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Application of Game Theoretical Model for Water Management in Rajkot City, Gujarat, India

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Abstract—Water is essential for the existence of society and needs in all human activities. In the present study, a game theoretical model developed to encourage the maximization of the public benefits in water distribution management in Rajkot city for different players. The present study concludes that Rajkot city demand-supply gap is 33 MLD and is expected to increase to 119 MLD and 236 MLD by 2035 and 2050 respectively. By 2035, when the population of Rajkot city grows to 21 lakh, the present water capacity would be insufficient to provide almost 50% of the city. This gap is continually increasing day by day as population increases and will become double in the year 2050. In the present study, five user game-theoretical model formulated for five players (industry, public, institutional, domestic and UFW) to minimize water demand supply gap based on current water supply in Rajkot city from all five water sources i.e. Ajii-I, Narmada, Nyari-I, Bhadar, Nyari-II. Asymmetric Nash bargaining approach is used to develop a game-theoretic model for the optimization of the weighted Nash product to obtain the actual solutions. From the study, it concluded that there is no water distribution strategy which satisfies the need of domestic users with current water distribution system. It calls for the establishment of new sources of water supply, their conservation, and optimal utilization.

Keywords—Water demand-supply gap, Water management, Game theory, Game theoretic model

I. INTRODUCTION

Water is essential for the existence of society and needs in all human activities. Access to safe freshwater now regarded as a worldwide human right (United Nations Committee on Economic, Social and Cultural Rights, 2003) [1]. It predicted that by 2025, one-third of the inhabitants of the developing world would face severe water shortages [2]. The availability of safe drinking fresh water is essential, and probably it will become critically insufficient due to the increasing demand for water, the continuous quick urban population growth, and expansion of agriculture and industrial actions. As populations are continuously growing so, user water demands will also be increasing [3]. Water resources are very under pressure from all over the world, particularly in drier countries, and this condition becomes worst because of climate change [4]. It has predicted that more than 2 billion people are affected by water shortages currently in over forty countries among which 1.1 billion people do not have adequate fresh drinking water [5]. There is enough water on earth, but it is distributed unevenly, polluted, unsustainably managed and wasted too much[6]. Growing consumption of water by aggressive water demands has resulted in severe water conflicts and succeeding ecological crisis. In the present study, a game theoretical model developed to encourage the maximization of the public benefits in water distribution management in Rajkot city for different players.

II. LITERATURE REVIEW

Game theory has applied to water resource management problems during the past decade. Becker and Easter (1995) used game theory to analyze the interdependency among eight states and two provinces concerning water diversions from the Great Lakes [7]. Carraro and Madani reviewed that the applicability of Game Theory in water resources management through a series of non-cooperative water resource games. Some cooperative game theoretic solutions (i.e., the core, Nash-Harsanyi, Shapley, and nucleolus) were formulated and applied by Madani and Dinar, through a numerical groundwater example. J.C Harsanyi developed an equilibrium solution for an n-person bargaining problem based on an initial Nash equilibrium solution for the two-person game [8]. Karamouz et al. used Nash product for the formulation of the objective function of a reservoir water allocation model and used resiliency and vulnerability indices to evaluate the performance of optimization algorithms [9]. Shouke Wei and Albrecht Gnauck (2007) explained that the water reservoirs management is complicated, and involved with multi-users. Water conflicts usually arise between multi-users due to their water use. They developed game theoretic models for water quality management of Danjiangkou Reservoir in Hanjiang River Basin in China as a game and used non-cooperative game theory results in a prisoner's dilemma and exacerbates the deterioration of water quality [10]. Madani, Kaveh (2010) reported that managing water resources systems usually involves conflicts in water resource management and game theory can identify and interpret such conflicts related to water resource problems. Outcomes predicted by game theory often differ from results suggested by optimization methods which assume all parties were willing to act towards the best system-wide outcome and reviewed the applicability of game theory in water resources management and conflict resolution throughout a series of non-cooperative water resource games[11]. Najmeh Mahjouri and Mojtaba Ardestani (2010) developed a new game theoretic optimization model to the competing users in water donor and receiving basins based on their water demands. For different coalitions of water users, the water shares of the coalitions determined by using an optimization model with economic objectives regarding the physical and environmental constraints of the system. Some cooperative game theoretic approaches utilized for transfer management to economic, equity, and environmental criteria, in a large-scale inter-basin water allocation problem in Iran [12]. Wang Zhanping and Tian Juncang (2012) discussed and analyzed the water resource demand of every area in different periods by economic development planning and possible development of super-normal conditions for Yinchuan City. The optimal water resource distribution model had established with optimization theory [13]. Ahmadi Ardeshir (2013) introduced the water distribution problem of the Mexican Valley and modeled as a three-person non-cooperative Game in which agriculture, industry, and domestic water users are the players and the total water Amounts supplied to the users were the payoff functions. A nonlinear optimization problem derived based on the Kuhn-Tucker necessary Conditions [14].

III. STUDY AREA

Rajkot city's primary water sources are surface water sources located near around the city. City's population as per 2011 is 1.2 million. As Rajkot city recognized as a developed town, so the future population estimation is done based on arithmetical increase method considering 30 years design period and 03 years time lag between design and completion of the project based on Manual on Water Supply & Treatment, published by Central Public Health and Environmental Engineering Organization (CPHEEO).

Table-1 Population Forecasting of Rajkot City

S. N	Year	Description	Population
1	2001	Census Department	1003015
2	2011	Census Department	1323363
3	2017	Design Year	1515572
4	2020	Base Year	1688560
5	2035	Middle of Design Period	2092198
6	2050	End of Design Period	2572720

In the present study, the projection for coming 30 years was considered to compares city's population with the water demand after every 15 years.

Table-2 Total Water Demand of Rajkot City

Need of Water	Water Demand (MLD)			
	Year			
	2017	2020	2035	2050
Domestic Demand	176	196	243	321
Institutional @ 7%	12.31	13.71	16.99	22.45
Industrial @ 20 %	35.16	39.17	48.54	64.15
Public Use @ 3%	5.27	5.88	7.28	9.62
UFW@28% to 20%	49.23	39.17	48.54	64.15
Total Demand	278	294	364	481

Rajkot Municipal Corporation (RMC) is daily withdrawing 245 MLD water from various surface water sources. The present water demand for the Rajkot City is 278 MLD. The present study concludes that the population of Rajkot city by 2035 would inhabit by over 21 lakh and would need about 364 MLD water and in further 15 years, that is by 2050 the population would reach 26 lakh and would need about 481 MLD water. Rajkot city demand-supply gap is 33 MLD and is expected to increase to 119 MLD and 236 MLD by 2035 and 2050 respectively. By 2035, when the population of Rajkot city grows to 21 lakh, the present water capacity would be insufficient to provide almost 50% of the city. This gap is continually increasing day by day as population increases and will become double in the year 2050.

Table-3 Water Demand Supply Gap of Rajkot

Water Demand Supply Gap (MLD)	Year			
	2017	2020	2035	2050
Water Demand	278	294	364	481
Water Supply	245	245	245	245
Demand Supply Gap	33	49	119	236

IV. MODEL FORMULATION

Game theory is a mathematical construction for analyzing the strategies of each decision maker or player to maximize each player's possibility of winning and to forecast possible outcomes of the game. Mathematically, five different users are competing for the water; those are Industrial (k=1), Unaccounted (k=2), Institutional (k=3), domestic (k=4) and Public (k=5). The common objective of all the users is to minimize water deficiency or to maximize water supply. However, there are limited freshwater sources in the city which results in a disagreement between these users. Let, index of the users is k = 1, 2, 3, 4, 5, for each of them the decision variables are:

1. Ajii-I (a_k)
2. Narmada (n_k)
3. Nyari-I (ny_k)
4. Bhadar (b_k^*)
5. Nyari-II (ny_k^*)

Here every water user, i.e., Industrial, Unaccounted for Water (UFW), Institutional, Domestic and Public wants to maximize its water supply i.e.

$$m\text{ax} \quad a_k + n_k + ny_k + b_k^* + ny_k^*$$

There are two general limitations for each user. Water supply cannot exceed the k user's water demand Dk. Each user required a minimum amount of water as $D_k^{\min}$. The users have two common constraints. The complete water quantity cannot go beyond the demand:

$$a_k + n_k + b_k^* + ny_k + ny_k^* \le D_k$$

Each user need a minimum amount of water as

$$a_k + n_k + b_k^* + ny_k + ny_k^* \ge D_k^{\min}$$

In addition to these constraints, each user has its conditions. Industrial users (k = 1) have two particular constraints. Let A_N Minimum quantity of Narmada water that is used by industry and A_{Ny} Maximum quantity of Nyari-I water that is used by industry.

$$\frac{n_1 + ny_1^*}{a_1 + n_1 + ny_1 + b_1^* + ny_1^*} \ge A_N$$

It can write as

$$A_N a_1 + (A_N - 1)n_1 + A_N ny_1 + A_N b_1^* + (A_N - 1)ny_1^* \le 0$$

And

$$\frac{ny_1}{a_1 + n_1 + ny_1 + b_1^* + ny_1^*} \le A_{Ny}$$

It can write as:

$$-A_{Ny}a_1 - A_{Ny}n_1 + (1 - A_{Ny})ny_1 - A_{Ny}b_1^* - A_{Ny}ny_1^* \le 0$$

Unaccounted for water (UAF) users (k = 2) have one additional constraint. Let A_u Maximum proportion of Nyari-I's water that can use for the unaccounted purpose and can write as follows:

$$\frac{ny_2}{a_2 + n_2 + ny_2 + b_2^* + ny_2^*} \le A_u$$

It can write as:

$$-A_u a_2 - A_u n_2 + (1 - A_u)ny_2 - A_u b_2^* - A_u ny_2^* \le 0$$

Institutional users (k = 3) have one additional constraint is used. Let AI , the maximum proportion of the Narmada and Nyari-I water that can use for institutional purpose. It can write as:

$$\frac{n_3 + ny_3}{a_3 + n_3 + ny_3 + b_3^* + ny_3^*} \ge A_I$$

It can write as:

$$A_I a_3 + (A_I - 1)n_3 + A_I ny_3^* + A_I b_3^* + (A_I - 1)ny_3 \le 0$$

Domestic users (k = 4) have one additional constraint is used. Let A_d, the maximum proportion of Nyari-I's water that can use for domestic purpose and can write as:

$$\frac{ny_4}{a_4 + n_4 + ny_4 + b_4^* + ny_4^*} \le A_d$$

It can write as:

$$-A_d a_4 - A_d n_4 + (1 - A_d)ny_4 - A_d b_4^* - A_d ny_4^* \le 0$$

Public users (k = 5) have one additional constraint is used. Let A_P, the maximum proportion of Nyari-I's water that can use for public purpose and can write as follows:

$$\frac{ny_5}{a_5 + n_5 + ny_5 + b_5^* + ny_5^*} \le A_P$$

It can write as:

$$-A_p a_5 - A_P n_5 + (1 - A_P)ny_5 - A_P b_5^* - A_P ny_5^* \le 0$$

The additional constraints can represent the total water availability of all sources:

$$a_1 + a_2 + a_3 + a_4 + a_5 = S_a$$

$$n_1 + n_2 + n_3 + n_4 + n_5 = S_n$$

$$ny_1 + ny_2 + ny_3 + ny_4 + ny_5 = S_{ny}$$

$$b_1^* + b_2^* + b_3^* + b_4^* + b_5^* \le S_{b^*}$$

$$ny_1^* + ny_2^* + ny_3^* + ny_4^* + ny_5^* \le S_{ny^*}$$

As it stated previously, each user wants to minimize its water deficiency which is equivalent to maximize the total quantity of water supply.

$$Max \quad a_k + n_k + b_k^* + ny_k + ny_k^* \qquad (k = 1,2,3,4,5)$$

This problem can consider as a five-person game, in which the water users are the players, the strategy of player k is the decision vector. If $p = (p_1, p_2, p_3, p_4, p_5)$ is the simultaneous strategy vector of the three players, then it has to satisfy conditions. The below equation give the payoff function of user. Observe that all constraints and payoff functions are linear. Hence the dilemma can be rewritten in the form of the matrix as follows:

$$\begin{aligned} &Maximum \quad z_k^T p_k \qquad (k = 1,2,3,4,5) \\ &sub\ to \quad p \geq 0 \\ &Xp \leq y \end{aligned}$$

Where, $z_k^T = (1,1,1,1,1)$ and the elements of vector y and matrix X are resolved by the constraints. Also, we require that

$$w_1 + w_2 + w_3 + w_4 + w_5 = 1$$

Then the non-symmetric Nash bargaining solution can be obtained as the optimal solution of the following nonlinear optimization problem.

$$\begin{aligned} \max \quad & (z_k^T p_1 - D_1^{\min})^{w_1} (z_k^T p_2 - D_2^{\min})^{w_2} (z_k^T p_3 - D_3^{\min})^{w_3} \\ & (z_k^T p_4 - D_4^{\min})^{w_4} (z_k^T p_5 - D_5^{\min})^{w_5} \\ & sub\ to \quad p \geq 0 \\ & Xp \leq y \end{aligned}$$

All the users treated in the same manner which is supposed by the symmetry axiom, but it is not possible if the users have different factors. The Nash bargaining approach is a method for utility distribution. In the present study, the asymmetric Nash bargaining approach is used to satisfy all other axioms. An optimal distribution of additional benefits can obtain through the Nash bargaining game. There are two assumptions was used to allocate water to the different users. First, it is a game of complete information. It means that all the information for each user treated as ordinary knowledge; such as water demand and benefit functions. Second, differences exist over the bargaining power of players, and the differences will affect the player's share. Therefore, the asymmetric Nash bargaining approach utilized in analyzing this game.

V. RESULT ANALYSIS

In the result analysis, Rajkot city water demand data used for the analysis of water distribution. The current water distribution shows that total water distribution from the first source Aji-I river is 6.612 MLD for domestic need, 1.656 MLD for institutional need, 10.447 MLD for industrial need, 7.815 MLD for unaccounted for waste, and 0.920 MLD for public demand. From the second source Narmada, the distribution is 99.922 MLD for domestic need, 6.010 MLD for institutional need, 11.005 MLD for industrial need, 8.683 MLD for unaccounted for waste, and 3.380 MLD for public demand. Bhadar river water is distributed 46 MLD for domestic need only. From Nyari-I total 11.412 MLD water is distributed for domestic purpose, 1.769 MLD water for the institutional purpose, 10.696 MLD water for industrial need, 7.129 MLD water waste unaccountably and 0.994 MLD water distributed for public use. From Nyari-II water is distributed for domestic purpose only. It means that the total available water in Nyari-I is 32(=S_{ny}) MLD, Nyari-II is 11 MLD (=S_{ny*}), Ajii-I is 27 MLD (=S_a), Bhadar is 46 MLD (=S_{b*}) and Narmada canal is 129 MLD (=S_n).

The total water demand for the industrial user is 35MLD (=D_1), the Unaccounted user is 49.00 MLD (=D_2), Institutional use is 12.31 MLD (=D_3), Domestic user is174.496 MLD (=D_4), and the Public user is 5.00 MLD (=D_5). However, minimum water demand for the industrial user is 32.148 MLD (= $D_1^{\min}$), the institutional user is 9.420 MLD (= $D_3^{\min}$), the domestic user is 90.00 MLD (= $D_4^{\min}$),the public user is 0.050 MLD(= $D_5^{\min}$), unaccounted demand is 23.627 MLD. (= $D_2^{\min}$). The water supply in industry from Narmada is 34% and from Nyari-I it is 33%. Total Water supply to institutional demand is from Narmada and Nyari - I is 83%. Domestic water demand from Nyari-1 is 6.5% only.

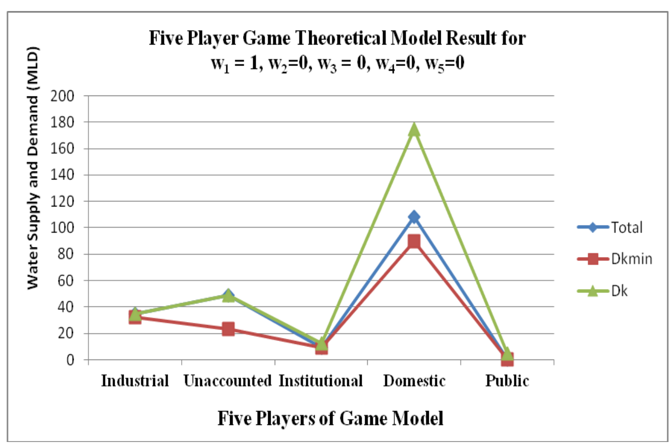

Figure 1. Five Player game Model Result for w_1=1, w_2=0,w_3=0,w_4=0,w_5=0

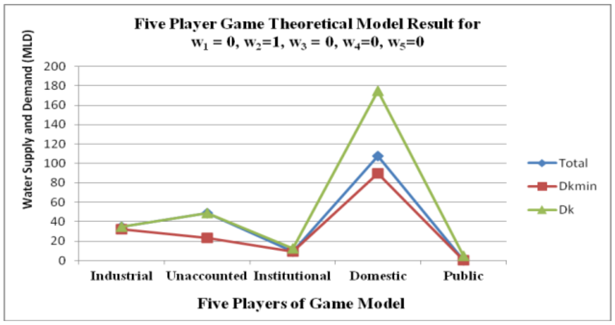

Figure 2. Five Player game Model Result for $w_1=0, w_2=1, w_3=0, w_4=0, w_5=0$

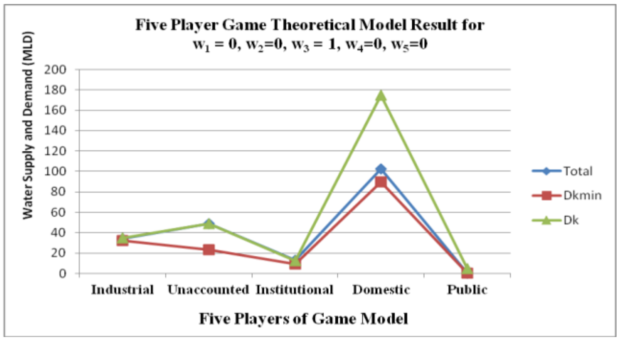

Figure 3. Five Player game Model Result for $w_1=0, w_2=0, w_3=1, w_4=0, w_5=0$

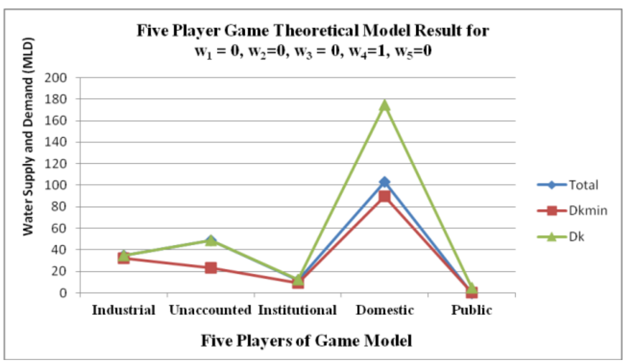

Figure 4. Five Player game Model Result for $w_1=0, w_2=0, w_3=0, w_4=1, w_5=0$

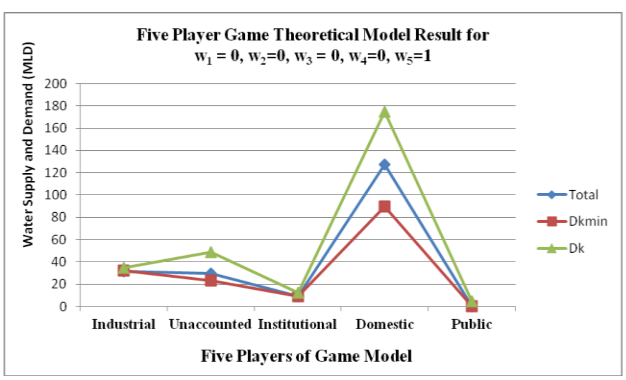

Figure 5. Five Player game Model Result for $w_1=0, w_2=0, w_3=0, w_4=0, w_5=1$

On comparing the five users' game theoretical model results in Figure-6, it shows that some users cannot fulfill their total water demand, but they can fulfill their minimum water demand.

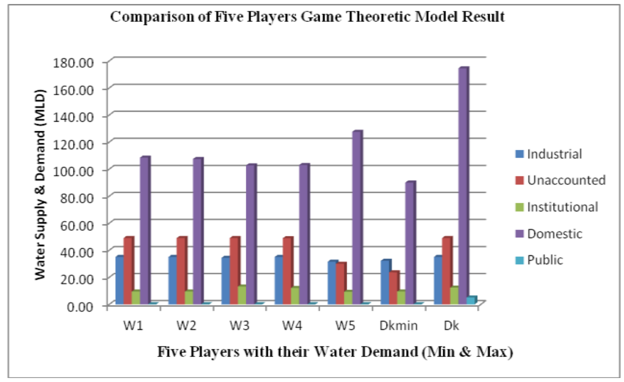

Figure 6. Comparison of Five Player Game Theoretical Model Results

VI. CONCLUSION

In the present study, five users game-theoretic model developed based on the present water demand-supply of the Rajkot city from five water resources to minimize water demand-supply gap. In this study, water demand for design period 30 years is also estimated based on the population forecasting of Rajkot city for the years 2035 and 2050. After getting current water demand and supply, water supply-demand gap identified for the study area. It concludes that the present water demand for the Rajkot City is 278 MLD considering per capita water demand of 135 lpcd and 40 lpcd for stand post while water supply is 245 MLD. The demand-supply gap is continually increasing day by day as population increases. Present water demand supply gap is 33 MLD for Rajkot city and is expected to increase to 119 MLD and 236 MLD by 2035 and 2050 respectively. By 2035, when the population of Rajkot city grows to 21 lakh, the present water supply would be insufficient to provide almost 50% of the city and expected to almost double by 2050.

In the study, five users game-theoretic model formulated for five players (Industry, public, institutional, domestic and Unaccounted for Water) using five water resources (Ajii-I, Narmada, Nyari-I, Bhadar, Nyari-II). RMC is providing freshwater to all five users and competing with each other in maximizing their freshwater supply. In the study, asymmetric Nash bargaining approach is used to develop a game-theoretic model for the optimization of the weighted Nash product to obtain the actual solutions. From the study, it concluded that there is no water distribution strategy which satisfies the need of domestic users with current water distribution system. It calls for the establishment of new sources of water supply, their conservation, and optimal utilization.

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