International Journal of Scientific Research in Mathematical and Statistical Sciences
Vol.**11**, Issue.**1**, pp.**06-10**, February **2024**
E-ISSN: 2348-4519
Available online at: www.isroset.org

Research Article

Utilizing CPM and PERT in project planning and scheduling, incorporating Linear Programming

Mansur Nuhu A.[1*], Suleiman K.[2], M. Hassan[3], Nazir Isma'il I.[4]

[1]Dept. of Mathematics, Federal College of Education (Technical), Bichi, Kano, Nigeria
[2,3]Dept. of Mathematics, Yusuf Maitama Sule University, Kano, Nigeria
[4]Dept. of Basic Studies, School of General studies, Kano State Polytechnic, Kano, Nigeria

**Corresponding Author: mansur.nuhu.alhassan@gmail.com*

Received: 26/Dec/2023; **Accepted**: 22/Jan/2024; **Published**: 29/Feb/2024

Abstract— Achieving the timely and cost-effective completion of a project presents a significant challenge, with the key hurdle residing in the effective planning and scheduling of the project. These aspects are crucial in accurately predicting both the time and cost dimensions. This research specifically concentrates on finding a balance between the cost of a project (crashing the project) and the minimum anticipated time required for the conclusion of a construction project. Information regarding the costs and durations of diverse activities was gathered from Bunyan Ltd., a construction firm situated in the Kano state of Nigeria. The study employed both the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT). Linear programming was applied to reduce both time and cost for various activities, leading to the identification of the critical path. The subsequent examination disclosed that the project's shortest possible duration is now 40 days, a substantial reduction from the initially expected 79 days. This underscores that precise scheduling of activities resulted in a noteworthy 39-day decrease in the anticipated completion time. However, this reduction incurred an additional cost of ₦ 142,175.27, increasing the initial projected cost for project completion from ₦ 4,305,920.71 to ₦ 4,448,095.99.

Keywords— Linear programming, Planning, CPM, PERT, Crashing, Critical path

1. Introduction

Accomplishing projects in the specified timeframe and budget is a challenging endeavor. Despite advancements in contemporary project management practices, a prevalent issue in many projects within Nigeria is the escalation of both cost and time, particularly as project complexity increases. Numerous factors contribute to delays, predominantly stemming from delays such as that of; client-related, consultant-related, labor-related, and various external factors [1]. Over time, these delays lead to delays in schedule, drawbacks in budget, conflicts, and arbitration. The critical nature of certain project activities exacerbates the situation, as delays in their initiation directly impact the overall timeline for project completion. Hence, effective planning and scheduling are crucial to address this issue [1,2]. Additionally, the conventional scheduling methods, the network based methods, which could potentially address most, if not all, challenges in construction settings, is struggling to gain widespread acceptance among project planners for project analysis. It is evident that the application of this method in project management has been long overdue, [2].

The continuous resistance among Nigerian public project executors to embrace network based planning schedule systems, along with the frequent halting of crucial projects due to time constraints, inadequate quality, and performance standards associated with out-dated traditional methods, highlight the urgency of this study. The on-going inefficiencies contribute to the persistent waste of national resources and hinder the development of Nigeria.

1.1 Organization of the paper/article

Rest of the paper is organized as follows,
Section 1 contains the introduction of the network based project scheduling methods, Section 2 contain the related work of CPM and PERT, Section 3 contain materials and methods used in the research, Section 4 contain results and discussions of the research, and section 5 explains the conclusion and recommendation of the findings.

2. Related Work

A project can be characterized as a collection of numerous activities or tasks conducted in a specific, logically or technologically determined sequence, with the goal of completion within a designated time and budget while meeting performance standards, [2]. Projects encompass a wide range of endeavors, such as software program development, construction of buildings, creation of a new medicine, execution of a campaign for a product sale, and various others, [3]. Network models serve as traditional methods for determining the optimal way to connect multiple activities directly or indirectly, ensuring the fulfillment of supply and demand requirements across different activity locations and facilitating project scheduling, [4]. For an extended period, two established approaches that have proven effective in planning, scheduling, and controlling construction projects are the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT), [5]. These methodologies empower project managers to assess the earliest and latest times for the initiation and completion of activities, compute activity float (slack), identify critical activities, and gauge the repercussions of alterations in duration, logical relationships, and costs on the overall project timeline. Both CPM and PERT operate on a network-based framework, facilitating the programming and monitoring of stage progress to ensure project completion within the designated deadline. In this process, they specify pivotal segments of the project, the delay of which beyond the normal timeframe would extend the overall project completion time. Additionally, these techniques aid in resource allocation such as labor and equipment, contributing to minimizing the total cost of the construction project by determining an optimal trade-off between various costs and time considerations [6]. Although there are some differences in terminology and network construction between PERT and CPM, their overarching objectives align [7]. Additionally, the analytical methods employed in both techniques exhibit substantial similarity. The primary distinction lies in CPM's assumption that activity times are directly proportional to the allocated resources, allowing for variations in activity times and overall project completion time by adjusting resource levels. CPM relies on prior experience with similar projects to establish relationships between resources and job times [8]. Conversely, PERT incorporates uncertainties in activity times into its analysis. It assesses the probabilities of completing various project stages by specified deadlines and calculates the expected time for project completion. A valuable byproduct of PERT analysis is the identification of potential "bottlenecks" in project activities with a high likelihood of causing delays in adhering to the project schedule. Consequently, even before the project commences, the project manager gains insight into potential delay points, enabling proactive measures to mitigate possible delays and maintain the project schedule. In fact, both PERT and CPM techniques were developed almost simultaneously. Project managers frequently face the challenge of needing to accelerate the scheduled completion time to expedite project execution. Shortening the project duration involves tactics such as introducing additional resources through overtime, allocating extra labor, or providing supplementary resources [9]. However, the managerial decision to employ these additional resources, including overtime and labor, inevitably raises the overall project cost while reducing the duration of critical path activities. This project management strategy, which entails investing extra budget to minimize the duration and meet the targeted completion date, is known as "crashing." The primary goal of expediting a project through crashing is to recover delays, thereby avoiding liquidated damages. The objective is always to strike a balance between cost and time, aiming for an optimal schedule [10]. This study, therefore, seeks to investigate the time and cost implications of various activities involved in a prototype 3-bedroom house project in Kano, Nigeria. The goal is to determine the optimal completion time using CPM and PERT techniques, incorporating linear programming.

3. Materials and Method

To arrange the activities within the network, it is essential to estimate the time required for each activity when performed under normal conditions. These estimates were provided by the project manager at Bunyan Ltd., a construction company located in the Kano state of Nigeria. Table 1 delineates the breakdown of activities involved in the construction process of a 3-bedroom house situated in Kumbotso Local Government Area, of Kano state, Nigeria.

The sequence of construction activities spans from activity A to activity N, illustrating the project's activity distribution in terms of the actual number of days required for each task and their corresponding cost implications in Nigerian Naira for the construction of a 3-bedroom building. The costs primarily entail labour costs, assuming that materials are already available for use. This assumption stems from the understanding that, once materials are accessible, any reduction in the number of days required to complete a specific activity is influenced solely by the cost of hiring additional labour.

Table1. Description of activities involved for the construction process of a house by Bunyan Ltd.

Activity	Description	Cost ₦	Predecessors	Duration
A	Clearing of Site	137,740.07	-	2 days
B	Foundation	94,568.77	A	4 days
C	Block Laying	395,014.64	B	10 days
D	Roofing	311,590.62	C	6 days
E	Plumbing	375,325.09	C	4 days
F	Electrical work	394,038.54	E	5 days
G	Plastering	477,909.85	D	7 days
H	Fittings (doors / Windows)	268,562.18	E,G	9 days
I	Ceiling	306,647.29	C	7 days
J	Flooring	317,100.16	F,I	8 days
K	Interior Fixtures	356,355.84	J	4 days
L	Exterior Fixtures	253,467.40	J	5, 2 days
M	Painting	310,699.78	H	6 days
N	Landscaping	276,100.21	K,L	

Construct a network diagram is shown in Figure 1.

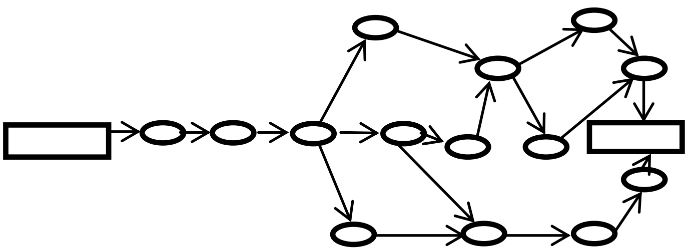

Fig 1. The Network diagram

We shall use the following equations to estimate the; Earliest start (ES), Earliest finish (EF), Latest start (LS), Latest finish (LF), Project completion time, slack of the activities and the critical path

$$EFj = ESj + tj ------------------------ (1)$$
$$ESj = max\ EFi ------------------------ (2)$$
$$Project\ completion\ time\ = \ ESFinish ------------ (3)$$
$$LSj = LFj - tj -------------------------(4)$$
$$LFi = min\ LSj -------------------------(5)$$
$$Slack\ of\ activity\ j = LSj - ESj ----------------(6)$$

The concept of slack revolves around the notion that if an activity possesses positive slack, it implies that the activity has some flexibility, [6]. In other words, it could commence a bit later without impeding the project's progress. Furthermore, its duration could be extended by the amount of its slack without causing any delays to the overall project [7]. Conversely, when an activity has zero slack, any extension of its duration invariably results in a delay to the entire project. Consequently, the critical path comprises activities with zero slack. The computation is illustrated in Table 2

Table 2: Estimations

Activity	(ES)	(EF)	(LS)	(LF)	Slack =LS-ES	Critical
A	0	2	0	2	0	Yes
B	2	6	2	6	0	Yes
C	6	16	6	16	0	Yes
D	16	22	20	26	4	No
E	16	20	16	20	0	Yes
F	20	25	20	25	0	Yes
G	22	29	26	33	4	No
H	29	38	33	42	4	No
I	16	23	18	25	2	No
J	25	33	25	33	0	Yes
K	33	37	34	38	1	No
L	33	38	33	38	0	Yes
M	38	40	42	44	4	No
N	38	44	38	44	0	Yes

In the context of Critical Path Method (CPM), the time estimates for all activities are initially treated as single values, by assuming a precise knowledge of each activity's duration, [7]. However, the reality of project execution introduces uncertainties in activity durations [8]. To address this, the Program Evaluation and Review Technique (PERT) employ a probabilistic approach, utilizing a beta distribution. Three time estimates optimistic (a), pessimistic (b), and most likely (m) are employed to calculate the expected time (mean) and variance of the distribution, [9]. The expected time is computed as a weighted average of these three estimates. To assess the feasibility of meeting the scheduled date, it becomes crucial to estimate the probability associated with achieving this timeframe. Table 3 illustrates the variations in estimates and their impact on construction activities. Using the formulae:

Mean of expected time $(x) = \frac{a+4m+b}{6}$ and Variance σ= $(\frac{b-a}{6})^2$

Where; a = Optimistic Estimate, b = Pessimistic Estimate and m = Most Likely Estimate

Table 3 Probabilistic computations

Activity	Predecessors	a	m	b	Mean	Variance
A	-	1	2	3	2	$\frac{1}{9}$
B	A	2	3,5	8	4	4
C	B	6	9	18	10	1
D	C	4	5,5	10	6	$\frac{4}{9}$
E	C	1	4,5	5	4	1
F	E	4	4	10	5	1
G	D	5	6,5	11	7	4
H	E,G	5	8	17	9	1
I	C	3	7,5	9	7	1
J	F,I	3	9	9	8	0
K	J	4	4	4	4	1
L	J	1	5,5	7	5	$\frac{1}{9}$
M	H	1	2	3	2	$\frac{4}{9}$
N	K,L	5	5,5	9	6	

While the project is projected to finish in 44 days, there is no assurance that it will indeed be completed within this timeframe. Unforeseen circumstances leading to delays in individual activities may hinder the project from adhering to the intended schedule. Consequently, it becomes essential to assess the likelihood of adhering to the project deadline. To initiate this evaluation, determining the variance and standard deviation of the total time along the critical path is crucial. This total time is equivalent to the sum of the variances of activity times on the critical path. Thus,

$$varT = \frac{1}{9} + 1 + 4 + \frac{4}{9} + 1 + 1 + 1 + \frac{4}{9} = 9.0$$

Standard Deviation T = $\sqrt{varT} = \sqrt{9} = 3.0$

Then, the probability that the project will be completed given the deadline to be 47 days. Hence, the probability using the normal distribution is:

$$Prob\ \{T \le 47\} = Prob\left\{\frac{T-44}{3} \le \frac{47-44}{3}\right\}$$
$$= Prob\{Z \le 1.0\}$$
$$= 1 - Prob\{Z > 1.0\} = 1 - 0.1587 = 0.84$$

Therefore, the chance that the critical path will be completed in less than 47 days is 84%.

4. Results and Discussion

Examining the potential cost associated with reducing the anticipated project duration from the initial 44 days. The construction company has set a deadline, and the maximum duration for which the company can allocate additional resources to the construction is 40 days. The critical setback for any operations researcher is to determine the optimal approach for expediting the project by investing extra funds, aiming to achieve the target completion time. Critical Path Method (CPM) offers an effective method for exploring time cost trade-offs, and in this case, we will employ the crashing process through linear programming. This involves special strategies like overtime implementation, employing extra workforce, utilizing time-efficient materials, and deploying specialized equipment, etc. [10].

The Table below provides details on project activities, cost, crash, durations, normal and crash costs, along with the calculations of crash costs the activities

$$\text{Crash cost (r)} = \frac{\text{Crash Cost} - \text{Normal}}{\text{CostNormal time} - \text{Crash time}} = \frac{\Delta C}{\Delta t} --------- (7)$$

Table 4: Crashing the data

Activity	Normal days	Normal Cost (₦)	Crash days	Crash Cost (₦)	ΔC	Δt	$r = \frac{\Delta C}{\Delta t}$
A	2	138,732.43	1	183,319.93	44,587.51	1	44,587.51
B	4	95,250.10	2	159,240.34	63,990.25	2	31,995.12
C	10	397,860.54	7	531,109.27	133,248.73	3	44,416.50
D	6	313,835.48	4	376,435.03	62,599.55	2	31,299.77
E	4	378,029.14	3	421,670.43	43,641.29	1	43,641.29
F	5	396,877.41	3	525,744.61	61,064.96	2	30,532.86
G	7	481,352.97	4	623,889.86	142,536.89	3	47,512.04
H	9	270,497.04	6	403,215.41	132,718.37	3	44,239.46
I	7	308,856.54	5	451,083.04	66,890.68	2	33,445.34
J	8	319,384.72	6	400,493.53	81,108.81	2	40,554.78
K	4	358,923.22	3	393,393.88	34,466.89	1	34,466.89
L	5	255,293.52	3	354,772.21	99,478.70	2	49,739.72
M	2	312,938.23	1	360,154.21	47,215.97	1	47,215.97
N	6	278,089.39	3	374,287.21	96,197.82	3	32,065.94

A linear programming model is formulated as follows:

$$\text{Minimize } Z = 44{,}587.51X_A + 31{,}995.12X_B + 44{,}416.50X_C + 31{,}299.77X_D + 43{,}641.29X_E + 30{,}532.86X_F + 47{,}512.04X_G + 44{,}239.46X_H + 33{,}445.34X_I + 40{,}554.78X_J + 34{,}466.89X_K + 49{,}739.72X_L + 47{,}215.97X_M + 32{,}065.94X_N$$

Subject to:

$$X_A \le 1, X_B \le 2, X_C \le 3, X_D \le 2, X_E \le 1, X_F \le 2, X_G \le 3,$$
$$X_H \le 3, X_I \le 2, X_J \le 2, X_K \le 1, X_L \le 2, X_M \le 1, X_N \le 3$$

Non-negativity

$$X_A \ge 0, X_B \ge 0, X_C \ge 0, X_D \ge 0, X_E \ge 0, X_F \ge 0, X_G \ge 0,$$
$$X_H \ge 0, X_I \ge 0, X_J \ge 0, X_K \ge 0, X_L \ge 0, X_M \ge 0, X_N \ge 0,$$
$$Y_A \ge 0, Y_B \ge 0, Y_C \ge 0, Y_D \ge 0, Y_E \ge 0, Y_F \ge 0, Y_G \ge 0,$$
$$Y_H \ge 0, Y_I \ge 0, Y_J \ge 0, Y_K \ge 0, Y_L \ge 0, Y_M \ge 0, Y_N \ge 0,$$
$$Y_{FINISH} \ge 0$$

Start time:

$$Y_B + X_A \ge 2 \qquad Y_C - Y_B + X_B \ge 4$$
$$Y_D - Y_C + X_C \ge 10 \qquad Y_E - Y_C + X_C \ge 10$$
$$Y_F - Y_E + X_E \ge 4 \qquad Y_G - Y_D + X_D \ge 6$$

$$Y_H - Y_E + X_E \ge 4 \qquad Y_H - Y_G + X_G \ge 7$$
$$Y_I - Y_C + X_C \ge 10 \qquad Y_J - Y_F + X_F \ge 3$$
$$Y_J - Y_I + X_I \ge 7 \qquad Y_K - Y_J + X_J \ge 6$$
$$Y_L - Y_J + X_J \ge 6 \qquad Y_M - Y_H + X_H \ge 9$$
$$Y_N - Y_K + X_K \ge 4 \qquad Y_N - Y_L + X_L \ge 5$$
$$Y_{FINISH} - Y_M + X_M \ge 2 \qquad Y_{FINISH} - Y_N + X_N \ge 7$$

Projection duration:

$$Y_{FINISH} \le 40$$

The construction needs to be crashed by 4 days.
LINDO software is used to obtain the solution of the model in the table below.

Table 5: Solution of the model

Activity	Start Time Y_i	Time Reduction X_i	Finish Time
A	0	0	2
B	2	0	6
C	6	0	16
D	16	0	22
E	16	0	20
F	20	2	23
G	22	0	29
H	29	0	38
I	16	0	23
J	23	2	29
K	30	0	34
L	29	0	34
M	38	0	40
N	34	0	40

End Time = 40 days and Overall Cost = ₦ 4,448,095.99
The shortest possible time for the completion of the building project is 40 days instead of the expected duration of 79 days. The completion time was reduced by 39 days. The additional cost is ₦ 142,175.27 which added the initial cost required to complete the job from ₦ 4,305,920.71 to ₦ 4,448,095.99.

5. Conclusion

This paper examines the utilization of project scheduling in the construction of a house by Bunyan Ltd., in Kumbotso Local Government Area of Kano State, focusing on the CPM and PERT methods. According to our CPM analysis, the projected finish time for the building is 44 days. However, by utilizing linear programming to crash project activities, the project duration can be reduced to 40 days, representing a 10% decrease. It's noteworthy that achieving this accelerated timeline incurs an additional cost of 3.30%, which, while appreciable, needs consideration. The findings suggest that the scheduling approach we propose significantly shortens the completion time compared to the actual project duration, indicating that CPM scheduling can be a lucrative and viable option for building construction job.

This research specifically concentrates on finding equilibrium between the cost of a project and the minimum anticipated time required for the conclusion of a construction project. Information regarding the costs and durations of diverse activities was gathered from Bunyan Ltd., a construction firm situated in the Kano state of Nigeria. The study employed both the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT). Linear programming was applied to reduce both time and cost for various activities, leading to the identification of the critical path. The subsequent examination disclosed that the project's shortest possible duration is now 40 days, a substantial reduction from the initially expected 79 days. This underscores that precise scheduling of activities resulted in a noteworthy 39-day decrease in the anticipated completion time.

Data Availability

The data used in this project, titled "Utilizing CPM and PERT in Project Planning and Scheduling, Incorporating Linear Programming, has been sourced from various repositories, databases, and literature. The specific datasets and sources are outlined below:

- **CPM and PERT Data:**

The project utilized industry-supplied datasets which was employed in the application of Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT).

- **Linear Programming Data:**

The Linear Programming (LP) component of this project involved the formulation and solution of optimization problems based on project constraints and objectives. The coefficients, constraints, and objective functions used in the LP models were generated for illustrative purposes and were not derived from proprietary or confidential sources.

- **Literature and Published Material:**

Relevant literature, research papers, and textbooks in the fields of project management, operations research, and linear programming were consulted. Any data or information derived from these sources is appropriately cited in the project documentation.

- **Software Tools and Algorithms:**

The project has utilized specific software tools or algorithms for implementing CPM, PERT, and Linear Programming. The details of these tools and algorithms, including versions and sources, are provided in the project documentation.

Conflict of interest: All authors declare no conflict of interest.

Funding Sources: The research required no funding as all data needed where provided by the construction company, which is resident in the same state as the researchers.

Author's contribution: I thank, Suleiman K., M. Hassan and Nazir Ismail for their valuable insights and assistance in verification of the computations as well as making adjustments where necessary.

Acknowledgement: The authors would like to express their gratitude to Ibrahim Yusuf of Bayero University; Kano for taking time to proof read the work. Special thanks to Mansur Hassan for valuable discussions and Yusuf Maitama Sule University, Kano for providing access to their Mathematics Laboratory where the needed software for the research was provided free of charge. We are also thankful to Farouk Tijjani Sa'ad, Dean Faculty of Science, Yusuf Maitama Sule University, Kano for his guidance throughout the research process.

References

[1] Wallace Agyei., "Project Planning and Scheduling Using PERT and CPM Techniques with Linear Programming," International Journal of Scientific & Technology Research, Vol.4, Issue.8, pp. 222-227, 2019.

[2] A. H. Taha, "Operations research: An Introduction," Pearson Education, Inc. and Dorling Kindersley Publishing Inc. Eight edition, pp. 295-313. 2007.

[3] K. Sunita, B. Snigdha, "CPM Analysis of Rolai-Rinjlai Road Construction," Research Journal of Mathematical and Statistical Sciences, Vol.1, No.2, pp. 7-15, 2017.

[4] C. Gueret, M. Sevaux, "Applications of Optimization with Xpress-MP," translated and revised by Susanne Heipke, Dash Optimization Ltd., London. First Edition, pp. 159-178, 2002.

[5] R.T. Ahuja, J. Orlin, "Network Flows: Theory, Algorithms and Applications" Prentice Hall, Upper Saddle River, NJ. Revised Edition, pp. 217-243, 2001.

[6] S. A. Adebowale, E. D. Oluboyede, "Network analysis and building construction: Implications for timing and costing of activities," Journal of Civil Engineering and Construction Technology, Vol. 2, No. 5, pp. 90-100, 2022.

[7] A. M. Aliyu, "Use of CPM in project management: A pragmatic study," Global Journal of Pure and Applied Sciences, Vol. 18, No. 3, pp. 197-206, 2022.

[8] P. Weaver, "A Brief History of Scheduling: Back to the Future" (PDF). Retrieved 2 December, 2023.

[9] G. E. Nworuh, "Network planning in implementation of public projects in Nigeria, empirical study," Interworld journal of management development studies, Vol. 1, No. 1, pp. 265-275, 2021.

[10] K. J. Bassey, P. E. Chigb, "On building decisions based model via coherent pluralism", Global journal of pure and applied sciences, Vol. 8, No. 3, pp. 347-355, 2020.

AUTHORS PROFILE

Mansur Nuhu Alhassan earned his NCE (Matthematics/Computer)., Bsc (Ed) Mathematics,. and Currently pursuing Msc in Mathematics from Yusuf Maitama Sule University, Kano in 2011, 2021, and 2024, respectively. He is currently working as a Lecturer in Department of Mathematics from F.C.E (T) Bichi, Kano State, since 2021. He is a member of TRCN since 2011. He has 14 years of teaching experience in Secondary Schools and Tertiary institutions combined and 2 years of research experience.

Kabiru Suleiman earned his Bsc., M. Sc., and Ph.D. in Mathematics from UDUS Sokoto in 1995, 2003, and 2010, respectively. He is currently working as an Associate Professor in Department of Mathematics, YUMUK, Kano since 2020. He has published more than 16 research papers in reputed international journals including International Journal of Operations Research and conferences and it's also available online. His main research work focuses on Operations Research. He has 15 years of teaching experience and 10 years of research experience.

Mansur Hassan earned his Bsc., from BUK Kano, M. Sc., from Jordan University of Science and Technology and Ph.D from University Sains Malasia. in Mathematics from in 2007, 2011, and 2017, respectively. He is currently working as a Senior Lecturer in Department of Mathematics from YUMSUK, Kano since 2013. He has published more than 10 research papers in reputed international journals and conferences including IEEE and it's also available online. His main research work focuses on Operations Research. He has 11 years of teaching experience and 8 years of research experience.

Nazir Isma'il Ibrahim earned his BSc., and currently studying M. Sc., in in Mathematics from YUMSUK Kano in 2014, and 2024, respectively. He is currently working as a Lecturer in Department of Basic Studies from Kano State polytechnic, Kano since 2015. His main research work focuses on Operations Research. He has 10years of teaching experience and 2 years of research experience.

