An EPQ Model under Constant Amelioration, Different Deteriorations with Exponential Demand Rate and Completely Backlogged Shortages

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Abstract: In the present paper, we have developed an economic production quantity inventory model for ameliorating and deteriorating items. At the inception of production activity, the constant amelioration, two parameter Weibullly distributed deterioration rate and exponential demand rate have been considered. At the conclusion stage of production activity, deterioration of items has been assumed to follow Pareto Type-I distribution with the same amelioration and demand rate as at the inception stage. The time dependent inventory holding cost is assumed to be a linear function of time. Shortages are acceptable and backlogged completely. The aim of this study is to find the optimal solution for minimizing the total inventory cost. The model, a numerical illustration has been presented along with a sensitivity analysis for studying the impact of parameters on assessment variables and total cost of the model.

Keywords: Weibull distribution, Pareto type – I distribution, Amelioration rate

I. INTRODUCTION

Several published theses have addressed the importance of the deterioration phenomenon in field applications; as a result, many deterioration models have been consequently developed. Yet we have not observed much appreciation of ameliorating consideration. Due to lack of consideration for the influence of demand, the ameliorating items assuming duration for the amount of inventory will gradually increase; meanwhile, in the traditional inventory model dealing with deteriorating items, the amount of inventory will gradually decrease. Among the published literatures, scholars and researchers have not pay much attention to the ameliorating problems and items. To cope with this deficiency, lately a few studies are concerned with the problems of amelioration, because they do exist in the real world such as the farming, fishery, and poultry industries. The fast growing animals like ducks, pigs, and broilers in poultry farms, highbred fishes in ponds, and the cultivation of vegetables and fruits in farms are typical field applications. This is quite different from the deteriorating items and deserves a comprehensive study. Weibull distribution has been used to address the problems of product life cycle in recent years due to its capacity to effectively describe various product life spans by imbibing the variations in parameters’ value. Inventory models for ameliorating and deteriorating items are applicable when the effects of deterioration and amelioration occur simultaneously. Such inventory models are for items like fruits, vegetables, flowers and some dairy products where the combined effect of amelioration and deterioration is observed.

Hwang [5] is likely to have initiated the study of inventory systems comprising both ameliorating and deteriorating items where he assumed that items ameliorate at a breeding yard, such as aquaculture facilities and deteriorate at distribution centres. Law and Wee [14] have studied the EPQ model with ameliorating and deteriorating items by allowing shortages along with time discounting and integrating the manufacturer-retail co-operation. L.Tadj et al.[8] have also done a similar study on production inventory model with both ameliorating and deteriorating items.

Lan, Yu, Lin, Tung, Yen and Deng [3] have depicted a study on an enriched algebraic method for the EPQ model with stochastic lead time. Shamsi, Haji, Shadrokh and Nourbakhsh [13] published their study on EPQ in reworkable production systems including inspection errors, scraps and backlogging. Jain, Sharma and Rathore [9] have presented an EPQ model with involvement of shortage,
price and stock dependent demand for deteriorating items. Integrated inventory models for decaying items with exponential demand under inflation have been given by Bansal and Ahalawat [6]. Kawale and Bansode [7] have published an EPQ model using Weibull distribution for deterioration items along with time varying holding cost. Gothi, Shah and Khatri [16] recently developed two warehouses inventory model for deteriorating items with power demand and time varying holding cost where shortages are permissible and are a mixture of partial backlog and lost sales. Parmar and Gothi [11] have developed an EPQ model for deteriorating items assuming three parameter Weibull distribution with constant production rate and time-varying holding cost. Also Kirtan Parmar and U.B. Gothi [10] have presented an EPQ model for deteriorating items under three parameter Weibull distribution and time dependent inventory holding cost with shortages. Mishra, Raju, U.K. Misra and G. Misra [15] have presented their study on optimal control of an inventory system consisting of variable demand and ameliorating/deteriorating demand. Inventory Models for both ameliorating and deteriorating items have also been presented by Ankit Bhojak and U. B. Gothi [1] and [2]. Pooja D. Khatri and U.B.Gothi [12] have recently published an inventory system for deteriorating items having power demand pattern and time-dependent inventory holding cost. An EPQ model for deteriorating items following two parameter Weibull distribution along with exponential demand rate has been presented by Devyani Chatterji and U. B. Gothi [4].

In the present paper, we have redeveloped the above inventory model by considering an EPQ model with constant amelioration rate and demand following exponential pattern. For different time periods, Weibull and Pareto Type-I distribution with two parameters are considered for the deterioration rate. We employ Weibull distribution because it is widely used in reliability and survival analysis and Pareto Type-I Distribution because it helps test severity of large casualty losses for certain lines of business. In this model shortages are allowed to occur and they are completely backlogged. In this model inventory holding cost is time dependent and a linear function of time while production rate is more than demand rate. Numerical illustration and sensitivity analysis presented here have been carried out by varying the parameter values one after the other.

II. ASSUMPTIONS

For development of the model, below mentioned assumptions have been considered:

1. The inventory system comprises of only one item and one stocking point.
2. Replenishment rate for the item is infinite and there is no lead time.
3. As soon as a unit is produced, it is available to satisfy the demand.
4. Holding cost is a linear function of time and it is \( C_h = h + rt \) \((h, r > 0)\).
5. Item deteriorates or ameliorates only when it is effectively in stock.
6. The annual demand rate is an exponential function of time and it is \( R(t) = \alpha e^{\beta t} \).
7. Amelioration rate \( \theta(t) = \theta_1 \) is constant which is derived from exponential distribution.
8. The deterioration rate is given by
\[
\theta(t) = \begin{cases} 
\alpha \beta t^{\alpha-1} & ; 0 \leq t \leq \mu \\
\theta & ; \mu \leq t \leq t_1 
\end{cases}
\]

where \( \alpha \) is scale parameter \((0 < \alpha \ll 1), \beta \) is shape parameter \((\beta > 0), \theta \) and \( \mu \) are parameters of Pareto type-I distribution taking positive real values.
9. Shortages are allowed and they are fully backlogged.
10. The second and higher power of \( \alpha \) and \( \theta \) are neglected in the analysis of the derived model.
11. Per unit cost for amelioration, deterioration, production, ordering and shortage are known and constant.
12. Total cost for the inventory is a real and continuous function, bowed to the origin.

III. NOTATIONS

The following notations are used to develop the model:

1. \( Q(t) \): Instantaneous rate of the Inventory level at any time \( t(0 \leq t \leq T) \).
2. \( R(t) \): Demand rate varying over time.
3. \( \theta(t) \): Deterioration rate.
4. \( A(t) \): Amelioration rate.
5. \( p \): Production rate.
The differential equations describing the instantaneous state of \( Q(t) \) over the period \([0, T]\) are given by

\[
\frac{dQ(t)}{dt} + \alpha \beta e^{\alpha t}Q(t) = \theta Q(t) + p - ae^{\alpha t} \quad 0 \leq t \leq \mu \tag{1}
\]

\[
\frac{dQ(t)}{dt} + \theta Q(t) = \theta Q(t) - ae^{\alpha t} \quad \mu \leq t \leq t_1 \tag{2}
\]

\[
\frac{dQ(t)}{dt} = -ae^{\alpha t} \quad t_1 \leq t \leq t_2 \tag{3}
\]

\[
\frac{dQ(t)}{dt} = p - ae^{\alpha t} \quad t_2 \leq t \leq T \tag{4}
\]

Using the boundary conditions \( Q(\mu) = S_1 \) and \( Q(t_2) = -S_2 \) the solutions of above four equations are given by

\[
Q(t) = (p - a)t + \{(p - a)\theta_1 + a\lambda\} \frac{t^2}{2} - (p - a)\alpha \beta \mu^{\alpha t+1} \tag{5}
\]

\[
Q(t) = -a \left[ \frac{1}{\theta + 1} (t - \mu)i^{\alpha i} \right] + \frac{\lambda - \theta_1}{\theta + 2} (i^2 - \mu^{\alpha i}) + \frac{\lambda - \theta_2}{\theta + 2} \theta_1 (i^2 - \mu^{\alpha i}) \tag{6}
\]

\[
Q(t) = -a \left[ (t - t_1) + \frac{\lambda}{2} (t^2 - t_1^2) \right] \tag{7}
\]

\[
Q(t) = (p - a)(t - T) - \frac{a\lambda}{2} (t^2 - T^2) \tag{8}
\]

Substituting \( Q(\mu) = S_1 \) in equation (5), we get

\[
S_1 = (p - a)\mu + \{(p - a)\theta_1 + a\lambda\} \frac{T^2}{2} - (p - a)\alpha \beta \mu^{\alpha t+1} \tag{9}
\]

Substituting \( Q(t_2) = -S_2 \) in equation (7) and (8), we get

\[
S_2 = a \left[ (t_2 - t_1) + \frac{\lambda}{2} (t_2^2 - t_1^2) \right] \tag{10}
\]

\[
S_2 = (p - a)(T - t_2) - \frac{a\lambda}{2} (T^2 - t_2^2) \tag{11}
\]

From equations (10) and (11), we get

\[
t_2 = \frac{1}{p} \left[ a\lambda + \frac{a\lambda}{2} (t_2^2 - T^2) + (p - a)T \right] \tag{12}
\]

Therefore, as \( t_2 \) can be expressed in terms of \( t_1 \) and \( T \), it is not taken as a decision variable.
V. COST COMPONENTS

Considering the aforementioned model description and underlying assumptions, the total cost is made up of following components:

(1) Ordering Cost (OC)
The operating cost over the period [0, T] is
\[ OC = A \]
(13)

(2) Deterioration Cost (DC)
The deterioration cost during the period [0, t₁] is
\[ DC = C_d \left[ \int_0^{t₁} \alpha t^\mu t Q(t) dt + \frac{h}{t} \int_0^{t₁} Q(t) dt \right] \]
(14)

(3) Shortage Cost (SC)
The cost of shortage during the interval [t₁, T] is given by:
\[ SC = -C_s \left[ \int_{t₁}^T Q(t) dt + \frac{r}{T} \int_{t₁}^T Q(t) dt \right] \]
(15)

(4) Production Cost (PC)
The production cost per cycle is
\[ PC = C_p \left[ \mu + (T - t₂) \right] p \]
(16)

(5) Amelioration Cost (AMC)
The amelioration cost over the period [0, t₁] is
\[ AMC = C_a \left[ \int_0^\mu O(t) dt + \int_{t₁}^\mu O(t) dt \right] \]
(17)

\[ = C_a \theta \left\{ (p - a) \frac{\mu^2}{2} - a \right\} \left[ \frac{1}{2(\theta + 1)} \left( t₁^2 - \mu^2 \right) - \frac{t₁ \mu^3}{3(1 - \theta)} \left( t₁^2 - \mu^2 \right) \right] + \]
(13)

\[ \left\{ \frac{\lambda - \theta}{\theta + 2} \left[ \frac{1}{3} \left( t₁^2 - \mu^2 \right) - \frac{t₁ \mu^3}{3(1 - \theta)} \left( t₁^2 - \mu^2 \right) \right] \right\} \]

(6) Inventory Holding Cost (IHC)
The cost of holding the inventory over the time period [0, t₁] is given by:
\[ IHC = \int_0^\mu (h + rt) Q(t) dt + \int_{t₁}^\mu (h + rt) Q(t) dt \]
(14)

\[ = h \left[ (p - a) \frac{\mu^2}{2} + \left( (p - a) \theta_1 + a \lambda \right) \frac{\mu^3}{6} - \frac{(p - a) \alpha \beta}{(\beta + 1)(\beta + 2)} \mu^{\mu + 2} \right] + \]
(13)

\[ \left\{ \frac{\lambda - \theta}{\theta + 2} \left[ \frac{1}{3} \left( t₁^2 - \mu^2 \right) - \frac{t₁ \mu^3}{3(1 - \theta)} \left( t₁^2 - \mu^2 \right) \right] \right\} \]

(15)

\[ r \left[ (p - a) \frac{\mu^2}{3} + \left( (p - a) \theta_1 + a \lambda \right) \frac{\mu^3}{8} - \frac{(p - a) \alpha \beta}{(\beta + 1)(\beta + 2)} \mu^{\mu + 3} \right] - \]
(16)
(7) **Total Cost (TC)**

Hence, the average total cost for the period \([0, T]\) is given by

\[
TC = \frac{1}{T} \left[ OC + DC + SC + AMC + PC + IHC \right]
\]

(19)

Using the expressions (12), \(t_2\) is eliminated from equation (19) of total cost TC.

Hence, TC becomes a function of \(\mu, t_1\) and \(T\) only, which are the decision variables and have \(\mu^*, t_1^*\) and \(T^*\) as respective optimum values, which minimize the cost function TC and they are the solutions of the equations

\[
\frac{\partial TC}{\partial \mu} = 0, \quad \frac{\partial TC}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial T} = 0
\]

such that

\[
\begin{align*}
\frac{\partial^2 TC}{\partial \mu^2} & \quad \frac{\partial^2 TC}{\partial \mu \partial t_1} & \quad \frac{\partial^2 TC}{\partial \mu \partial T} \\
\frac{\partial^2 TC}{\partial t_1 \partial \mu} & \quad \frac{\partial^2 TC}{\partial t_1 \partial T} & \quad \frac{\partial^2 TC}{\partial T \partial t_1} & \quad \frac{\partial^2 TC}{\partial T^2} \\
\end{align*}
\]

\[\mu = \mu^*, \ t_1 = t_1^*, \ T = T^*\]

(20)

The optimal values \(\mu^*, t_1^*\) and \(T^*\) can be obtained with the help of a mathematical software.

**VI. NUMERICAL EXAMPLE**

Below numerical illustration is used to demonstrate the above mentioned inventory model with the values of parameters being \(A = 400, p = 4, a = 2, \alpha = 0.0001, \beta = 2, \lambda = 0.0002, \theta = 0.22, \theta_1 = 0.11, \ h = 8, \ r = 4, \ C_p = 10, \ C_d = 2, \ C_s = 8\) and \(C_a = 3\) (With appropriate units).

The optimal values of \(\mu, t_1\) and \(T\) are

\[
\begin{align*}
\mu^* & = 1.063004997, \ t_1^* = 3.729283836 \quad \text{and} \\
T^* & = 11.67971141. \text{The optimal total cost per unit time TC} \\
& = 83.5993432 \text{units.}
\end{align*}
\]

**VII. SENSITIVITY ANALYSIS**

Sensitivity analysis is a technique to study the impact of variation in an independent variable’s value, on a particular dependent variable. Here, we’ve tried to capture the sensitivity of TC per unit time for every change in the value of parameters \(A, p, a, \lambda, \theta, \beta, \theta_1, h, r, C_p, C_d, C_s\) and \(C_a\).

The below analysis is carried out by considering an increase and decrease variation of 10% and 20% for the value of each parameter, keeping other parameters static. The results are presented in the Table-1 at the end and the last column shows the % variation in TC as compared to the original value, for each of the parameter.

**VIII. GRAPHICAL RENDITION**

Graphical rendition facilitates presentation of data in a simple, clear and effective manner along with easy comparison of values, trends and relationships.

Graphical rendition of the sensitivity analysis is shown in Fig. 2 and Fig. 3 at the end.

**IX. CONCLUSION**

- From the Table – 1, we observe that the parameters \(A, p, a, \lambda, \theta, h, r, C_p, C_d, C_s\) and \(C_a\) have a linear relationship with average total cost while the parameters \(\alpha, \beta, \theta, \text{ and } \theta_1\) have an inverse relation with average total cost.

(20)

- From Table-1 and Figure-2, a continuum scale is observed for the average total cost. On one hand it is highly sensitive by the values of \(A, p, a, C_p, C_s\) and on the other hand it is less sensitive by the values of \(\alpha, \beta, \lambda, \theta, \text{ and } \theta_1\) and in between it shows the moderate sensitivity due to the values of \(h, r, C_d\) and \(C_a\).

- Figure – 3 shows the effect of decision variables \(t_1\) and \(T\) on average total cost TC.

**REFERENCES**


Table-1: Sensitivity analysis of parameters considered in defining inventory model

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<th>Parameter</th>
<th>Value</th>
<th>$\mu$</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$TC$</th>
<th>% change</th>
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Fig. 2: Graphical rendition of the sensitivity analysis

Fig. 3: