Alpha Weakly Semi Closed Sets in Topological Spaces

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Abstract – N. Levine introduced the concept of generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets and they characterized that sets. In this paper, we introduce a new class of closed sets which is called Alpha weakly semi closed sets in topological spaces and we study the relationships of this set with some other generalized closed sets. Also we study some of its basic properties.

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I. INTRODUCTION

In 1970, Levine [1] introduced generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets. In 2000, M. Sheik John [2] introduced and investigated w-closed sets in topology. In 2017, Veeresh A Sajjanar [3] introduced weakly semi closed sets and investigated some of their properties. In this paper, Section I contains the concept of Alpha Weakly semi-closed (briefly αws-closed) set is introduced and their properties are investigated. Section II contains the Certain preliminary concepts. Section III contain the concept of αws-closed set is studied and a diagram also includes which states the relationships among the generalized closed sets in topological spaces and Section IV contains the conclusions and Section V contains the references.

II. PRELIMINARIES

Throughout this paper X and Y represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, clA and intA denote the closure of A and the interior of A respectively. X – A denotes the complement of A in X. We recall the following definitions.

Definition 2.1: A subset A of a space X is called

(i) pre-open [4] if A ⊆ int clA and pre-closed if cl intA ⊆ A.
(ii) semi-open [5] if A ⊆ cl intA and semi-closed if int clA ⊆ A.
(iii) semi-pre-open [6] if A ⊆ cl int clA and semi-pre-closed if int cl intA ⊆ A.
(iv) a-open [7] if A ⊆ int cl intA and a-closed if cl int clA ⊆ A.
(v) regular open [8] if A = int clA and regular closed if cl intA = A.
(vi) b-open [9] if A ⊆ cl intA ∪ int clA and b-closed if cl intA ∩ int clA.
(vii) π-open [10] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The alpha-closure (resp. semi-closure, resp. semi-pre-closure, resp. pre-closure, resp. b-closure) of a subset A of X is the intersection of all alpha-closed (resp. semi-closed, resp. semi-pre-closed, resp. pre-closed, resp. b-closed) sets containing A and is denoted by aclA (resp. sclA, resp. spclA, resp. pclA, resp. bclA).

Definition 2.2: A subset A of a space X is called
Lemma 2.4: [34] In an extremally disconnected space X,

(i) $pclA = spclA$.
(ii) $aclA = sclA$.

Lemma 2.5: [34] In an extremally disconnected sub maximal space X,

$clA = aclA = sclA = pclA = spclA$. 

The complements of the above mentioned closed sets are their respective open sets. For example a subset B of a space X is generalized open (briefly g-open) if $X - B$ is g-closed.

**Definition 2.3:** A space X is called a

(i) $T_b$ space [32] if every gs-closed set is closed.
(ii) $\alpha$-space [7] if every $\alpha$-closed set is closed.
(iii) door space [33] if every subset is either open or closed.
Lemma 2.6: [6] For any subset \( A \) of \( X \), the following results hold:

(i) \( sclA = A \cup int clA \).
(ii) \( pclA = A \cup cl intA \).
(iii) \( spclA = A \cup int clA \).
(iv) \( aclA = A \cup cl int clA \).

III. ALPHA WEAKLY SEMI CLOSED SETS

In this section, we introduce a new type of closed sets namely \( \alpha ws \)-closed sets in topological spaces and study some of their properties.

Definition 3.1: A subset \( A \) of a space \( X \) is called Alpha Weakly Semi closed (briefly \( \alpha ws \)-closed) if \( aclA \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( ws \)-open.

Proposition 3.2:

(i) Every closed set is \( \alpha ws \)-closed.
(ii) Every \( \alpha \)-closed set is \( \alpha ws \)-closed.
(iii) Every \( \pi \)-closed set is \( \alpha ws \)-closed.
(iv) Every regular closed set is \( \alpha ws \)-closed.
(v) Every \((gsp)^*\)-closed set is \( \alpha ws \)-closed.

Proof:

(i) Let \( A \) be a closed set in \( X \). Let \( A \subseteq U \) and \( U \) is \( ws \)-open. Since \( A \) is closed, \( clA = A \).

But \( aclA \subseteq clA \). Therefore \( aclA \subseteq U \). Hence \( A \) is \( \alpha ws \)-closed in \( X \).

(ii) Let \( A \) be a \( \alpha \)-closed set in \( X \). Let \( A \subseteq U \) and \( U \) is \( ws \)-open. Since \( A \) is \( \alpha \)-closed, \( aclA = A \).

Therefore \( aclA \subseteq U \). Hence \( A \) is \( \alpha ws \)-closed in \( X \).

(iii) Let \( A \) be a \( \pi \)-closed subset of \( X \). Since every \( \pi \)-closed set is closed [19] and by (i), we have \( A \) is \( \alpha ws \)-closed.

(iv) Let \( A \) be a regular-closed subset of \( X \). Since every regular-closed set is closed [8] and

By (i), we have \( A \) is \( \alpha ws \)-closed.

(v) Let \( A \) be a \((gsp)^*\)-closed set in \( X \). Let \( A \subseteq U \) and \( U \) is \( ws \)-open. Since every \( ws \)-open set is \( gsp \)-open and \( A \) is \((gsp)^*\)-closed, \( clA \subseteq A \). But \( aclA \subseteq clA \). Therefore \( aclA \subseteq U \). Hence \( A \) is \( \alpha ws \)-closed in \( X \).

The reverse implications are not true as shown in Examples 3.3, 3.4, 3.5 and 3.6.

Example 3.3: Let \( X = \{a, b, c\} \) with topology \( \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \). Then \( \{b\} \) is \( \alpha ws \)-closed but not regular closed.

Example 3.4: Let \( X = \{a, b, c\} \) with topology \( \tau = \{\emptyset, \{a\}, \{a, b\}, X\} \). Then \( \{b\} \) is \( \alpha ws \)-closed but not closed.

\( \{b\} \) is \( \alpha ws \)-closed but not \((gsp)^*\)-closed.

Example 3.5: Let \( X = \{a, b, c, d\} \) with topology \( \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\} \). Then \( \{c\} \) is \( \alpha ws \)-closed but not \( \pi \)-closed.

Example 3.6: Let \( X = \{a, b, c, d\} \) with topology \( \tau = \{\emptyset, \{a, b\}, \{a, b, c\}, X\} \). Then \( \{a, c, d\} \) is \( \alpha ws \)-closed but not \( \alpha \)-closed.

Proposition 3.7:

(i) Every \( \alpha ws \)-closed set is \( \alpha g \)-closed.
(ii) Every \( \alpha ws \)-closed set is \( gpr \)-closed.
(iii) Every \( \alpha ws \)-closed set is \( gb \)-closed.
(iv) Every \( \alpha ws \)-closed set is \( rgb \)-closed.
(v) Every $\alpha ws$-closed set is $gp$-closed.
(vi) Every $\alpha ws$-closed set is $gs$-closed.
(vii) Every $\alpha ws$-closed set is $agr$-closed.
(viii) Every $\alpha ws$-closed set is $gab$-closed.
(ix) Every $\alpha ws$-closed set is $sg$-closed.
(x) Every $\alpha ws$-closed set is $sbg$-closed.

**Proof:**

(i) Let $A$ be a $\alpha ws$-closed subset of a space $X$. Let $A \subseteq U$ and $U$ is open. Since every open set is $ws$-open in $X$ and $A$ is $\alpha ws$-closed, $aclA \subseteq U$. Hence $A$ is $ag$-closed.

(ii) Let $A$ be a $\alpha ws$-closed set in $X$. Let $A \subseteq U$ and $U$ is regular open. Since every regular open set is $ws$-open in $X$ and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $pclA \subseteq aclA$. Therefore $pclA \subseteq U$. Hence $A$ is $gpr$-closed.

(iii) Let $A$ be a $\alpha ws$-closed set in $X$. Let $A \subseteq U$ and $U$ is open. Since every open set is $ws$-open in $X$ & Since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $bclA \subseteq aclA$. Therefore $bclA \subseteq U$. Hence $A$ is $gb$-closed in $X$.

(iv) Let $A$ be a $\alpha ws$-closed set in $X$. Let $A \subseteq U$ and $U$ is regular open. Since every regular open set is $ws$-open in $X$ and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $bclA \subseteq aclA$. Therefore $bclA \subseteq U$. Hence $A$ is $gprf$-

(v) Let $A$ be a $\alpha ws$-closed set in $X$. Let $A \subseteq U$ and $U$ is open. Since every open set is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $pclA \subseteq aclA$. Therefore $pclA \subseteq U$. Hence $A$ is $gp$-closed.

(vi) Let $A$ be a $\alpha ws$-closed set. Let $A \subseteq U$ and $U$ is open. Since every open set is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $aclA \subseteq aclA$. Therefore $sclA \subseteq U$. Hence $A$ is $gs$-closed.

(vii) Let $A$ be a $\alpha ws$-closed set. Let $A \subseteq U$ and $U$ is regular open. Since every regular open set is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. Hence $A$ is $agr$-closed.

(viii) Let $A$ be a $\alpha ws$-closed set. Let $A \subseteq U$ and $U$ is $a$-open. Since every $a$-open set

is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $bclA \subseteq aclA$. Therefore $bclA \subseteq U$. Hence $A$ is $gab$-closed.

(ix) Let $A$ be a $\alpha ws$-closed set. Let $A \subseteq U$ and $U$ is semi-open. Since every semi-open set is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $aclA \subseteq aclA$. Therefore $sclA \subseteq U$. Hence $A$ is $sg$-closed.

(x) Let $A$ be a $\alpha ws$-closed set. Let $A \subseteq U$ and $U$ is semi-open. Since every semi-open set is $ws$-open and since $A$ is $\alpha ws$-closed, $aclA \subseteq U$. But $bclA \subseteq aclA$. Therefore $bclA \subseteq U$. Hence $A$ is $sbg$-closed.

The reverse implications are not true as shown in Example 3.8, 3.9 And 3.10

**Example 3.8:** Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then

- $\{a, c\}$ is $ag$-closed but not $\alpha ws$-closed.
- $\{a\}$ is $gpr$-closed but not $\alpha ws$-closed.
- $\{a, c\}$ is $gb$-closed but not $\alpha ws$-closed.
- $\{a\}$ is $rgb$-closed but not $\alpha ws$-closed.
- $\{a, c\}$ is $gp$-closed but not $\alpha ws$-closed.
- $\{a\}$ is $gs$-closed but not $\alpha ws$-closed.
- $\{a\}$ is $agr$-closed but not $\alpha ws$-closed.

**Example 3.9:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a, b\}, \{a, b, c\}, X\}$. Then

- $\{a\}$ is $gab$-closed but not $\alpha ws$-closed.

**Example 3.10:** Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. Then

- $\{a\}$ is $sg$-closed but not $\alpha ws$-closed.
\{a\} is sgb-closed but not \(\alpha\)-closed.

The concept “\(\alpha\)-closed” is independent from the concepts “g-closed”, “gr-closed”, “\(g^*\)-closed”, “\(rg\)-closed”, “\(g^p\)-closed”, “\(*g\)-closed”, “\(\pi g\)-closed” as seen in the following Examples 3.11 & 3.12.

**Example 3.11:** Let \(X = \{a, b, c\}\) with topology \(\tau = \{\emptyset, \{a\}, \{a, b\}, X\}\).
- \(b\) is \(\alpha\)-closed but not g-closed and \(\{a, c\}\) is g-closed but not \(\alpha\)-closed.
- \(b\) is \(\alpha\)-closed but not gr-closed and \(\{a, c\}\) is gr-closed but not \(\alpha\)-closed.
- \(b\) is \(\alpha\)-closed but not \(g^p\)-closed and \(\{a, c\}\) is \(g^p\)-closed but not \(\alpha\)-closed.

**Example 3.12:** Let \(X = \{a, b, c, d\}\) with topology \(\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}\).
- \(c\) is \(\alpha\)-closed but not \(rg\)-closed and \(\{a, b\}\) is \(rg\)-closed but not \(\alpha\)-closed.
- \(c\) is \(\alpha\)-closed but not \(g^p\)-closed and \(\{b, d\}\) is \(g^p\)-closed but not \(\alpha\)-closed.
- \(c\) is \(\alpha\)-closed but not \(\pi g\)-closed and \(\{a, b, d\}\) is \(\pi g\)-closed but not \(\alpha\)-closed.

Thus the above discussions lead to the following diagram. In this diagram, “A→B” means A implies B but not conversely and “A ↔ B” means A and B are independent of each other.

![Diagram](image-url)

**Figure 1**

**Theorem: 3.13**

The union of two \(\alpha\)-closed subsets of \(X\) is \(\alpha\)-closed set.

**Proof:**

Let A and B be any two \(\alpha\)-closed sets in \(X\). Let \(A \subseteq U \cup U\) is ws-open, \(B \subseteq U \cup U\) is ws-open. Then \(acl(A) \subseteq U\) and \(acl(B) \subseteq U\). since \(A \subseteq U\) and \(B \subseteq U\), then \(A \cup B \subseteq U\). \(\Rightarrow acl(A) \cup acl(B) \subseteq U\). we know that \(acl(A \cup B) = acl(A) \cup acl(B)\) [35].
Hence \( \text{acl} (A \cup B) = \text{acl}(A) \cup \text{acl}(B) \subseteq U \). Hence \( \text{acl} (A \cup B) \subseteq U \). Therefore \( A \cup B \) is \( \alpha ws \)-closed in \( X \).

**Theorem 3.14**

If a subset \( A \) of \( X \) is \( \alpha ws \)-closed in \( X \), then \( \text{acl} (A) \subseteq A \). Therefore \( A \subseteq \text{acl} (A) \). Hence \( A \subseteq \text{acl} (A) \).

**Proof:**

Let \( A \) be a \( \alpha ws \)-closed set in \( X \) and \( F \) be a \( \alpha cl \)-closed subset of \( \text{acl}(A) \). Then \( F \subseteq \text{acl}(A) \cap (X-A) \Rightarrow F \subseteq \text{acl}(A) \cap F \subseteq X-A \Rightarrow A \subseteq X-F \).

Since \( A \) is \( \alpha ws \)-closed set and \( X-F \) is \( \alpha cl \)-open, then \( \text{acl}(A) \subseteq X-F \) \( \Rightarrow \) \( F \subseteq X-\text{acl}(A) \).

We have \( F \subseteq \text{acl}(A) \). Therefore, \( F \subseteq (X-\text{acl}(A)) \cap \text{acl}(A) = \phi \). Thus \( F \subseteq \phi \).

Hence \( \text{acl}(A) \subseteq A \). Therefore \( A \subseteq \text{acl}(A) \).

**Theorem 3.15**

If a subset \( A \) is \( \alpha ws \)-closed set in \( X \) and, \( A \subseteq B \subseteq \text{acl}(A) \), then \( B \) is also \( \alpha ws \)-closed set.

**Proof:**

Let \( A \) be a \( \alpha ws \)-closed set in \( X \) such that \( A \subseteq B \subseteq \text{acl}(A) \). To prove \( B \) is also \( \alpha ws \)-closed set in \( X \). It is enough to prove \( \text{acl}(B) \subseteq U \). Let \( U \) be a \( \alpha cl \)-open set in \( X \) such that \( B \subseteq U \).

Since \( A \subseteq B \subseteq U \). Also since \( A \) is \( \alpha ws \)-closed, \( \text{acl}(A) \subseteq U \). Now, \( B \subseteq \text{acl}(A) \).

\( \Rightarrow \text{acl}(B) \subseteq \text{acl}(\text{acl}(A)) = \text{acl}(A) \subseteq U \). Therefore, \( B \) is \( \alpha ws \)-closed set in \( X \).

**Theorem 3.16**

For every point \( x \) in a space \( X \), \( X-\{x\} \) is \( \alpha ws \)-closed or \( \alpha cl \)-open.

**Proof:**

**Case (i)**

Suppose \( X-\{x\} \) is not \( \alpha cl \)-open. Then \( X \) is the only \( \alpha cl \)-open set containing \( X-\{x\} \).

Then using Definition 3.1 \( \text{acl}(X-\{x\}) \subseteq X \). Hence \( X-\{x\} \) is \( \alpha ws \)-closed.

**Case (ii)**

Suppose \( X-\{x\} \) is not \( \alpha ws \)-closed. Then there exists a \( \alpha cl \)-open set \( U \) containing \( X-\{x\} \) such that \( \text{acl}(X-\{x\}) \not\subseteq U \).

Therefore \( \text{acl}(X-\{x\}) \) is either \( X-\{x\} \) or \( X \). Therefore Take

\( \text{acl}(X-\{x\}) = X-\{x\} \), then \( X-\{x\} \) is \( \alpha cl \)-closed. By Preposition 3.2 (i) every \( \alpha cl \)-closed set is \( \alpha ws \)-closed, \( X-\{x\} \) is \( \alpha ws \)-closed. This is contradiction to our assumption. Therefore \( \text{acl}(X-\{x\}) = X \). To prove \( X-\{x\} \) is \( \alpha cl \)-open. Suppose \( X-\{x\} \) is not \( \alpha cl \)-open. By case (i) \( X-\{x\} \) is \( \alpha ws \)-closed. Which is contradiction to our assumption. Therefore \( X-\{x\} \) is \( \alpha cl \)-open.

**Theorem 3.17**

Let \( X \) and \( Y \) be topological spaces and \( A \subseteq Y \subseteq X \). Suppose that \( A \) is \( \alpha ws \)-closed set in \( X \) then \( A \) is \( \alpha ws \)-closed relative to \( Y \).

**Proof:**

Given \( A \subseteq Y \subseteq X \) and \( A \) is \( \alpha ws \)-closed in \( X \). To prove that \( A \) is \( \alpha ws \)-closed relative to \( Y \).
Let $A \subseteq Y \cap U$, where $U$ is $ws$-open in $X$. Since $A$ is $aws$-closed, then $acl A \subseteq U$. This implies $Y \cap acl A \subseteq Y \cap U$, where $Y \cap acl A$ is the $a$-closure of $A$ in $Y$ and $Y \cap U$ is $ws$-open in $Y$. Therefore $acl A \subseteq Y \cap U$ in $Y$. Hence, $A$ is $aws$-closed set relative to $Y$.

**Theorem: 3.18**

Let $A$ be $aws$-closed in $X$. Then $A$ is $a$-closed iff $acl A - A$ is $ws$-closed.

**Proof:**

Suppose $A$ is a $a$-closed set. Then $acl A = A \Rightarrow acl A - A = \emptyset$ which is $ws$-closed. Conversely, suppose $acl A - A$ is $ws$-closed. Since $A$ is $aws$-closed, Then by Theorem 3.14, $acl A - A = \emptyset$, (ie) $acl A = A$. Hence $A$ is $a$-closed.

**Theorem: 3.19**

Suppose $A$ is $ws$-open and $A$ is $aws$-closed. Then $A$ is $a$-closed.

**Proof:**

Given that $A$ is $ws$-open and $A$ is $aws$-closed. Then $A \subseteq A \Rightarrow acl A \subseteq A$

Hence $A$ is $a$-closed.

**Theorem: 3.20**

In a topological space if $X aO(X) = \{X, \emptyset\}$ then every subset of $X$ is a $aws$-closed set.

**Proof:**

Given that $X$ is a topological space and $aO(X) = \{X, \emptyset\}$. Let $A$ be a subset of $X$. Suppose $A = \emptyset$, then by Theorem 3.4, $\emptyset$ is $aws$-closed set. Suppose $A \neq \emptyset$, then $X$ is only $a$-open set containing $A$. Therefore $acl A \subseteq X$. Hence $A$ is $aws$-closed set in $X$.

**Theorem: 3.21**

If $A$ is regular open and $agr$-closed set then $A$ is $aws$-closed set in $X$.

**Proof:**

Suppose $A$ is a regular open set and $agr$-closed. Let $U$ be any $ws$-open set in $X \exists: A \subseteq U$.

Since $A$ is regular open and $agr$-closed set in $X$, by Definition $acl A \subseteq A$. then $acl A \subseteq A \subseteq U$. Hence $A$ is $aws$-closed.

**Definition: 3.22**

The intersection of all $ws$-open subsets of $X$ containing $A$ is called the $ws$-kernel of $A$ and is denoted by $ws$-ker ($A$).

**Theorem: 3.23**

If $A$ is a subset of $X$ is $aws$-closed iff $acl A \subseteq ws$-ker ($A$).

**Proof:**

Suppose $A$ is $aws$ closed. Then $acl A \subseteq U$ whenever $A \subseteq U$ & $U$ is $ws$-open.

To prove $acl(A) \subseteq ws$-Ker ($A$). Take $x \in acl(A)$. To prove $x \in ws$-ker ($A$)

Suppose $x \notin ws$-ker($A$) then there exist a $ws$-open set $U$ containing $A$ such that $x \notin U$. Since $A$ is $aws$-closed, then $acl A \subseteq U \Rightarrow x \notin acl(A)$, Which is a contradiction to our assumption. Therefore $acl A \subseteq ws$-ker ($A$). Conversely, Suppose $acl A \subseteq ws$-ker ($A$). To prove $A$ is
aws-closed. If \( U \) is any ws-open set containing \( A \), then \( \text{ws-ker } A \subseteq U \Rightarrow acl A \subseteq U \). Hence \( A \) is aws-closed in \( X \).

**Note: 3.24 [37]**

Let \( x \) be a point of \( X \). Then \( \{ x \} \) is either nowhere dense or pre-open.

**Remark: 3.25 [37]**

By the above note we take the following decomposition of a given topology \( X \), namely

\[
X = X_1 \cup X_2
\]

Where \( X_1 = \{ x \in X; \{ x \} \text{ is nowhere dense} \} \)

\[
X_2 = \{ x\in X; \{ x \} \text{ is pre-open} \}
\]

This is called Jankovic-Reilly Decomposition.

**Theorem: 3.26**

For any subset \( A \) of \( X \), \( X_2 \cap acl A \subseteq \text{ws-ker } (A) \)

**Proof:**

To prove \( X_2 \cap acl (A) \subseteq \text{ws-ker}(A) \). Consider \( x \in X_2 \cap acl (A) \). To prove \( x \in \text{ws-ker}(A) \)

Suppose \( x \notin \text{ws-ker}(A) \), then there is a ws-open set \( U \) containing \( A \) such that \( x \notin U \).

If \( F = X - U \), then \( F \) is ws-closed. Now, \( x \in acl (A) \Rightarrow acl (\{ x \}) \subseteq acl (acl (A)) \subseteq acl (A) \)

Since \( acl (\{ x \}) \subseteq acl (A) \), we get \( int (acl (\{ x \})) \subseteq int (acl (A)) \subseteq A \cap int (acl (A)) \)

Therefore \( int (acl (\{ x \})) \subseteq A \cap int (cl (A)) \). Now, take \( x \in X_2 \). We have \( x \notin X_1 \) and so

\( int (cl (\{ x \})) \neq \emptyset \). Let \( y \in int (cl (\{ x \})) \). Consider a point \( y \in A \cap int (cl (\{ x \})) \)

\( \Rightarrow y \in A \cap cl (\{ x \}) \Rightarrow y \in A \cap F \) which is a contradiction to \( x \notin \text{ws-ker}(A) \) [38]. Therefore \( x \in \text{ws-ker}(A) \). Hence \( X_2 \cap acl (A) \subseteq \text{ws-ker}(A) \).

**Theorem 3.27:**

A subset \( A \) of \( X \) is aws-closed iff \( X_1 \cap acl (A) \subseteq A \)

**Proof:**

Consider \( A \) is aws-closed. To prove \( X_1 \cap acl (A) \subseteq A \). Let \( x \in X_1 \cap acl (A) \), then \( x \in X_1 \) and \( x \in acl (A) \).

Since \( x \in X_1 \), \( int (cl (\{ x \})) \neq \emptyset \). Hence \( \{ x \} \) is semi-closed. Every semi closed set is ws-closed in \( X \) [15]. \( \{ x \} \) is ws-closed. If \( x \notin A \), then \( U = X - \{ x \} \) ws-open set containing \( A \) and \( acl A \subseteq U \). Since \( x \in acl (A), x \in U \) which is a contradiction to \( x \notin U \).

Hence \( X_1 \cap acl (A) \subseteq A \). Conversely, let \( X_1 \cap acl (A) \subseteq A \). To prove \( A \) is aws-closed

Since \( X_1 \cap acl (A) \subseteq A \), \( X_1 \cap acl (A) \subseteq \text{ws-ker}(A) \).

Now, \( acl (A) = X \cap acl (A) = (X_1 \cup X_2) \cap acl (A) = (X_1 \cap acl (A)) \cup (X_2 \cap acl (A)) \)

\( acl (A) \subseteq \text{ws-ker}(A) \). Then by Theorem 3.23 \( A \) is aws-closed.

**Theorem 3.28**

Arbitrary intersection of aws-closed set is aws-closed.

**Proof:**

Let \( \{ A_i \} \) be the collection \( aws \)-closed sets of \( X \). Let \( A = \bigcap A_i \). Since \( A \subseteq A_i \) for each \( i \),

then \( acl (A) \subseteq acl (A_i) \Rightarrow X_1 \cap acl (A) \subseteq X_1 \cap acl (A_i) \) for each \( i \).
Since each $A_i$ is $\alpha$ws-closed, then by theorem 3.28, $X_1 \cap acl(A_i) \subseteq A_i$ for each $i$.

Thus $X_1 \cap acl(A) \subseteq X_1 \cap acl(A_i) \subseteq A_i \subseteq A$ for each $i$. By Theorem 3.27, $A$ is $\alpha$ws-closed.

**Theorem: 3.29**

In a door space $X$, every $\alpha$ws-closed set is $\alpha$-closed

**Proof:**

Let $A$ be a $\alpha$ws-closed set in $X$. Since $X$ is a door space, by Definition 2.3(iii), $A$ is either open or closed. If $A$ is closed, then $A$ is $\alpha$-closed. If $A$ is open, then $A$ is ws-open.

Since $A$ is $\alpha$ws-closed & $A$ is ws-open, by Theorem 3.19, $A$ is $\alpha$-closed.

**Theorem: 3.30**

In an extremally disconnected space $X$, every $\alpha$ws-closed set is gs-closed.

**Proof:**

Let $X$ be an extremally disconnected space and $A$ be a $\alpha$ws-closed subset of $X$.

Let $A \subseteq U \cup U$ be open. Since every open set is ws-open in $X$ and since $A$ is $\alpha$ws-closed,

$aclA \subseteq U$. Since $X$ is extremally disconnected space, by Lemma 2.4(ii), $sclA = aclA \subseteq U$,

$\Rightarrow sclA \subseteq U$. Hence $A$ is gs-closed.

**Theorem: 3.31**

In an extremally disconnected sub maximal space $X$, every $\alpha$ws-closed set is $w$-closed.

**Proof:**

Let $X$ be an extremally disconnected space and $A$ be a $\alpha$ws-closed subset of $X$.

Let $A \subseteq U \cup U$ be semi-open. Since every semi open set is ws-open & since $A$ is $\alpha$ws-closed,

$aclA \subseteq U$. Since $X$ is extremally disconnected submaximal space, by Lemma 2.5 $clA \subseteq U$.

Hence $A$ is $w$-closed.

**Theorem: 3.32**

In a $T_b$ space $X$, Every gs-closed set is $\alpha$ws-closed

**Proof:**

Let $A$ be a $\alpha$closed set. Since $X$ is $T_b$ space, by Definition 2.3(i), $A$ is closed. By Preposition 3.2(i) $A$ is $\alpha$ws-closed.

**Theorem: 3.33** In a $\alpha$-space $X$, every $\alpha$-closed set is $\alpha$ws-closed.

**Proof:** Let $A$ be a $\alpha$-closed set in $X$. Since $X$ is a $\alpha$-space, by Definition 2.3(ii), $A$ is closed. By Preposition 3.2(ii), $A$ is $\alpha$ws-closed.

**IV. CONCLUSION**

In this paper, we have focused on Alpha Weakly Semi closed sets in topological spaces and found some important properties.

In future this concept can be extended to bitopological and ideal topological spaces.

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