})))
\]
\[
\leq r^n d(x_1, x_0)
\]

For \(n > m\) we have
\[
d(x_n, x_m) \leq d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + \ldots + d(x_{m+1}, x_m)
\]
\[
\leq [r^{n-1} + r^{n-2} + \ldots + r^m]d(x_1, x_0)
\]
\[
\leq \frac{r^m}{1-r}d(x_1, x_0)
\]
Let \(0 < c\) be given, choose a natural number \(N_1\) such that 
\[
\frac{r^m}{1-r}d(x_1, x_0) < c\quad \text{for all } m \geq N_1\] 

\(c\) implies \(d(x_n, x_m) < c\) for all \(m \geq N_1\) this implies \(d(x_n, x_m) < c\). For \(n > m\), \(\{x_n\}\) is a Cauchy sequence in \((X, d)\) is complete cone metric space, there exists \(p \in X\) such that \(x_n \to p\). Choose a natural number \(N_2\) such that 
\[
d(x_n, p) < \frac{c}{3}\quad \text{for all } n \geq N_2.
\]
Hence for \(n \geq N_2\) we have \(d(x_n, p) < \frac{c}{3}\)
\[
\psi(d(Tp, P)) \leq \psi(H(Tx_n, Tx_0) + d(Tx_n, p))
\]
\[
\leq F(\psi(r \max\{d(x_n, p), d(x_n, Tx_n), d(p, Tx_n)\}), \varphi(r \max\{d(x_n, p), d(x_n, Tx_n), d(p, Tx_n)\}))
\]
\[
\leq F(\psi(r \max\{d(x_n, x_{n+1}), d(p, x_{n+1})\}), \varphi(r \max\{d(x_n, x_{n+1}), d(p, x_{n+1})\}))
\]
\[
\leq F(\psi(rd(x_n, x_{n+1})), \varphi(rd(x_n, x_{n+1})))
\]
\[
\leq r^n d(x_1, x_0)
\]

For all \(n \geq N_2\), \(d(Tp, P) \leq \frac{c}{m}\) for all \(m \geq 1\), we get 
\[
c - d(Tp, P) \in P \quad \text{and } m \to \infty \quad \text{we get } \frac{c}{m} \to 0
\]

and \(P\) is closed \(-d(Tp, P) \in P\)
\[
\therefore d(Tp, P) = 0 \quad \text{and so } p \in Tp.
\]

**Corollary 2.2:** Let \((X, d)\) be a complete cone metric space and the mapping \(T : X \to CB(X)\) be multivalued map satisfy the condition
\[
H(Tx, Ty) \leq k d(x, y)
\]
For all \(x, y \in X\) and \(k \in [0, 1]\). \(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively, \(F \in C\) such that 
\[
\psi(t + s) \leq \psi(t) + \psi(s).
\]
Then \(T\) has a fixed point in \(X\).

**Proof:** The proof of the corollary immediately follows by taking \(d(x, y)\) as maximum value in previous theorem.

**Note 2.3:** We prove the above theorems in the setting of \(P\) is a normal cone with normal constant \(K\)

**Theorem 2.4:** Let \((X, d)\) be a complete cone metric space and \(P\) a normal cone with normal constant \(K\). Suppose the mapping \(T : X \to CB(X)\) be multivalued map satisfy the condition
\[
\psi(H(Tx, Ty)) \leq F(\psi(r \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(Tx, y)\}), \varphi(r \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(Tx, y)\}))
\]
For all \(x, y \in X\) and \(r \in [0, 1]\). \(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively, \(F \in C\) such that 
\[
\psi(t + s) \leq \psi(t) + \psi(s).
\]
Then \(T\) has a fixed point in \(X\).

**Proof:** for every \(x_0 \in X\) and \(n \geq 1\), \(x_1 \in Tx_0\) and \(x_{n+1} \in Tx_n\)
\[
\psi(d(x_{n+1}, x_n)) \leq \psi(H(Tx_n, Tx_{n-1})
\]
\[
\leq F(\psi(r \max\{d(x_n, x_{n-1}), d(x_n, Tx_n), d(x_{n-1}, Tx_{n-1})\}), \varphi(r \max\{d(x_n, x_{n-1}), d(x_n, Tx_n), d(x_{n-1}, Tx_{n-1})\}))
\]
\[
\leq F(\psi(r \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_{n+1})\}), \varphi(r \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_{n+1})\}))
\]
\[
\leq F(\psi(rd(x_n, x_{n-1})), \varphi(rd(x_n, x_{n-1})))
\]
\[
\leq r^n d(x_1, x_0)
\]
\[\leq \psi(r \max\{d(x_n, x_{n-1}), d(x_{n+1}, x_{n-1})\})\]

**Case (i)** If \(d(x_{n+1}, x_0) \leq rd(x_n, x_{n-1})\) then we get, \(d(x_{n+1}, x_n) \leq r^n d(x_1, x_0)\) for \(n > m\)

\[d(x_n, x_m) \leq \psi(r^{n-1} + r^{n-2} + \cdots + r^m) d(x_1, x_0)\]

\[\leq \frac{r^m}{(1-r)} d(x_1, x_0)\]

We get \(\|d(x_n, x_m)\| \leq \frac{r^m}{(1-r)} d(x_1, x_0)\). \(d(x_n, x_m) \to 0\) as \(n, m \to \infty\). Hence \(\{x_n\}\) is a Cauchy sequence. By the completeness of \(X\), there is \(p \in X\) such that \(x_n \to p\) as \(n \to \infty\)

\[\psi(d(Tp, P)) \leq \psi(H(Tx_n, P) + d(Tx_n, p))\]

\[F(\psi(r \max\{d(x_n, x_0), d(x_n, Tx_n), d(p,Tp), d(x_n, Tx_n), d(Tx_n, P)\} + d(x_{n+1}, p)),\]

\[\varphi(r \max\{d(x_n, x_0), d(x_n, Tx_n), d(p,Tp), d(x_n, Tx_n), d(Tx_n, P)\} + d(x_{n+1}, p))\]

\[\leq F(\psi(r \max\{d(x_n, x_{n+1}), d(p,Tp), d(x_n, Tx_n), d(x_{n+1}, p)\} + d(x_{n+1}, p)),\]

\[\varphi(r \max\{d(x_n, x_{n+1}), d(p,Tp), d(x_n, Tx_n), d(x_{n+1}, p)\} + d(x_{n+1}, p))\]

\[\leq F(\psi(rd(p, Tp)), \varphi(rd(p, Tp)))\]

\[d(Tp, P) = 0\). Hence \(P \in Tp\)

**Case (ii)** \(d(x_{n+1}, x_n) \leq rd(x_n+1, x_{n-1})\) then we get

\[d(x_{n+1}, x_n) \leq rd(x_n, x_{n-1}) + d(x_{n-1}, x_n)\]

\[\leq \frac{r}{1-r} [d(x_n, x_{n-1})]\]

\[h[d(x_n, x_{n-1})]\quad \text{where} \quad h = \frac{r}{1-r} < 1\]

For \(n > m\)

\[d(x_n, x_m) \leq d(x_n, x_{n-1}) + d(x_{n-1}, x_n)\]

\[\leq \frac{h^m}{(1-h)} d(x_1, x_0)\]

We get \(\|d(x_n, x_m)\| \leq \frac{h^m}{(1-h)} d(x_1, x_0)\). \(d(x_n, x_m) \to 0\) as \(n, m \to \infty\). Hence \(\{x_n\}\) is a Cauchy sequence. By the completeness of \(X\), there is \(p \in X\) such that \(x_n \to p\) as \(n \to \infty\)

\[\psi(d(Tp, P)) \leq \psi(H(Tx_n, P) + d(Tx_n, p))\]

\[F(\psi(r \max\{d(x_n, x_0), d(x_n, Tx_n), d(p,Tp), d(x_n, Tx_n), d(Tx_n, P)\} + d(x_{n+1}, p)),\]

\[\varphi(r \max\{d(x_n, x_0), d(x_n, Tx_n), d(p,Tp), d(x_n, Tx_n), d(Tx_n, P)\} + d(x_{n+1}, p))\]

\[\leq F(\psi(r \max\{d(x_n, x_{n+1}), d(p,Tp), d(x_n, Tx_n), d(x_{n+1}, p)\} + d(x_{n+1}, p)),\]

\[\varphi(r \max\{d(x_n, x_{n+1}), d(p,Tp), d(x_n, Tx_n), d(x_{n+1}, p)\} + d(x_{n+1}, p))\]

\[\leq F(\psi(rd(p, Tp)), \varphi(rd(p, Tp)))\]

\[d(Tp, P) = 0\). Hence \(P \in Tp\)

\[\psi(d(p, q)) = \psi(H(Tp, Tq))\]

\[\leq F(\psi(r \max\{d(x, y), d(p, Tp), d(q, Tq), d(p, Tp), d(Tq, Tp)\} + d(x, y)),\]

\[\varphi(r \max\{d(x, y), d(p, Tp), d(q, Tq), d(p, Tp), d(Tq, Tp)\} + d(x, y))\]

\[\leq F(\psi(rd(p, q)), \varphi(rd(p, q)))\]

\[\leq F(\psi(r \max\{d(p, q), d(p, q), d(p, q), d(p, q)\}),\]

\[\varphi(r \max\{d(p, q), d(p, q), d(p, q), d(p, q)\})\]

\[\leq F(\psi(rd(p, q)), \varphi(rd(p, q)))\]

This is contradiction and hence \(T\) has a unique fixed point in \(X\)

**Corollary 2.3:** Let \((X, d)\) be a complete cone metric space and \(P\) a normal cone with normal constant \(K\). Suppose the mapping \(T: X \to CB(X)\) be multivalued map satisfy the condition

\[\psi(H(Tx, Ty)) \leq F(\psi(r \max\{d(x, y), d(x, Tx), d(y, Ty)\}), \varphi(r \max\{d(x, y), d(x, Tx), d(y, Ty)\}))\]

For all \(x, y \in X\) and \(r \in [0, 1]\). \(\psi\) and \(\varphi\) are altering distance and ultra altering distance functions respectively. \(F \in C\) such that \(\psi(t + s) \leq \psi(t) + \psi(s)\). Then \(T\) has a fixed point in \(X\)

**Proof:** The proof of the corollary immediately follows since

\[\max\{d(x, y), d(x, Tx), d(y, Ty)\} \leq \max\{d(x, y), d(x, Tx), d(y, Ty), d(Tx, y)\}\]

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