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Research Article

Numerical Simulation of Mixed Convection in Three Sided Lid Driven Cavity

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Abstract—In this article mixed convection in an incompressible, 2D, steady state, Newtonian fluid flow in a three sided lid driven cavity is studied using finite volume method and Semi Implicit Method for Pressure Linked Equations (SIMPLE) technique. The top and bottom walls (North and South walls) move in same direction whereas the right vertical wall (East wall) moves in upward direction. The left vertical sidewall (West wall) is stationary. A detailed analysis of the fluid flow has been carried out for different values of density and viscosity of fluid for varied values of wall velocity. The results are displayed in terms of velocity profiles, pressure and temperature contours and in accord with the set of literature.

Keywords—Three sidedlid driven cavity, incompressible flow, mixed convection flow, SIMPLE algorithm.

1. Introduction

Flow inside a 2D lid driven cavity is a captivating research problem that has attracted investigators over the years. It is a popular classical problem of fluid dynamics which not only has simple geometry but also has wide variety of practical applications such as polymers, production of high grade paper, processingfood etc.

After the benchmark results on lid driven cavity problem published by Ghia et al. [1] and Schreiber and Keller [2], one-sided lid driven cavity flows has been significantly studied and researched[3,4,5,6,7] .

One-sided lid driven cavityproblem was extended byKuhlmann et al. to a two-sided problem, in which the flow is induced by two opposite walls moving tangentially in opposite directions[8].Alleborn et al furthered the study and investigated heat and mass transport in a 2D flow in a shallow lid driven cavity with a moving heated lid and a moving cooled lid[9].

Albensoeder et al adopted finite volume method and numerically studied 2D, steady incompressible flow in rectangular cavities with various cavity aspect ratios[10]. For same set of parameters they obtained seven solutions, when side walls are moving in opposite directions. Blohm and Kuhlmann experimentally investigated the flow of incompressible fluid in a rectangular cavity with two facing sidewalls moving in opposite directions for Reynolds numbers up to 1200[11].

Mixed convection problem of steady, 2D flow in a vertical two sided lid driven cavity with different temperatures of left and right walls was studied numerically by Oztop and Dagtekin[12] . They found that fluid flow and heat transfer in the cavity are affected by both Richardson number and direction of moving walls.Heat transfer in a two-sided lid-driven cavity with an aspect ratio of 1.96 was studied by Luo and Yang[13]. They presented a continuation method for calculating flow bifurcation and studied the flow for different temperature gradients.Numerical simulation of incompressible flow in two-sided and four-sided lid driven cavities was done byWahba [14]. He presented three possible solutions and also constructed flow bifurcation diagrams.By taking oscillating angular frequency of lid into consideration Zulhidayat Noor et al numerically studied the flow inside a square cavity with double-sided oscillating lids for different Reynolds number [15]. They found that at higher Reynolds number, the heat transfer along the lids is increased whereas increase in angular frequency of lid motion results in decreased heat transfer. Combined effects of the buoyancy force and mechanically driven lid within rectangular enclosures on fluid flow and heat transfer was investigated by Waheed[16]. He employedthe profiles for vertical and horizontal components of velocity, temperature, local heat fluxand hydrodynamic and thermal fields to present his results.

Bifurcations were studied numerically for two specific geometries in two-dimensional lid-driven cavity by Cadou et al [17]. In first geometry upper and the left side of the cavity were moving and in second geometry all the four sides had a prescribed motion.A continuation method was employed by Chen et al for calculation of flow bifurcation in a two-sided lid-driven cavity with different aspect ratios for opposite motion[18]. Hammami et al employed finite volume method to numerically investigate the effect of Reynoldsnumber ($100 \le \mathrm{Re} \le 1500$) on flow inside two-sided lid-driven cubical cavity induced by a cylindrical shape at the center[19]. They achieved a perfect bifurcation for the examined geometry and presented results by using velocity profiles, particle trajectories, Kinetic energy and isocontours of velocity.

Based on the above literature review, it can be concluded that numerical studies for solving problems based on either one sided or two sided lid driven cavity are extensively done by researchers. The objective of this study is to investigate mixed convection in three sided lid driven square cavities in which North, South and East walls are moving and West wall is adiabatic. Numerical simulations are performed using SIMPLE algorithm for varied values of density and viscosity of fluid taking different values of wall velocity. The results obtained for u and v velocity, pressure and temperature are shown through quiver plot and contour plots.The results obtained in the study provide a significant source to researchers for verification of their results. The MATLAB code employed in this study can also be used for solvingvarious fluid flow problems with justifiable results.

This article is organized as follows. In section 2, some recent related researches are being discussed. In section 3 description of problemispresented. In Section 4, the methodology to solve the problem is given along with the governing equations. In Section 5, results obtained are discussed. Finally, in Section 6conclusion of the research is given.

2. Related work

Several researches for studying three sided and four sided lid driven cavity problem have been done.
The flow of a Newtonian fluid in a three-sided lid-driven cavity was studied by Kamel et al. [20].They employed finite difference method and alternating direction implicit technique to study an incompressible, two-dimensional, time-dependent, flow in two cases.They deduced that increase inReynolds number give rise tomore secondary vortices near the corners of cavity and the primary vortex go nearer to the cavity center.Azzouz & Houatnumerically investigated the flow in a three-sided lid-driven cavity having aspect ratios as unity [21]. They analyzed 2D, steady, incompressible flow of fluid in detail using finite volume method and coupled algorithm to deal with the pressure–velocity coupling for Reynolds number as high as 5000.Chen et alstudied incompressible flow in a four-sided lid-driven square cavity and adopted continuation method to investigate the flow field states and flow bifurcation [22]. By taking Reynolds number of the top and bottom plates as 250 and changing the driving velocity of two side plates, they located the existence of two non-crossing branches of solution in the flow bifurcation.Li and Maaemployed a multiple relaxation time lattice Boltzmann equation to study the transverse aspect ratio for a three-dimensional, incompressible flow in a four-sided lid driven cavity [23]. Using 2D numerical models they found that to replicate flow patterns, both symmetric and asymmetric flows, an aspect ratio greater than 5 is required.

3. Problem Statement

Consider2D, steady and incompressible laminar flow of Newtonian fluid in a square cavity of unit height and unit length. Three walls of the cavity, North, South and East walls are allowed to move in same plane at a constant same speed and the West wall have no-slip condition. The North and Southmoving walls of the cavity are kept at same temperature $T_h$whereas East wall is at lower temperature T_c. The West wall is adiabatic. All fluid properties are taken to be constant, excluding the density in the buoyancy term. We also assume density to vary linearly with temperature as $(\rho - \rho_c) = g\beta(T - T_c)$, where g, β, ρ are acceleration due to gravity, coefficient of thermal expansion and fluid density respectively. Viscous dissipation is also assumed to be neglected.

4. Methodology

The governing conservations equations in Cartesian coordinates for the model are Continuity equation (1), Navier Stokes Equations (2)and (3) and Energy Equation (4), which aregiven by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta\left(T-T_c\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where
u and v: x- and y components of fluid velocity
p: Pressure,
T: Temperature
g: Acceleration due to gravity
β: Volumetric coefficient of thermal expansion
υ : Kinematic viscosity
ρ: Density of the fluid and
α: Thermal diffusivity.
We introduce the Dimensionaless quantities

$$X=\frac{x}{L}\ ,\ Y=\frac{y}{L}\ , U=\frac{u}{U_0}\ ,\ V=\frac{v}{U_0}, \qquad \theta=\frac{T-T_c}{T_h-T_c},\ \ \mathrm{P}=\frac{p}{\rho U_0{}^2} \tag{5}$$

Where, L is characteristic length, U_0 is freestream velocity. Then, non dimensionalised Eqn. (1)-(4) are

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (6)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (7)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Gr}{\mathrm{Re}^2}\theta \qquad (8)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (9)$$

Where *Re*, *Gr* and *Pr* defined as

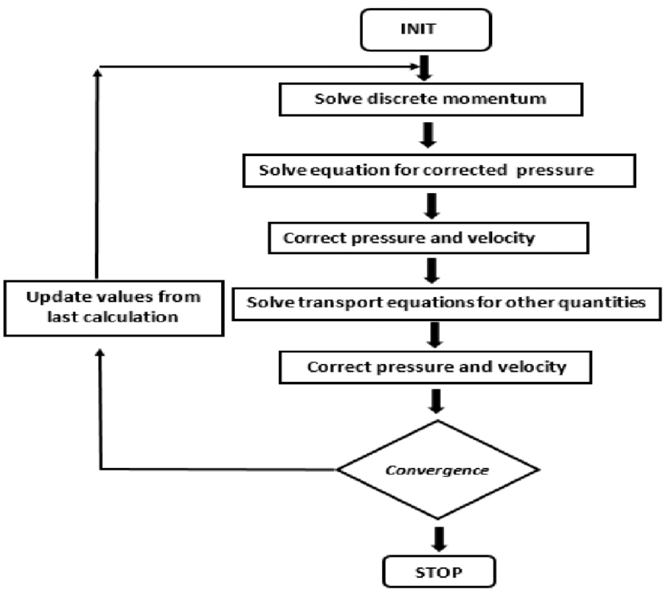
$$\mathrm{Re}=\frac{U_0L}{\upsilon}3\ ,\ Gr=\frac{g\beta\Delta TL^3}{\upsilon^2},\ \mathrm{Pr}=\frac{\upsilon}{\alpha}$$

are non- dimensionalised Reynolds number, Grashof number and Prandtl number respectively. The initial and boundary conditions for non dimensionalised system of equations are

$$\begin{aligned}
&U=1\ ,V=0,\ \theta=1 && 0\le X\le 1,Y=1\\
&U=1\ ,V=0,\ \theta=1 && 0\le X\le 1,Y=0\\
&U=0\ ,V=1,\ \theta=0.5 && 0\le Y\le 1,X=1\\
&U=V=0\ ,\ \frac{\partial \theta}{\partial X}=0 && 0\le Y\le 1,X=0
\end{aligned} \qquad (10)$$

For the application of the SIMPLE algorithm, the computational domain/ cavity is divided uniformly into 30×30 grid nodes usingthe staggered grid arrangement.Using finite volume method, the nonlinear governing partial differential equations in dimensionless form are converted into a system of discretised equations.These discretised equations along with pressure correction equation are solved using SIMPLE algorithm to obtain the velocities, pressure and temperature at all node points of the grid.

The SIMPLE algorithm was originally put forward by Patankar and Spalding [24]. It is a guess and correct procedure for the calculation of pressure and velocities. The flowchart of the algorithm [25] is as follows:



5. Results and Discussion

The governing equations are discretised using central difference scheme of second order. Numerical solution of the discretised system of equation isdone to obtain unknown variables u, v, p and T for various values of ρ and µ for different values of fluid velocities. MATLAB programming language is employed to implement SIMPLE algorithm. Various values of under relaxation factors for u velocity and v velocity and pressure were tested for all cases during the process.

The study is done separately for a set of smaller values density of fluid and a set of higher values of density of fluid. The results obtained for both studies are as follows.

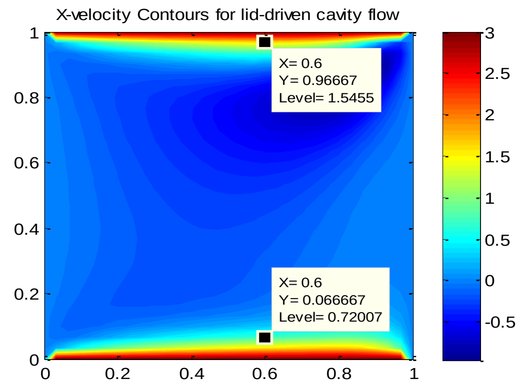


Fig 1(a)

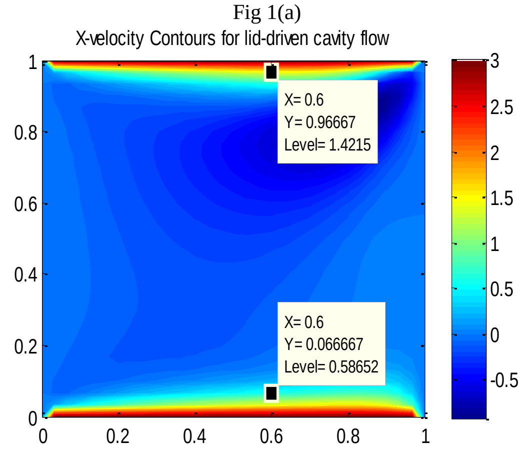


Fig 1(b)

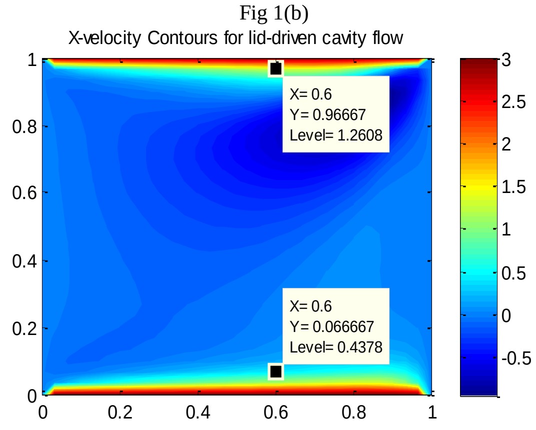


Fig 1(c)

Figure 1(a)-1 (c): u velocity for ρ=20, 30, 40

4.1 For smaller values of density

In Figure 1(a)-1(c) we can see that increase in density results in decreasing values of u velocity near the boundaries of cavity. In Figure 2(a)-2(c) we can see that with increase in density results in decrease in v velocity near East wall of cavity and increase in v velocity near West wall of the cavity.

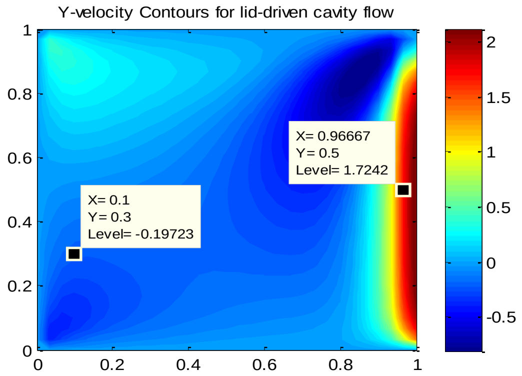


Fig 2(a)

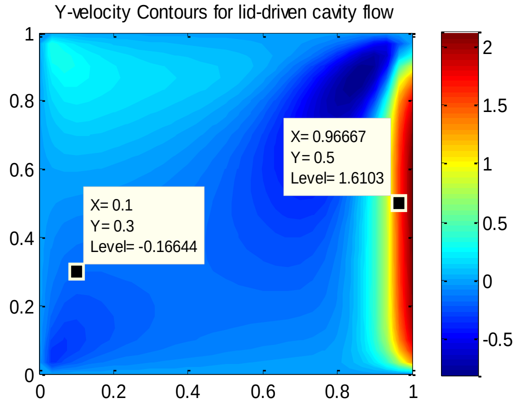


Fig 2(b)

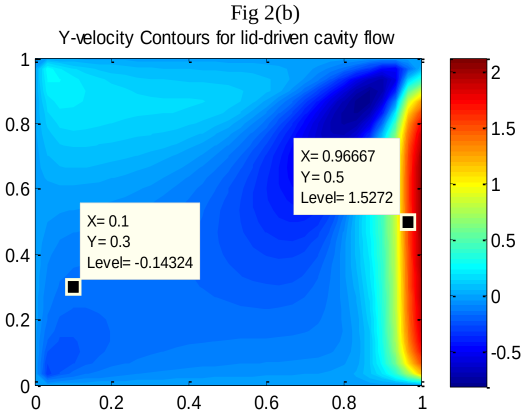


Fig 2(c)

Figure 2(a)- 2(c): v velocity for ρ=20, 30, 40

Figure3(a)-3(c) shows that as density increases pressure at North East corner and North West corner of cavity increases rapidly, as expected. It was found that, if we increase the viscosity of fluid, keeping all other variables same, the pattern of change in pressure changes with increase in density. The pressure increases at North East corner with increase in density, but decreases at South West corner of cavity. In 4(a)-4(c) it can be seen that temperature changes slightly near the boundary of the cavity.

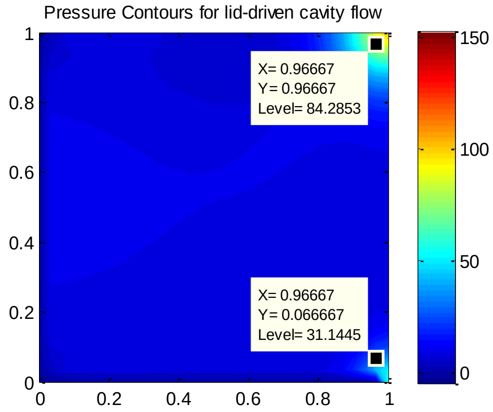


Fig 3(a)

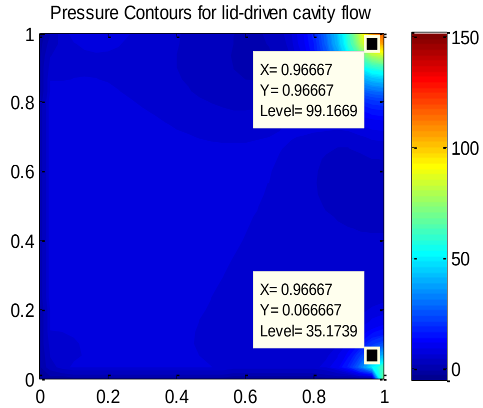


Fig 3(b)

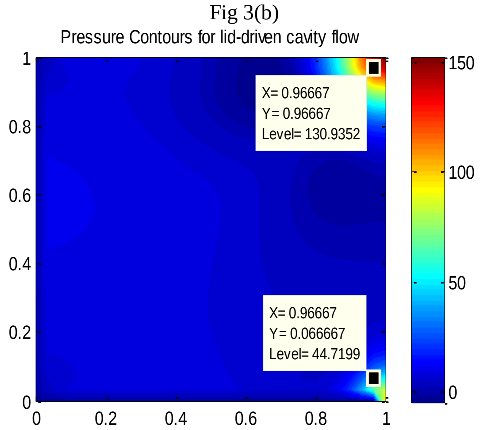


Fig 3(c)

Figure 3(a)-3(c): Pressure for ρ=20, 30, 40

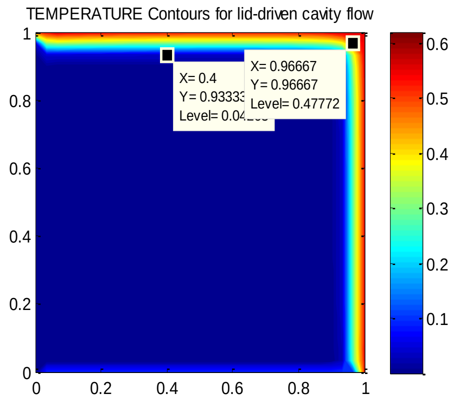


Fig. 4(a)

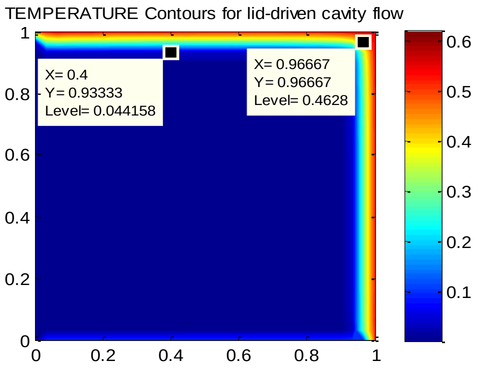


Fig 4(b)

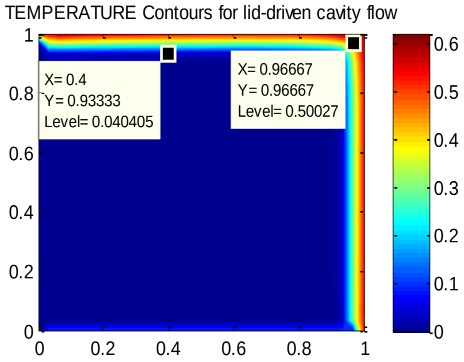


Fig 4(c)

Figure 4(a)-4(c): Temperature for ρ=20, 30, 40

In Figure5(a)-5(c) we can see that as the values of viscosity increases, the u velocity decreases inside the cavity, but increases near the North and South walls. As the viscosity increases, v velocity decreases everywhere except near the East wall and North West corner. The v velocity also decreases rapidly at South West corner. This can be seen in Figure 6(a) and 6(b).

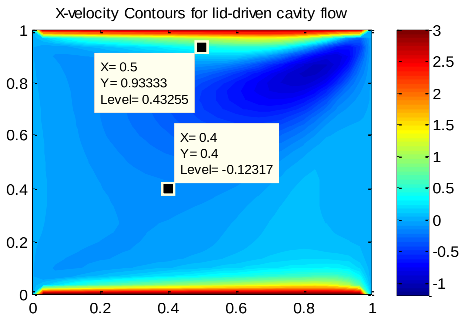


Fig. 5(a)

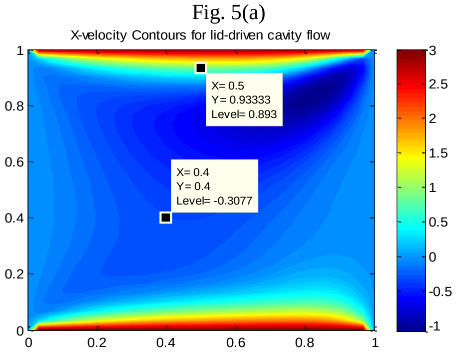


Fig. 5(b)

Figure5(a) -5(b): u velocity for μ=0.1, 0.5

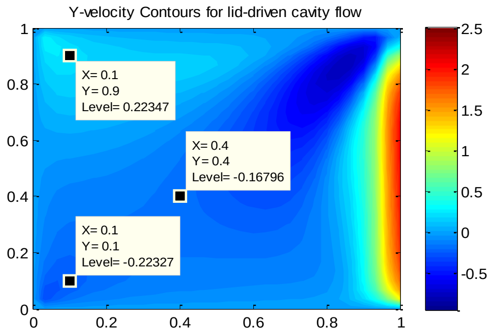


Fig 6(a)

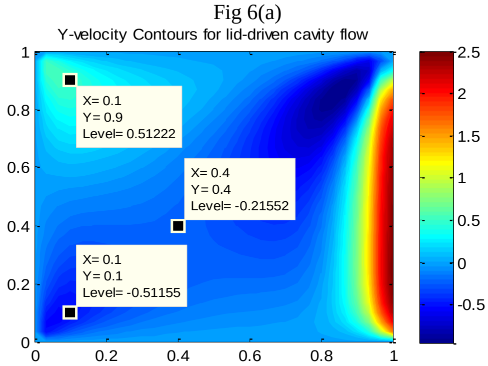


Fig 6(b)

Figure6(a) -6(b) : v velocity for μ=0.1, 0.5

In Figure7(a)-7(b) it is seen that the pressure inside the cavity increases drastically, with increase in viscosity of fluid. In Figure8(a)-8(b)we see that the temperature inside the cavity increases inside the cavity, but decreases near North and East walls of cavity as the viscosity increases

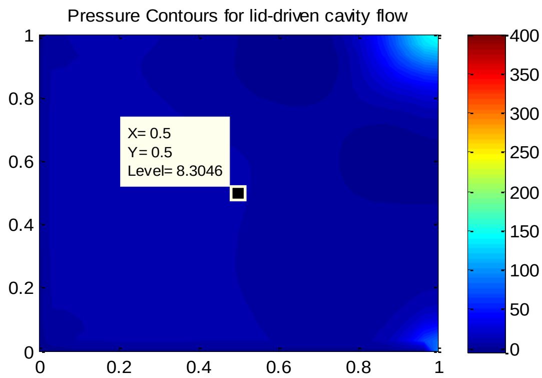


Fig 7(a)

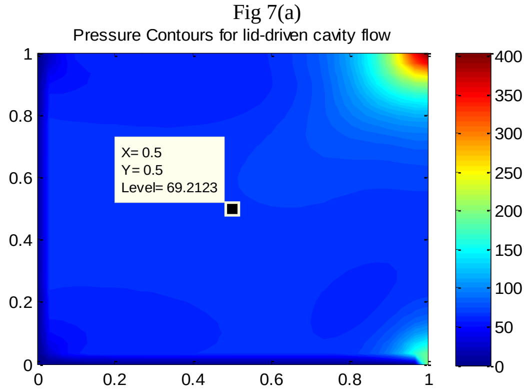


Fig 7(b)

Figure7(a)-7(b): Pressure for μ=0.1, 0.5

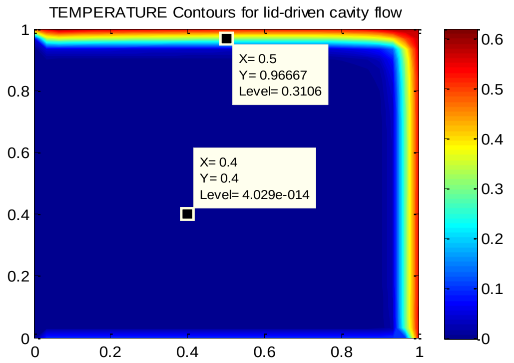


Fig 8(a)

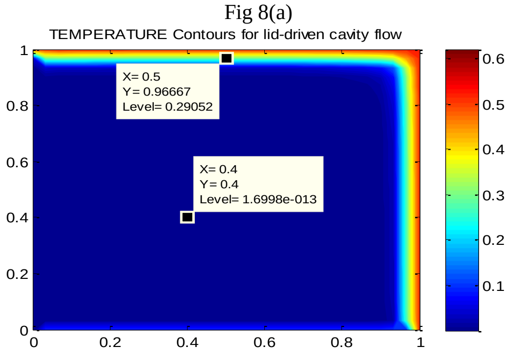


Fig 8(b)

Figure8(a)-8(b): Temperature for μ=0.1, 0.5

4.2 For higher values of density

In case of higher values of density, increase in density of fluid does not remarkably change u velocity and v velocity. But as density increases, the pressure increases near North East corner of cavity, whereas decreases in interior of cavity. This can be seen in Figure 9(a)-9(b).

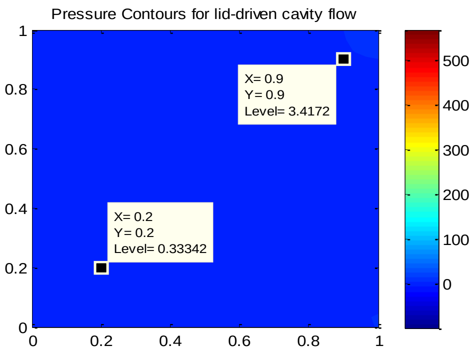


Fig 9(a)

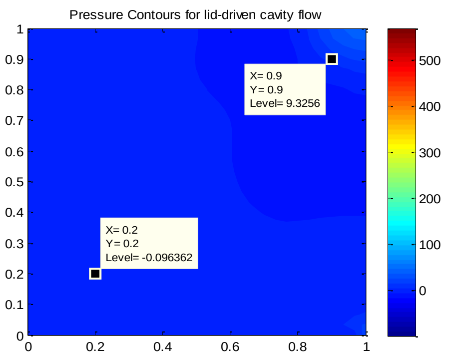


Fig 9(b)

Figure9(a)-9(b): Pressure for ρ=100, 1000

In Figure10(a)-10(b), it is seen that as density increases, the temperature near North and East wall decreases whereas increases up to double near South East corner of cavity.

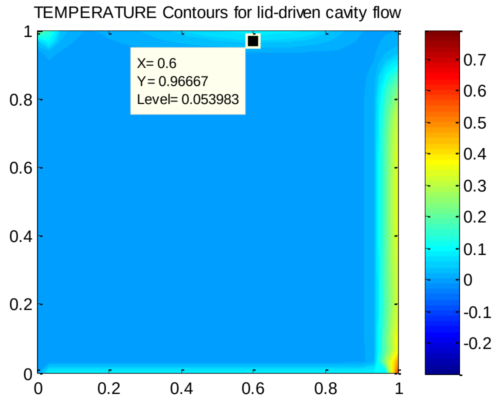


Fig 10(a)

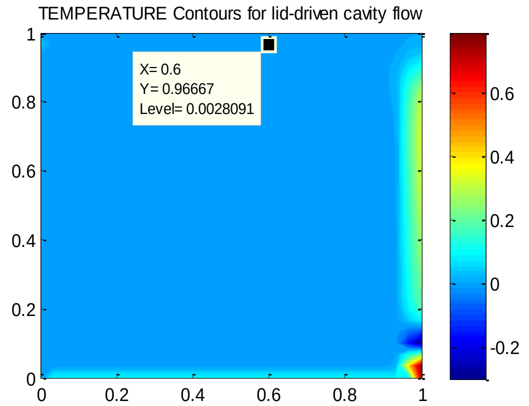


Fig 10(b)

Figue10(a) -10(b): Temperature for ρ=100,1000

If we increase the value of viscosity, keeping other variables same as in case of Figure 9 and 10, the increase in density increases u velocity everywhere except near North and South walls,where it is decreased. This can be seen in Figure 11(a) and 11(b).

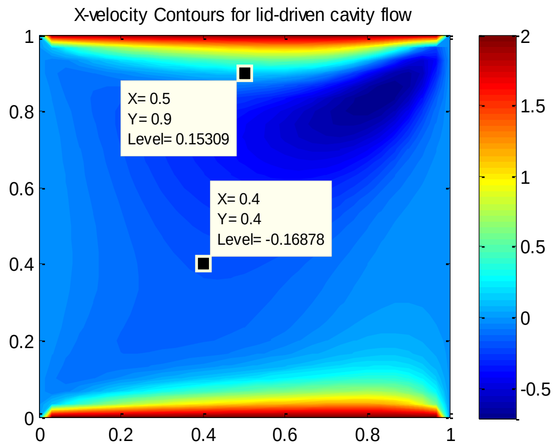


Fig 11(a)

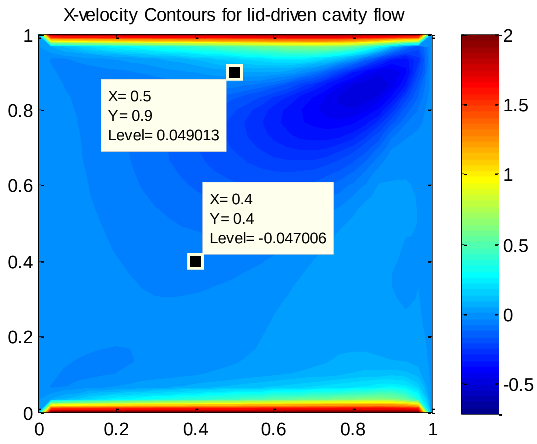


Fig 11(b)

Figure11(a) -11(b): u velocity for ρ=100, 1000

In Figure12(a) -12(b) we can see that increase in density decreases v velocity near North East wall except at North East and South West corners. In Figure 13(a) -13(b) the increase in pressure at North East corner, which is almost doubled with increase in density can be seen. Also the pressure at South East corner is increased.

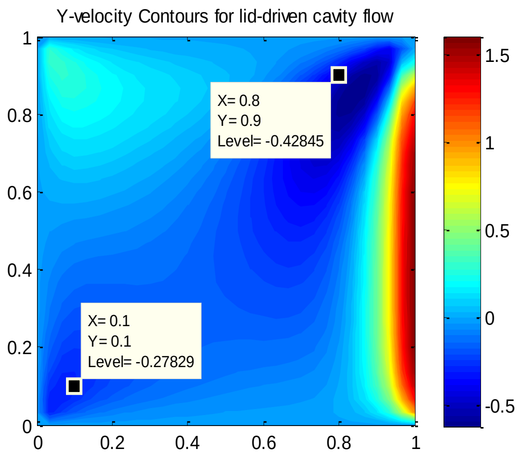


Fig 12(a)

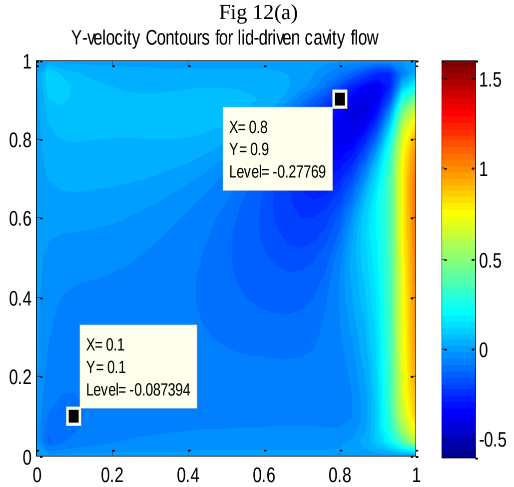


Fig 12(b)

Figure 12(a) -12(b) : v velocity for ρ=10, 1000

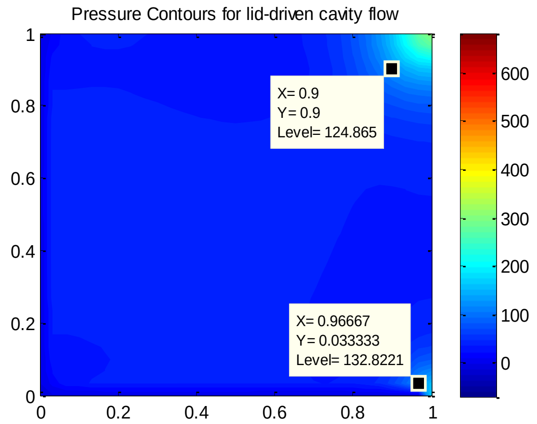


Fig 13(a)

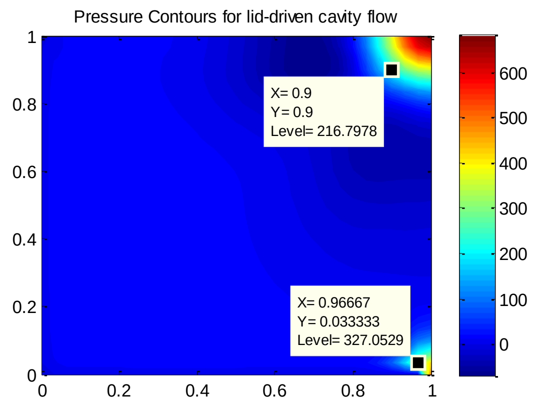


Fig 13(b)

Figure 13(a)-13(b): Pressure for ρ=10, 1000

In Figure 14(b)-14(b) we can see that the temperature everywhere in the cavity changes drastically with increase in density of fluid. Figure 15(a) and 15(b) shows the change in u velocity with change in viscosity. We can see as viscosity increases u velocity decreases everywhere except North and South walls of the cavity, where the u velocity is increasing with increase in viscosity.

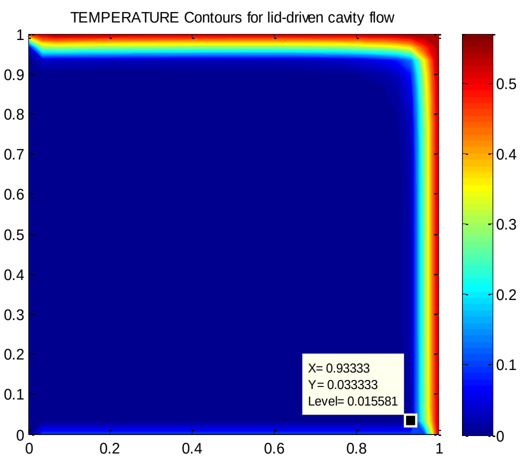


Fig 14(a)

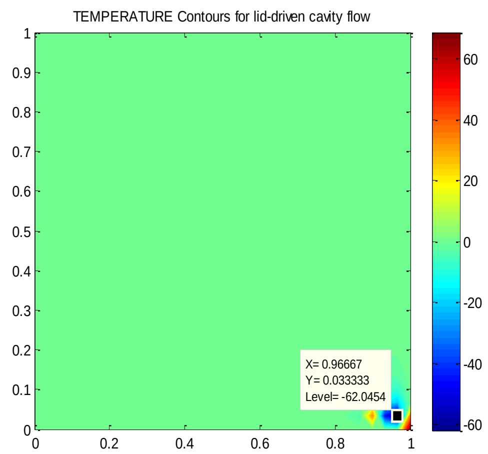


Fig 14(b)

Figure14(a)-14(b): Temperature for ρ=100,1000

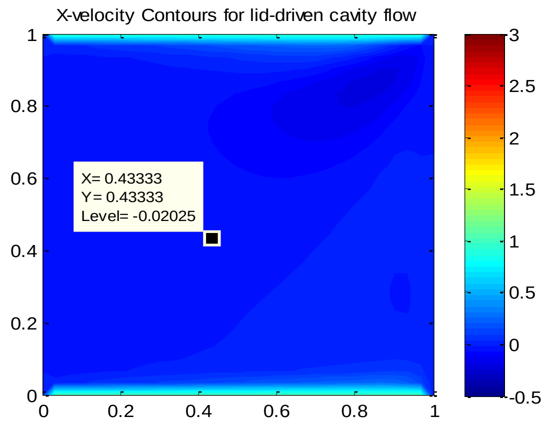


Fig 15(a)

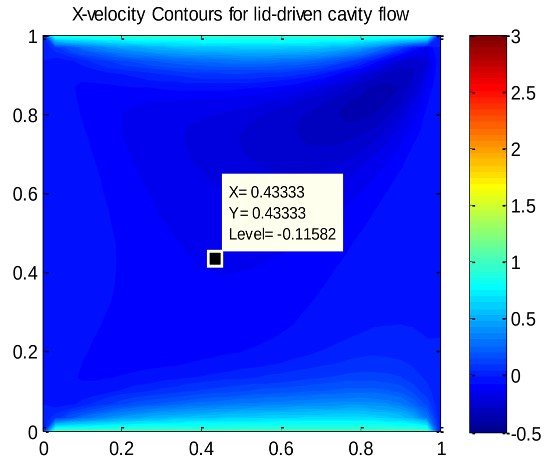


Fig 15(b)

Figure 15(b)-15(b): u velocity for μ=0.01, 0.5

In Figure 16(a)-16(b), we can see that as viscosity increases, v velocity decreases except at East wall and North West corner of cavity. In Figure17(a) and 17(b), we can see that as viscosity increases, pressure increases rapidly at all points of cavity. In Figure 18a and 18b, it can be seen that temperature increases drastically at all point of cavity especially near East and North boundaries.

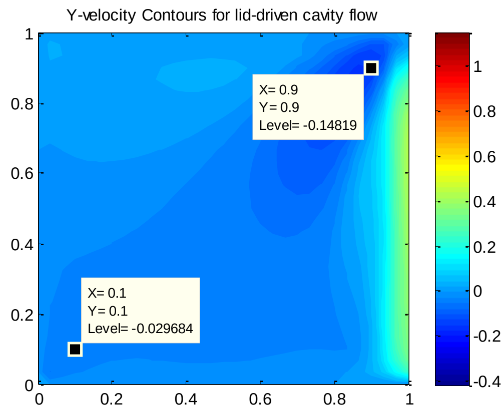


Fig 16(a)

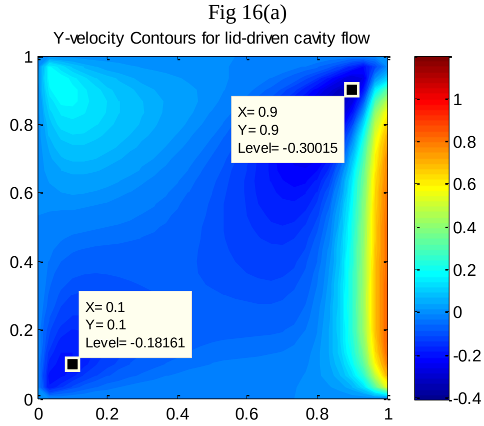


Fig 16(b)

Figure 16(a)-16(b): v velocity for μ=0.01, 0.5

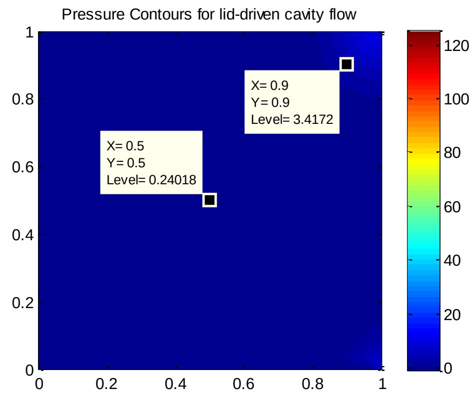


Fig 17(a)

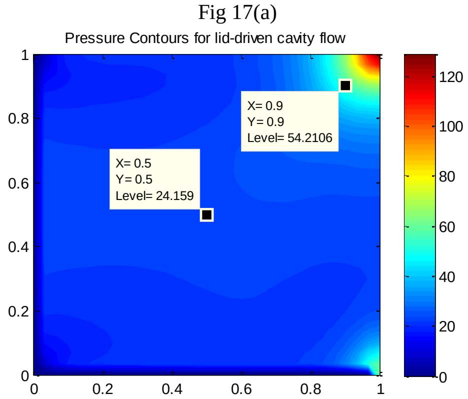


Fig 17(b)

Figure 17(a) -17(b): Pressure for μ=0.01, 0.5

It is also noted that if density of the fluid increases, the increase in viscosity results in decrease in temperature everywhere in cavity especially near South East corner of the cavity.

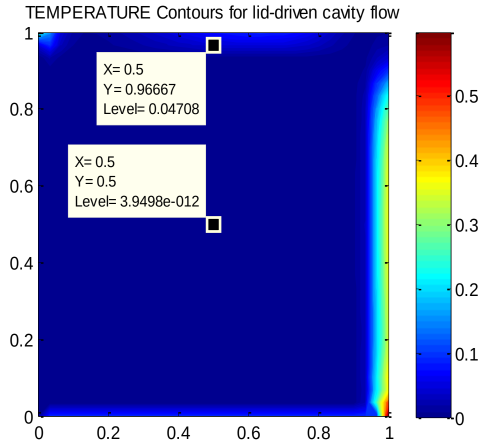


Fig 18(a)

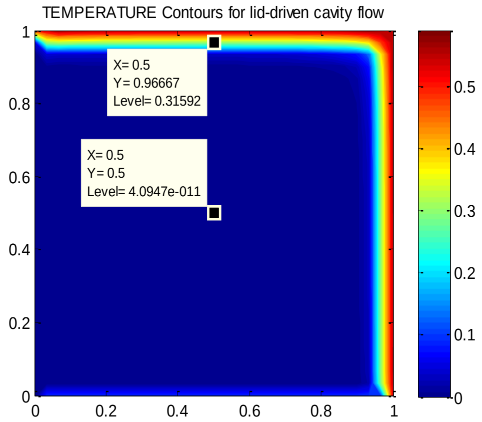


Fig 18(b)

Figure18(a)-18(b): Temperature for μ=0.01, 0.5

It is observed from Figure19(a) -19(c) that increase in wall velocity, increases u velocity near walls of the cavity as expected, whereas start to decrease towards the North East corner of cavity giving rise to primary vortex. Also, v velocity near North East corner decreases. This can be seen in Figure20(a)-20(b)

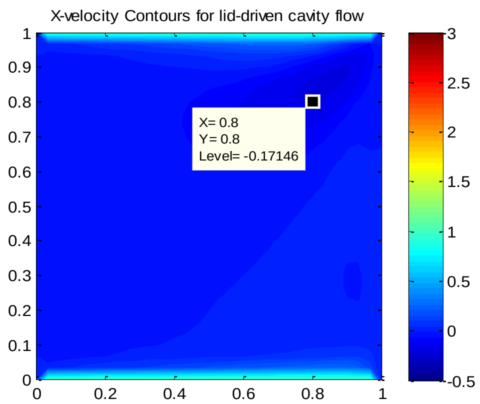


Fig 19(a)

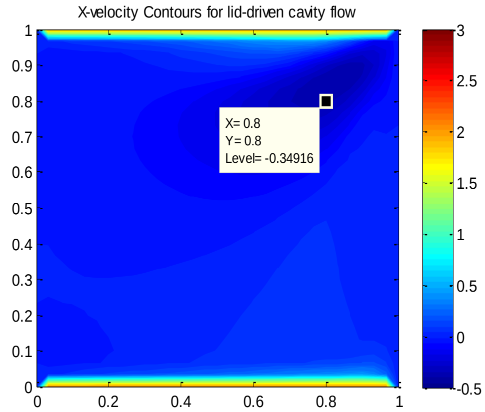


Fig 19(b)

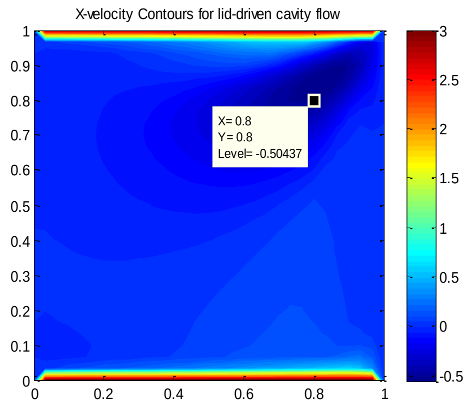


Fig 19(c)

Figure19(a)-19(c): u velocity for wall velocity=1, 2, 3.

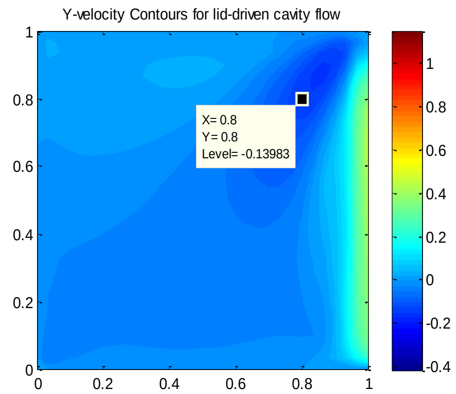


Fig 20(a)

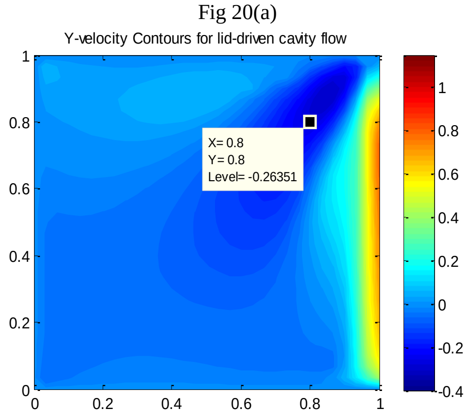


Fig 20(b)

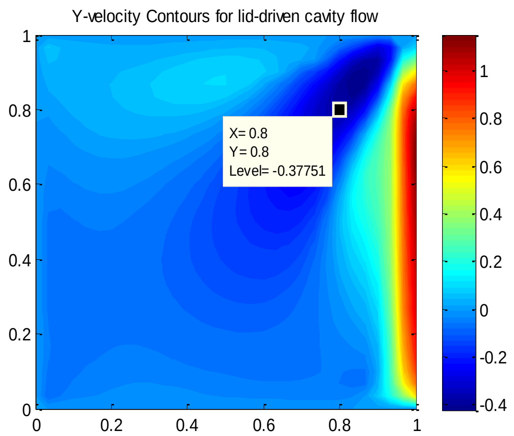


Fig 20(c)

Figure20(a)-20(c): v velocity for wall velocity=1, 2, 3.

In Figure 21(a)-21(c), it is observed that as wall velocity increases, pressure in North East corner and South East corner increases. Figure22(a)-22(c) show that as the wall velocity increases, the heat transfer shifts from towards East to towards North and then West wall of the cavity.

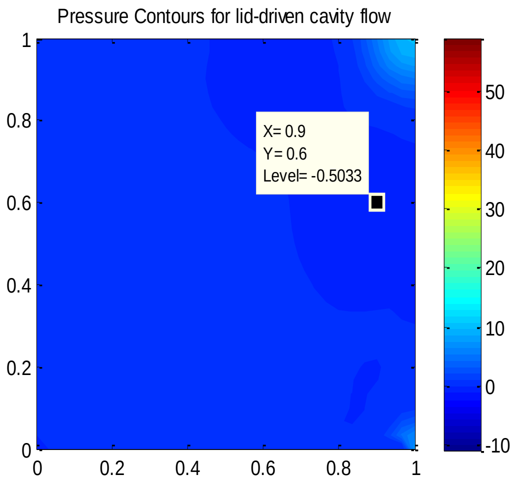


Fig 21(c)

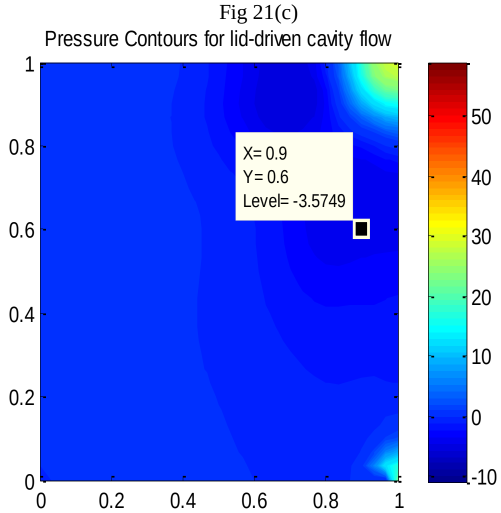


Fig 21(b)

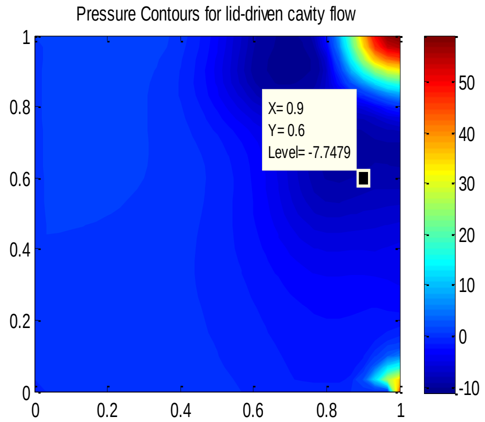


Fig 21(c)

Figure21(a)-21(c): Pressure for wall velocity=1, 2, 3.

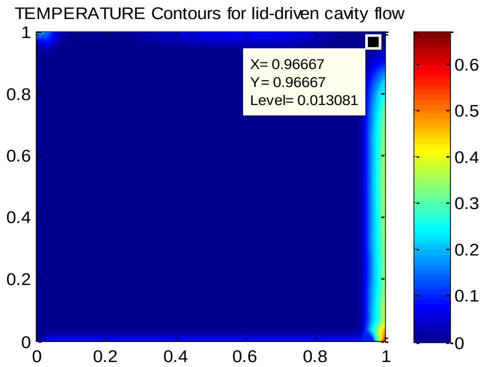


Fig 22(a)

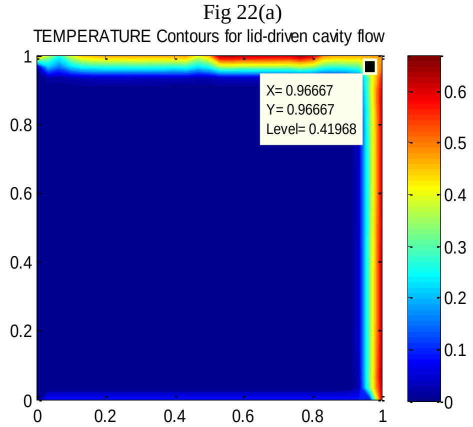


Fig 22(b)

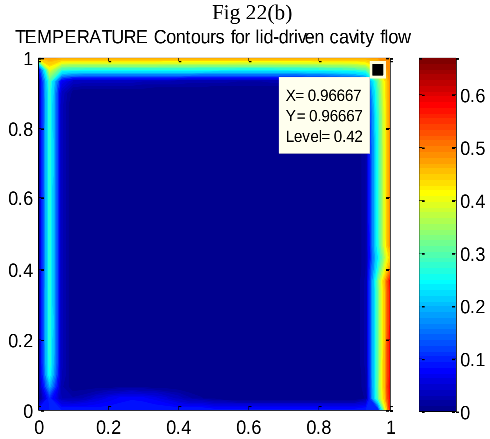


Fig 22(c)

Figure22(a)-22(c): Temperature for wall velocity=1, 2, 3.

It is interesting to see the change in velocity profile of the flow with increase in viscosity of the fluid. The decrease in u velocity and v velocity with increase in viscosity can be clearly seen in Figure23(a) and 23(b).

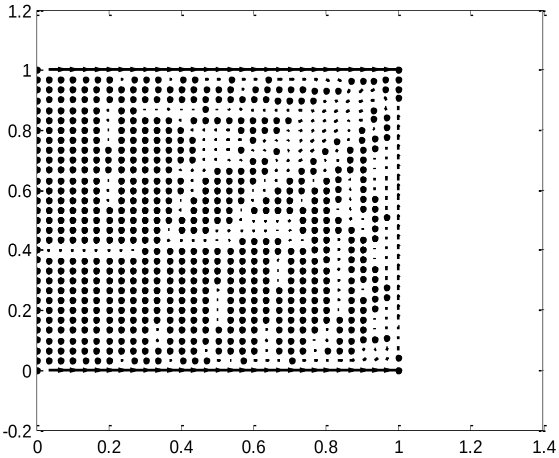


Fig 23(a)

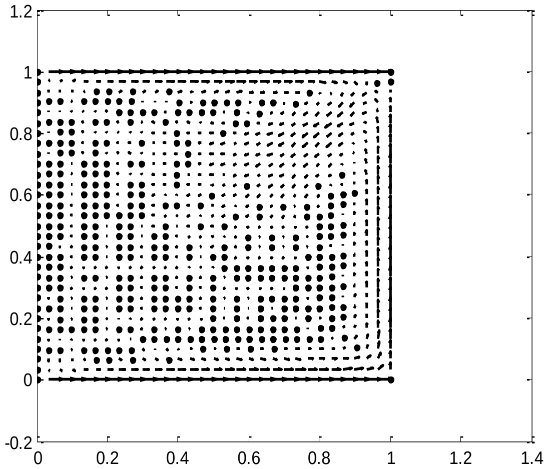


Fig 23(b)

Figure23(a)-2(b): velocity profile for μ=0.01, 0.5.

6. Conclusion and future scope

Numerical simulation of steady, 2-D incompressible flow through a three sided lid driven cavity is presented in this paper. The equations of motion governing the flow are solved for u velocity, v velocity, pressure and temperature using SIMPLE algorithm. The numerical computations were conducted using staggered grid system. Convergence of solutions were obtained for only few values of under relaxation factor for pressure. Results obtained from the study are as follows:

1. It was seen that the behaviors of u velocity and v velocity are different for smaller and larger values of density. For smaller values of densities, increase in density decreases both u velocity and v velocity. For larger values of densities, increase in density of fluid brings no remarkable change in u velocity or v velocity. However, if we increase the viscosity of fluid, then u velocity increases with increase in density of fluid. In this case, the v velocity decreases near North East wall of the cavity

2. With increase in density of fluid, the pressure decreases in cavity but increases in North East corner and North West corner of cavity. This holds for both lower and higher values of density. However, if viscosity of fluid is also increased, the pattern change as the pressure now decreases in South West corner.

3. Temperature near North and East walls of the cavity decreases with increase in density of fluid, whereas the temperature increases to nearly double near South East corner of the cavity. This holds for higher values of density. However, for smaller values of densities, there is no remarkable change in temperature inside the cavity with increase in density of fluid.

4. With increase in viscosity of fluid, both u velocity and v velocity decreases in the cavity. This holds for lower as well as higher values of densities. However, for larger values of densities, u velocity increases near North and South walls and v velocity increases near East wall and North West corner, with increase in viscosity.

5. With increase in viscosity of fluid, both temperature and pressure increases drastically in the cavity. This holds for both lower and higher values of densities.

By reviewing the results obtained, we can imply that the behavior of u and v velocity, pressure inside the cavity and temperature inside the cavity are greatly affected by the density as well as viscosity of fluid in the cavity. These results may be used for further studies which may include factors like compressible fluids, nanofluids, magnetic field etc.

Data availability

There is no data to provide other than that given in the Article. The MATLAB code used in research can be obtained from author on request.

Conflict of interest

Author declare no conflict of interest

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Authors' Contributions

The author has contributed for complete articleand nobody else has any role in its preparation. The article is prepared from author's own knowledge and research.

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