A Deterministic Inventory Model for Perishable Items with Lead Time and Price Dependent Demand

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Abstract- In the present paper, an attempt has been made to develop a deterministic inventory model for perishable items with lead time and price dependent demand. Shortages are allowed and completely backlogged. The problem of perishability or deterioration plays an important role in the field of inventory control and management. The purpose of our study is to minimize the total variable inventory cost during a given period of time. A numerical example is given to demonstrate the developed model.

Keywords- Inventory, Deterioration, Lead Time and Price Dependent Demand

I. INTRODUCTION

Academi Some researchers have great interest in the development of inventory control and their uses. There are many goods which either deteriorate or become obsolete with passage of time. For such perishable products different modeling techniques are applied. Perishable inventory forms a small part of total inventory and includes fashionable garments, electronic items, digital products and periodicals. The perishable products can be classified based on two categories: (1) deterioration (2) obsolescence. Deterioration is defined as damage, decay or spoilage of the items that are stored for future use and which always lose part of their value with passage of time. Obsolescence occurs due to the arrival of new and better products in the market.

In the existing literature, some inventory models which were developed by contemporary researchers considering some or all of the parameters related to constant demand rate, increasing/decreasing function of time, price and stock dependent have been quoted. The demand of newly arrived products in market is influenced by their prices, because the attractive prices or offers on the products motivate the customers to buy more. This situation increases the order quantity of the retailers or customers. In recent years some researchers also gave their attention towards a time dependent rate, because the demand of newly launched products such as fashionable garments, electronic items, motor vehicles, mobiles etc. increases with time and later it becomes constant.


II. ASSUMPTIONS NOTATIONS

We consider the following assumptions and notations

1. The demand rate is \( R(p) = ap^{-b}, \quad a, b > 0 \)
   
   Here \( p \) is the selling price.

2. The deterioration rate is taken as \( \theta(t) = \theta t \).

3. \( a_c \) is the ordering cost per order.

4. \( h_i \) is the holding cost per unit time.

5. \( s_i \) is the shortage cost per unit time.

6. \( p_c \) is the purchase cost per unit time.

7. \( T \) is the replenishment cycle length.

8. \( I(t) \) is the inventory level at any time \( t \) in \( [0, T] \).

9. \( T_i \) is the time at which inventory level becomes zero.

10. \( TC(L, T, T) \) is the total variable inventory cost per cycle.

11. The replenishment rate is infinite.

12. The lead time is \( L \).

13. There is no repair or replacement of the deteriorated items.

III. MATHEMATICAL FORMULATION

Suppose an inventory system contains the maximum inventory level \( Q + R(p) \) in the beginning of each cycle, where \( R(p) \) is the price dependent demand. During the interval \( [L, T_i] \), the inventory level decreases due to both demand and deterioration and it becomes zero at \( t = T_i \). During the shortage interval \( [T_i, T] \) the demand is unsatisfied. The instantaneous inventory level at any time \( t \) in \( [L, T] \) is given by the following differential equations:
The solutions of the equations (1) and (2) are given by the equations (3) and (4) respectively. By considering the first degree terms in $\theta$, we have

$$I = a p^{-b} \left[ T_1 - t + \frac{\theta}{6} T_1^3 + \frac{\theta}{3} t^3 - \frac{\theta}{2} T_1 t^2 \right]$$  

$$I = a p^{-b} \left[ T_1 - t \right]$$

(3)  

(4)

The maximum inventory level is obtained by putting $t = L$ in equation (3), so

$$Q = a p^{-b} \left[ T_1 - L + \frac{\theta}{6} T_1^3 + \frac{\theta}{3} L^3 - \frac{\theta}{2} T_1 L^2 \right]$$

$$Q = a p^{-b} \left[ T_1 - L \right]$$

(5)

The quantity $Q + LD(p)$ is ordered in the beginning of each cycle. The maximum back ordered quantity $I_B$ is obtained by putting $t = T$ in equation (4). Therefore

$$I_B = a p^{-b} \left[ T_1 - T \right]$$

(6)

The ordering cost per cycle is

$$O_c = o_c$$

(7)

The holding cost per cycle is

$$H_c = h_c \int_L^{T} I(t) dt$$

Or
\[ H_c = a h_c p^{-b} \left[ \frac{1}{2} T_i^2 - LT_i + \frac{1}{2} L^2 + \frac{\theta}{12} T_i^4 \right. \]
\[ \left. \frac{- \theta}{12} L^4 + \frac{\theta}{6} T_i L^3 \right] \quad (8) \]

The deterioration cost per cycle is

\[ D_c = d_c \left[ Q - \int R(t) dt \right] \]

Or

\[ D_c = a d_c p^{-b} \left[ \frac{\theta}{6} T_i^3 + \frac{\theta}{3} L^3 - \frac{\theta}{2} T_i L^2 \right] \quad (9) \]

The shortage cost per cycle is

\[ S_c = -s_c \int \frac{T}{T_i} I(t) dt \]

Or

\[ S_c = a s_c p^{-b} \left[ \frac{1}{2} T_i^2 + \frac{1}{2} T^2 - TT_i \right] \quad (10) \]

The purchase cost per cycle is

\[ P_c = p_c \left[ Q + I_B \right] \]

Or

\[ P_c = a p_c p^{-b} \left[ 2T_i - T - L + \frac{\theta}{6} T_i^3 + \frac{\theta}{3} L^3 - \frac{\theta}{2} T_i L^2 \right] \quad (11) \]

The total variable inventory cost per cycle is

\[ TC(L, T_i, T) = \frac{1}{T} \left[ O_c + H_c + D_c + S_c + P_c \right] \quad (12) \]

Putting the values of \( O_c, H_c, D_c, S_c \) and \( P_c \) in equation (12), we obtain
On solving these equations, we find the optimum values of \(L, T_1\) and \(T\) for which the total variable inventory cost is minimum.

The sufficient conditions for \(TC(L, T_1, T)\) to be minimum are that the principal minors of Hessian matrix or H matrix are positive definite. The Hessian matrix is defined as follows

\[
H = \left[ \begin{array}{ccc} \frac{\partial^2 TC(L, T_1, T)}{\partial L^2} & \frac{\partial^2 TC(L, T_1, T)}{\partial L \partial T_1} & \frac{\partial^2 TC(L, T_1, T)}{\partial L \partial T} \\ \frac{\partial^2 TC(L, T_1, T)}{\partial T_1 \partial L} & \frac{\partial^2 TC(L, T_1, T)}{\partial T_1^2} & \frac{\partial^2 TC(L, T_1, T)}{\partial T_1 \partial T} \\ \frac{\partial^2 TC(L, T_1, T)}{\partial T \partial L} & \frac{\partial^2 TC(L, T_1, T)}{\partial T \partial T_1} & \frac{\partial^2 TC(L, T_1, T)}{\partial T^2} \end{array} \right]
\]

Partially differentiating equation (13), we have

\[
\frac{\partial TC(L, T_1, T)}{\partial L} = \frac{a p^b}{T} \left[ -p_c + h_c L - h_c T_1 + \theta(d_c + p_c)L^2 - \theta(d_c + p_c)T_1L - \frac{\theta h_c}{3} L^2 + \frac{\theta h_c}{2} T_1^2 \right] \quad (14)
\]

\[
\frac{\partial TC(L, T_1, T)}{\partial T_1} = \frac{a p^b}{T} \left[ 2p_c + (h_c + s_c)T_1 - h_c L - s_c T + \frac{\theta(d_c + p_c)}{2} T_1^2 - \frac{\theta(d_c + p_c)}{2} L^2 + \frac{\theta h_c}{3} T_1^3 \\ + \frac{\theta h_c}{6} L^3 \right] \quad (15)
\]

\[
\frac{\partial TC(L, T_1, T)}{\partial T} = \frac{a p^b}{T} \left[ -p_c + s_c T - s_c T_1 \right] - \frac{1}{T^2} \left[ o_c + a p^b \left( 2p_c T_1 - p_c L - p_c T + \frac{(h_c + s_c)}{2} T_1^2 \right. \right.
\]

\[
+ \frac{h_c}{2} L^2 - h_c LT_1 + \frac{s_c}{2} T^2 - s_c TT_1 + \frac{\theta(d_c + p_c)}{6} T_1^3 + \frac{\theta(d_c + p_c)}{3} L^3 - \frac{\theta(d_c + p_c)}{2} T_1 L^2 \]
\[
\frac{\partial^2 TC(L,T_1,T)}{\partial L^2} = \frac{a \, p^{-b}}{T} \left[ h_c + 2\theta(d_c + p_c)L - \theta(d_c + p_c)T_1 - \theta h_c L^2 + \theta h_c T_1 L \right]
\]

IV. NUMERICAL EXAMPLE

Let us consider the following data for parameters in the appropriate units as follows:

\[
H = \begin{bmatrix}
-17.8959 & 17.1981 & 0.0131 \\
17.1981 & 29.5256 & 13.8449 \\
0.0131 & -14.2443 & 9.2057
\end{bmatrix}
\]
\( a = 300, \ b = 1, \ o_c = 100, \ h_c = 5, \ d_c = 2, \ s_c = 8, \ p_c = 10, \ p = 25, \ \theta = 0.05 \)

Table 1, variation in total inventory cost with respect to \( \theta \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( L )</th>
<th>( T_1 )</th>
<th>( T )</th>
<th>( TC(L,T_1,T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>15.2579</td>
<td>6.0163</td>
<td>10.4284</td>
<td>303.5640</td>
</tr>
<tr>
<td>0.10</td>
<td>12.4212</td>
<td>4.4046</td>
<td>8.8553</td>
<td>307.2574</td>
</tr>
<tr>
<td>0.15</td>
<td>11.2630</td>
<td>3.7017</td>
<td>8.4324</td>
<td>334.1419</td>
</tr>
<tr>
<td>0.20</td>
<td>10.6132</td>
<td>3.2903</td>
<td>8.3558</td>
<td>366.2981</td>
</tr>
<tr>
<td>0.25</td>
<td>10.1905</td>
<td>3.0164</td>
<td>8.4296</td>
<td>399.6628</td>
</tr>
</tbody>
</table>

From the table 1, we see that if we increase the deterioration parameter \( \theta \) then the values of \( L, T_1 \) and \( T \) are decreased, but the values of \( TC(L,T_1,T) \) get increased.

Table 2, variation in total inventory cost with respect to \( a \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>( L )</th>
<th>( T_1 )</th>
<th>( T )</th>
<th>( TC(L,T_1,T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>15.2579</td>
<td>6.0163</td>
<td>10.4284</td>
<td>303.5640</td>
</tr>
<tr>
<td>400</td>
<td>15.2507</td>
<td>6.0088</td>
<td>10.3936</td>
<td>401.0250</td>
</tr>
<tr>
<td>500</td>
<td>15.2449</td>
<td>6.0028</td>
<td>10.3660</td>
<td>498.3465</td>
</tr>
<tr>
<td>600</td>
<td>15.2423</td>
<td>6.0000</td>
<td>10.3531</td>
<td>595.7901</td>
</tr>
<tr>
<td>700</td>
<td>15.2404</td>
<td>5.9981</td>
<td>10.3439</td>
<td>693.2439</td>
</tr>
</tbody>
</table>

From this table, we see that if we increase the demand parameter \( a \), then the values of \( L, T_1 \) and \( T \) are decreased, but the values of \( TC(L,T_1,T) \) get increased.

Figure 2, variation in \( TC \) with respect to \( \theta \), Figure 3, variation in \( TC \) with respect to \( a \)

Table 3, variation in total inventory cost with respect to \( b \)

<table>
<thead>
<tr>
<th>( b )</th>
<th>( L )</th>
<th>( T_1 )</th>
<th>( T )</th>
<th>( TC(L,T_1,T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.2579</td>
<td>6.0163</td>
<td>10.4284</td>
<td>303.5640</td>
</tr>
<tr>
<td>2</td>
<td>15.8578</td>
<td>6.6203</td>
<td>13.4213</td>
<td>21.3155</td>
</tr>
<tr>
<td>3</td>
<td>19.9962</td>
<td>10.1909</td>
<td>40.6606</td>
<td>4.5005</td>
</tr>
<tr>
<td>4</td>
<td>30.4007</td>
<td>17.8134</td>
<td>183.5960</td>
<td>1.0255</td>
</tr>
</tbody>
</table>
From this table, we see that if we increase the demand parameter \( b \), then the values of \( L, T_1 \) and \( T \) are increased, but the values of \( TC(L, T_1, T) \) get decreased.

![Figure 4, variation in TC with respect to \( b \)](image)

![Figure 5, variation in TC with respect to \( L \)](image)

**V. CONCLUSION**

The results of the proposed model show that the total variable inventory cost is deeply impacted by the parameters \( a \) and \( b \) in comparison with the parameter \( \theta \). This is due to the reason that the newly arrived goods/products in the supermarket increase the demand. The cycle length and lead time are main components for optimizing the cost/profit of an organization. The products such as vegetables, milk, bakery products and newspapers are necessarily to be sold in the market as the cycle length decreases.

**REFERENCES**


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Mr. Sushil Kumar is an assistant professor in the department of mathematics & astronomy, Lucknow University, Lucknow- u.p. (India). His area of specialization is inventory modeling in operations research. He has published 15th research papers in national and international journals.

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