International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Volume-5, Issue-4, pp.09-21, August (2018)
E-ISSN: 2348-4519

Slip Boundary Condition Effects on the Rate of Heat Transfer in A Micro channel Including Viscous Dissipation

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Available online at: www.isroset.org

Accepted 19/Aug/2018, Online 30/Aug/2018

Abstract- This study analyzes laminar forced convection flow in parallel plate micro-channel with slip velocity and temperature jump. Closed-form solutions are obtained for temperature, bulk temperature and rate of heat transfer when both walls of the channel are kept at unequal temperatures. The effects of various controlling parameters such as rarefaction parameter, fluid-wall interaction parameter and Brinkman number on the thermal behavior and rate of heat transfer are discussed with the aid of line graphs. Interesting result from the present work is that increase in rarefaction parameter leads to enhancement in fluid temperature while increase in fluid-wall interaction parameter leads to increase in temperature jump on the walls of the channel. In addition, the rate of heat transfer represented as the Nusselt number at the both walls of the channel displays an unbounded swing which varies with increase in fluid-wall interaction and rarefaction parameters.

Keywords: Micro-channel, forced convection, velocity slip, temperature jump, viscous dissipation, Nusselt number.

NOMENCLETURE

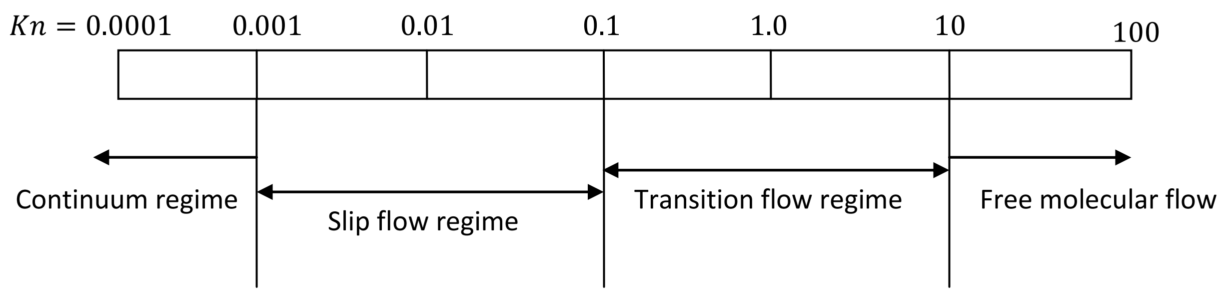
A	degree of asymmetry
Br	Brinkman number
D_h	Hydraulic Diameter
h_1	heat transfer coefficient at lower wall
h_2	heat transfer coefficient at upper wall
k	thermal conductivity of the fluid
Kn	Knudsen number
L	length of wall
ln	fluid-wall interaction parameter
Nu_1	Nusselt number at the lower wall of the channel
Nu_2	Nusselt number at the upper wall of the channel
P	pressure gradient
T	dimensional fluid temperature
T_b	dimensional bulk temperature
T	dimensional temperature
T_1	dimensional temperature at the lower wall
T_2	dimensional temperature at the upper wall
U	dimensionless velocity
u	dimensional velocity
u_{avg}	reference velovcity
X	dimensionless coordinate in axial direction
Y	dimensionless coordinate normal to the axis of the channel
y	dimensional coordinate normal to the axis of the channel

Greek symbols

θ	dimensionless temperature
θ_b	dimensionless mean temperature
β_v, β_t	dimensionless variables
λ	molecular mean free path
μ	dynamic viscosity
υ	kinematic viscosity
ρ	fluid density

I. INTRODUCTION

In the last few decades, research in micro-coolers, micro-fuel cells, micro-biochips and micro-reactors has been growing at a tremendous pace due to rapid developments in micro-electronics and biotechnologies. In micro-fluidic systems, micro-channel has been noticed to be one of the important elements in fluid flow within a miniature area. In addition to been a reactant delivery, the micro-channels are also used to connect different chemical chambers. The design and process controls of micro-fluidic and microelectromechanical systems (MEMS) involve the effect of geometrical configurations on the pressure, velocity and thermal distributions of the fluid on the micrometer $(10^{-6}m)$ scale. Knudsen number (Kn) is an important factor in the flow of fluids through micro-channels and it is defined as the ratio of molecular free path (λ) to characteristic length (L). In the slip flow regime, the gas at the surface has a tangential velocity which causes it to slip along the surface and, the temperature of the gas at the surface is finitely different from the surface temperature resulting to a jump in temperature between the surface and the adjacent gas. However, Beskok and Karniadakis [1] proposed four flow regimes in gases as shown below



Classification of different flow regimes

II. RELATED WORK

Extensive research has been carried out recently in the field of micro geometric flows. Weng and Cheng [2] studied analytically the fully developed natural convection in open ended vertical parallel plate micro-channel, taking into account the effects of rarefaction and fluid wall interaction parameters on the volume flow rate and heat transfer. Fully developed thermocreep-driven gas micro-flow has been investigated by Weng *et al* [3] Laminar convection in a vertical channel with viscous dissipation and buoyancy effects have been examined by Barletta [4]. Recently, Khadrawi [5] studied the transient hydrodynamic and thermal behaviors of fluid flow in a vertical porous micro-channel under the effect of hyperbolic heat conduction model. Using semi-implicit method, Biswal *et al.* [6] considered the hydrodynamic and heat transfer behavior in the developing region of a micro-channel. It was deduced that slip velocity and jump temperatures enhances the rate of heat transfer.

Similarly, the effect of viscous dissipation has received considerable attention owing to its importance in the heat transfer point of view. Citing few works, Mondal and Sanchayan [7] investigated viscous dissipation effect on the limiting value of Nusselt numbers for shear driven flow through asymmetrically heated parallel plates. Tso *et al.* [8] and Aydin and Avci [9] studied viscous dissipation effects of power law fluid flow within parallel plates with constant heat fluxes and its effects on the rate of heat transfer in a Poiseuille flow respectively. Barletta [10] investigated laminar mixed convection with viscous dissipation in a vertical channel. Recently, Ramjee and Satyamurty [11] studied the Nusselt number for viscous dissipation flow between parallel plates kept at unequal temperatures. In another related work, Hatton and Turton [12] investigated Heat transfer in the thermal entry length with laminar flow between parallel walls at unequal temperatures. The Nusselt number for laminar forced convection in asymmetrically heated annuli with viscous dissipation was presented by Kumar and Satyamurty [13].

Therefore, the objective of the present work is to analyze the influence of fluid-wall interaction parameter (ln), rarefaction parameter $(\beta_v Kn)$ and viscous dissipation on the steady forced convective flow through a micro-channel. The present study extends the work of Ramjee and Satymurty[11] by taking into account the effect of velocity slip and temperature jump boundary conditions on the thermal characteristics and rate of heat transfer at both walls of the channel.

III. MATHEMATICAL FORMULATION

Consider a fully developed flow of viscous incompressible fluid in a micro-channel formed by two parallel plates. The x coordinates coincides with the centerline of the channel and the y coordinate is normal to it and the distance between the two plates is L as shown in figure 1. The plates are positioned at $y = \pm 1/2$ and the average and inlet fluid temperatures are given as $\bar{T}$ and T_i respectively. The plates are kept at unequal temperatures of $T_1\ at\ y = -1/2$ and $T_2\ at\ y = +1/2$.

The mathematical model extended here is a generalization of Ramjee and Satymurty[11] in the presence of slip boundary conditions. Under the usual Boussinesq approximation and neglecting convective terms, the governing equations in dimensional form are

$$\frac{d^2u}{dy^2} - \frac{1}{\rho\nu}\frac{dp}{dx} = 0 \qquad (1)$$

$$\frac{d^2T}{dy^2} + \frac{\mu}{k}\left(\frac{du}{dy}\right)^2 = 0 \qquad (2)$$

The boundary conditions which describe velocity slip and temperature jump conditions at the fluid wall interface are

$$u = \beta_v\lambda\frac{du}{dy}\ , T = T_1 + \beta_t\lambda\frac{dT}{dy}\ at\ y = -\frac{L}{2}$$

$$u = -\beta_v\lambda\frac{du}{dy}\ , T = T_2 - \beta_t\lambda\frac{dT}{dy}\ at\ y = +\frac{L}{2} \qquad (3)$$

Using the following dimensionless quantities

$$U = \frac{u}{u_{avg}}, Y = \frac{y}{2L}, P = \frac{1}{\rho\nu}\frac{dp}{dx}\frac{4L^2}{u_{avg}}, \theta = \frac{T-\bar{T}}{T_i-\bar{T}}, \bar{T} = \frac{T_1+T_2}{2}$$

$$Br = \frac{\mu u_{avg}^2}{k(T_i-\bar{T})}, Kn = \frac{\lambda}{2L}, \beta_t = ln = \frac{\beta_t}{\beta_v} \qquad (4)$$

Equations $(1)-(3)$ in dimensionless form are given as

$$\frac{d^2U}{dY^2} = P \qquad (5)$$

$$\frac{d^2\theta}{dY^2} + Br\left(\frac{dU}{dY}\right)^2 = 0 \qquad (6)$$

Subject to the relevant boundary conditions

$$U = \beta_v Kn\frac{dU}{dY}\ , \theta = -\frac{(1-A)}{(1+A)} + \beta_v Knln\frac{d\theta}{dY}\ at\ Y = -\frac{1}{4}$$

$$U = -\beta_v Kn\frac{dU}{dY}\ , \theta = \frac{(1-A)}{(1+A)} - \beta_v Knln\frac{d\theta}{dY}\ at\ Y = +\frac{1}{4} \qquad (7)$$

Where $A = \frac{(T_2-T_i)}{(T_1-T_i)}$

Solving $(5) and\ (6)$ using (7) gives dimensionless velocity and temperature solutions respectively

$$U = \frac{P}{32}\{16Y^2 - 8\beta_v Kn - 1\} \quad (8)$$

$$\theta = \frac{BrP^2}{3072}\{16\beta_v Knln - 256Y^4 + 1\} + \frac{(1-A)}{(1+A)}a_0 Y \quad (9)$$

The definition for the heat coefficient h_1, based on bulk temperature as the reference fluid temperature at the wall $y = -{}^1/_2$ is given by

$$-k\left(\frac{\partial T}{\partial y}\right)\Big|_{y=-\frac{L}{2}} = h_1(T_1 - T_b) \quad (10)$$

Where T_b the bulk temperature is defined by

$$\theta_b = \int_{-\frac{1}{4}}^{\frac{1}{4}} U\theta dY \Bigg/ \int_{-\frac{1}{4}}^{\frac{1}{4}} U dY \quad (11)$$

Using equation (8) and (9) for U and θ respectively, equation (11) yields

$$\theta_b = \frac{BrP^2}{64a_3}\left\{\frac{a_3 a_4}{3} - \frac{a_1}{112} - \frac{a_2}{5}\right\} \quad (12)$$

The Nusselt number at the lower plate $(Y = -1/4)$ and upper plate $(Y = 1/4)$ are given by

$$Nu_1 = (h_1 D_h)/k = \left\{1\Big/\left(\theta_b - \theta|_{Y=-\frac{1}{4}}\right)\right\}\left(\frac{\partial\theta}{\partial Y}\right)\Big|_{Y=-\frac{1}{4}} \quad (13)$$

$$Nu_2 = (h_2 D_h)/k = \left\{1\Big/\left(\theta_b - \theta|_{Y=\frac{1}{4}}\right)\right\}\left(\frac{\partial\theta}{\partial Y}\right)\Big|_{Y=\frac{1}{4}} \quad (14)$$

Employing equation (9) for θ and (12) for θ_b in equations (13) and (14), Nu_1 and Nu_2 are obtained respectively as

$$Nu_1 = \frac{a_0\{(1-A)/(1+A)\} + \frac{BrP^2}{192}}{\frac{BrP^2}{64a_3}\left\{\frac{a_3 a_4}{3} - \frac{a_1}{112} - \frac{a_2}{5}\right\} + \{(1-A)/(1+A)\}(1-a_6) - BrP^2 a_5} \quad (15)$$

$$Nu_2 = \frac{a_0\{(1-A)/(1+A)\} - \frac{BrP^2}{192}}{\frac{BrP^2}{64a_3}\left\{\frac{a_3 a_4}{3} - \frac{a_1}{112} - \frac{a_2}{5}\right\} + \{(1-A)/(1+A)\}(a_6-1) - BrP^2 a_5} \quad (16)$$

IV. RESULTS AND DISCUSSION

The basic parameters that govern this flow are Brinkman number (Br), degree of asymmetry heating (A), rarefaction parameter $(\beta_v Kn)$ and fluid-wall interaction parameter (ln). To examine the effects of these basic parameters, the variations in velocity, temperature and Nusselt number at both walls of the channel are graphically presented and discussed. The present parametric study has been performed in continuum and slip flow regimes $(Kn \leq 0.1)$. Also, for air and various surfaces, the values of β_t and β_v range from near 1.64 to more than 10 and from near 1 to 1.667 respectively. Therefore this study has been performed over the reasonable ranges of $0 \leq ln \leq 10$ and $0 \leq \beta_v Kn \leq 0.1$. The chosen reference values of $\beta_v Kn$ and ln for this analysis are 0.05 and 1.64 respectively as given in Weng and Chen[2]. Furthermore, three different degrees of asymmetry parameter $A = -0.5, 0.0$ and 0.5 have been considered.

Hence the results can be discussed in the interval $-1 \leq A \leq 1$, where $A = 1$ corresponds to symmetric heating/cooling, $T_1 = T_2 = T$ and $A \neq 1$ shall be taken as asymmetric heating/cooling. Furthermore, positive values of Brinkman number $(Br > 0)$ are compatible with the fluid cooling case while negative values of Brinkman number $(Br < 0)$ are compatible with the fluid heating case with reference to $\bar{T}$.

Figure 2 shows the variation in velocity for different values of rarefaction parameter $(\beta_v Kn)$. It is observed from fig. 2 that as rarefaction parameter $(\beta_v Kn)$ increases, the velocity slip at both walls of the channel increases. The rarefaction parameter $(\beta_v Kn)$ reduces the retarding effects of the surface of the channel. This yields a significant increase in the fluid velocity near both surfaces of the channel.

Fig. 3 and 4 reveals the variations in temperature distribution for different values of rarefaction parameter $(\beta_v Kn)$ at $A = -0.5\ and\ 0.5$ respectively. It is observed from both figures that increase in rarefaction parameter $(\beta_v Kn)$ leads to increase in temperature jump on the walls of the channel. This is attributed to the weak interaction between the fluid molecules and the heated wall of the channel. Also, an increase in rarefaction parameter $(\beta_v Kn)$ leads to enhancement in fluid temperature. Viscous dissipation converts the kinetic energy of the moving fluid into internal energy thereby increasing the fluid temperature. It also accounts for the distortion on the temperature profile.

Fig 5 and 6 depicts the variations in temperature distribution for different values of fluid-wall interaction parameter (ln) at two degrees of asymmetric heating $A = -0.5\ and\ 0.5$ respectively. It is interesting to note that increase in fluid-wall interaction parameter (ln) brings about increase in temperature jump on both walls of the channel. In addition, when positive values of dissipation are considered, increase in fluid-wall interaction parameter (ln) leads to increase in temperature while at negative dissipation, increase in fluid-wall interaction parameter (ln) leads to decrease in temperature.

Fig 7 shows the rate of heat transfer at the wall $Y = -\frac{1}{4}$ plotted against the Brinkman number (br) for $A = -0.5$. It is interesting to note from the figure that increase in rarefaction parameter $(\beta_v Kn)$ leads to an increase in rate of heat transfer for negative values of Brinkman number (br) while a reverse trend is observed for positive values of Brinkman number (br).

Fig 8 and 9 depicts the rate of heat transfer at the wall $Y = -\frac{1}{4}$ for different values of rarefaction parameter $(\beta_v Kn)$ at $A = 0.0\ and\ 0.5$ respectively. It is observed from the figures that increase in rarefaction parameter $(\beta_v Kn)$ produces an unbounded swing on the rate of heat transfer for some Brinkman number $(br < 0)$. This phenomenon is attributed to the fact that the rate of heat transfer attains a larger value when the bulk temperature of the fluid equals the average wall temperature. The unbounded swing on the rate of heat transfer for $A = -0.5\ and\ A = 0.0$ not displayed in figures 7 and 8 occurs at some Brinkman number $(br < -2)$.

The effects of the fluid-wall interaction parameter (ln) on the rate of heat transfer at wall $Y = -\frac{1}{4}$ for $A = -0.5$ are shown in fig 10. It is evident that increase in fluid-wall interaction parameter (ln) leads to decrease in the rate of heat transfer for some Brinkman number $(br < 0)$ while a reverse effect of increasing the fluid-wall interaction parameter (ln) on the rate of heat transfer is observed for some Brinkman number $(br > 0)$.

Fig 11 and 12 illustrates the effect of fluid-wall interaction parameter (ln) on the rate of heat transfer at wall $Y = -\frac{1}{4}$ for $A = 0.0\ and\ A = 0.5$. It is noticed from both figures that increase in fluid wall interaction leads to decrease in the rate of heat transfer. In addition, it is evident that the rate of heat transfer is slowest at fluid wall interaction parameter $(ln = 10)$, for $A = 0.0$ while the rate of heat transfer is slowest at fluid wall interaction parameter $(ln = 0.0)$, for $A = 0.5$.

Fig 13 depicts the effects of rarefaction parameter $(\beta_v Kn)$ on the rate of heat transfer at the wall $Y = \frac{1}{4}$ for $A = -0.5$. I it noticed that increase in rarefaction parameter $(\beta_v Kn)$ leads to increase in rate of heat transfer for negative dissipation while an opposite trend of behavior is observed for negative dissipation.

The influence of rarefaction parameter $(\beta_v Kn)$ on the rate of heat transfer at wall $Y = \frac{1}{4}$ for $A = 0.0\ and\ 0.5$ is illustrated in figures 14 and 15. It is clear from the figures that increase in rarefaction parameter $(\beta_v Kn)$ generates an unbounded swings on the rate of heat transfer for positive dissipation while it has an insignificant effect for negative dissipation.

Fig 16 demonstrates the effects of fluid-wall interaction parameter (ln) on the rate of heat transfer at wall $Y = \frac{1}{4}$ for $A = -0.5$. It is evident that increase in fluid-wall interaction leads to decrease in rate of heat transfer for negative dissipation, while for positive dissipation, increase in fluid-wall interaction leads to increase in the rate of heat transfer.

Fig. 17 and 18 depicts the variation in rate of heat transfer versus Brinkman number (br) for different values of fluid-wall interaction parameter (ln) for $A = 0.0\ and\ 0.5$ respectively. It is observed from both figures that increase in fluid-wall interaction parameter (ln) leads to increase in rate of heat transfer at wall $Y = \frac{1}{4}$ for positive dissipation. In addition, it is evident that the rate of heat transfer is significantly influenced by high value of fluid-wall interaction parameter (ln) for

$A = 0.0$, while small value of fluid-wall interaction parameter (ln) has significant effect on the rate of heat transfer for $A = 0.5$.

Finally, it is worthy to note from equations (15) *and* (16) that the rate of heat transfer on both walls vary with degree of asymmetric heating (A) when dissipation is not neglected while the rate of heat transfer vary with dissipation when degree of asymmetric heating $(A \neq 1)$.

V. CONCLUSION

This study considered fully developed laminar forced convection flow through parallel plate micro-channel under slip and jump boundary conditions. Exact solutions for the mathematical model governing the present physical situation and the expressions for the rate of heat transfer at both walls of the channel have been obtained. This study conforms to the findings of Ramjee and Satyamurty[11] when $ln, \beta_v Kn \to 0$. The following deductions were made from the present work:

i. The increase in rarefaction parameter $(\beta_v Kn)$ and fluid-wall interaction parameter (ln) leads to increase in temperature jump.
ii. At positive dissipation, increase in fluid-wall interaction parameter (ln) leads to increase in temperature while at negative dissipation, fluid temperature decreases with increase in fluid-wall interaction parameter (ln).
iii. The rate of heat transfer at both walls of the channel depends strongly on the degree of asymmetric heating $(A \neq 1)$ and viscous dissipation $(br \neq 0)$.
iv. The rate of heat transfer at the wall $Y = -\frac{1}{4}$ increases with an increase in rarefaction parameter $(\beta_v Kn)$ for negative dissipation while for positive dissipation, increase in rarefaction parameter $(\beta_v Kn)$ has an insignificant effect on the rate of heat transfer.

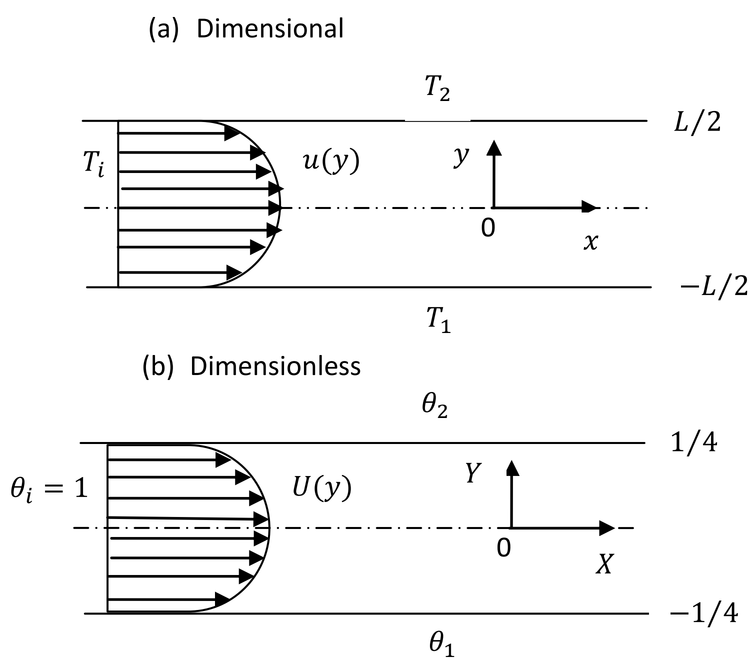


Fig. 1. Flow configuration and coordinate system

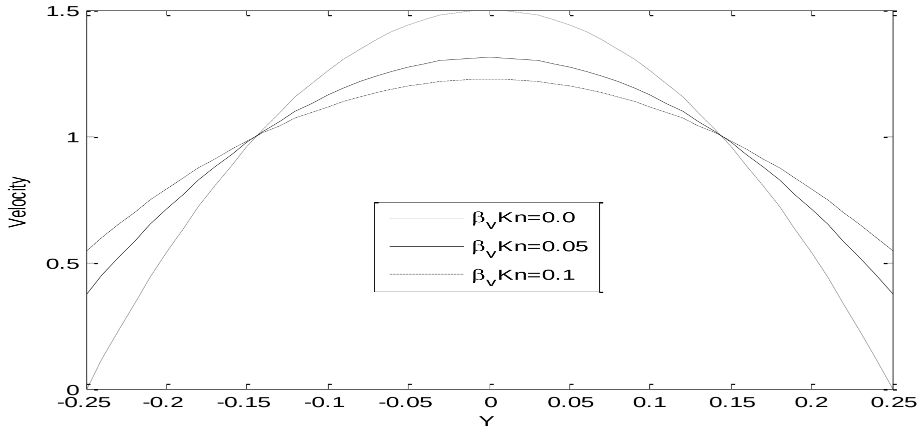


Figure 2. Velocity distribution for different values of rarefaction parameter $(\beta_v Kn)$

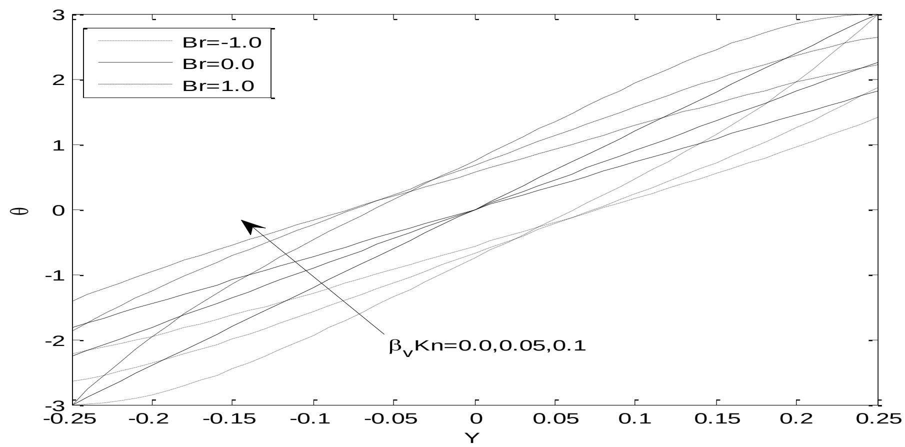


Figure 3 Temperature distribution for different values of $(\beta_v K_n)$ for $A = -0.5$, $ln = 1.64$.

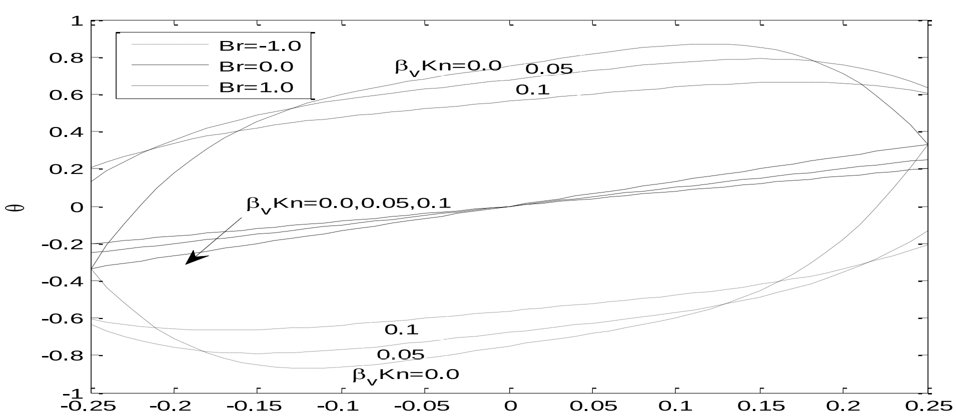


Figure 4 Temperature distribution for different values of $\beta_v Kn$ for $A = 0.5$, $ln = 1.64$.

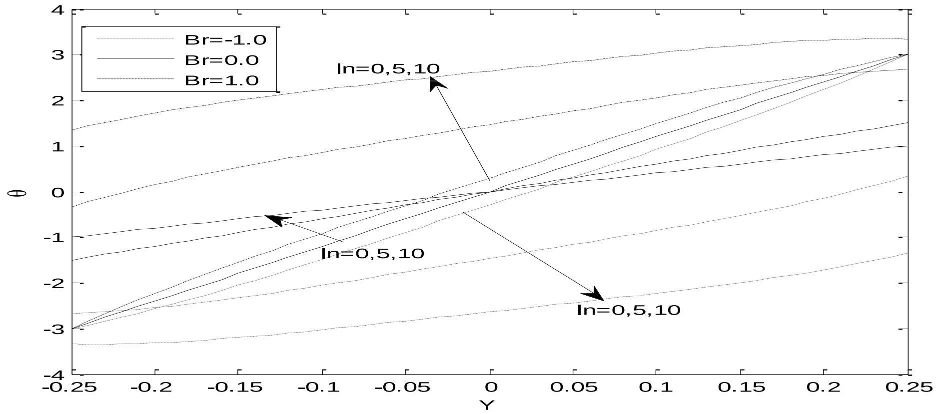


Figure 5 Temperature distribution for different values of ln for $\beta_v Kn = 0.05$, $A = -0.5$.

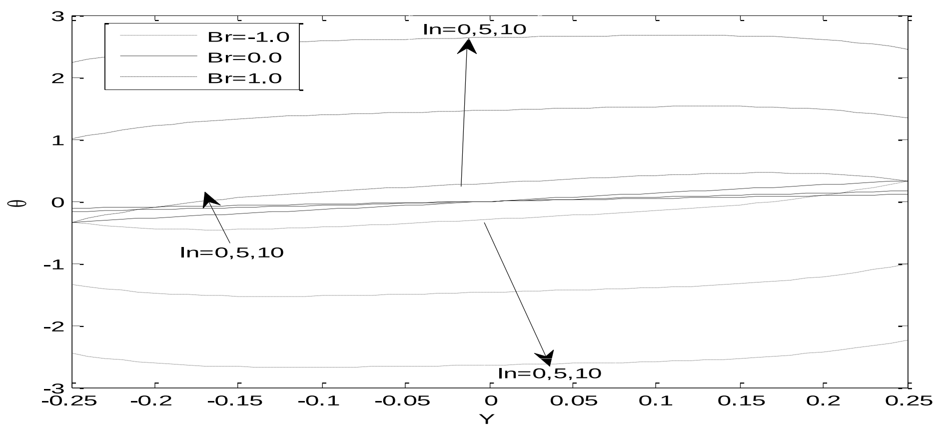


Figure 6 Temperature distribution for different values of ln for $\beta_v Kn = 0.05.$, $A = 0.5$.

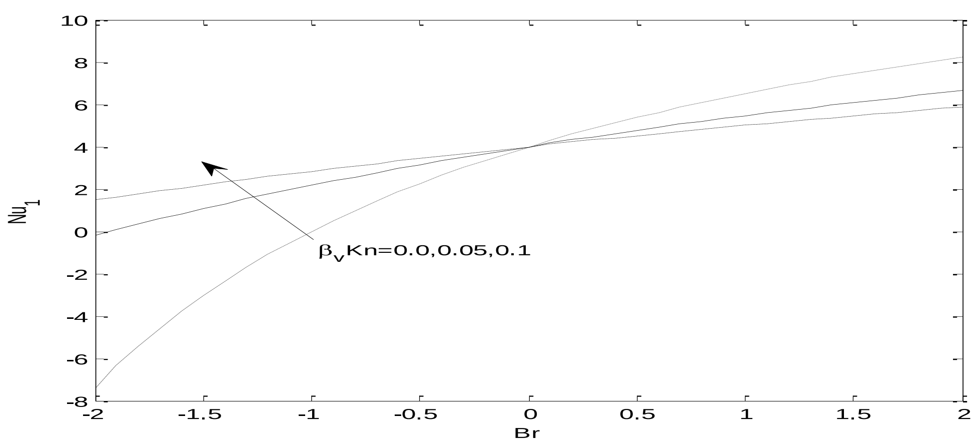


Figure 7 Variation of Nu_1 with Br for different values of $\beta_v Kn$ for $A = -0.5$, $ln = 1.64$

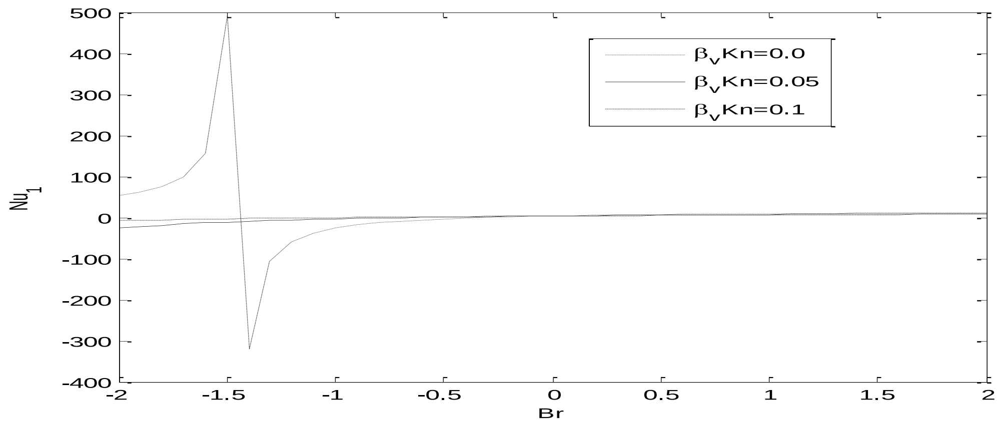


Figure 8 Variation of Nu_1 with Br for different values of $\beta_v Kn$ for $A = 0.0$, $ln = 1.64$

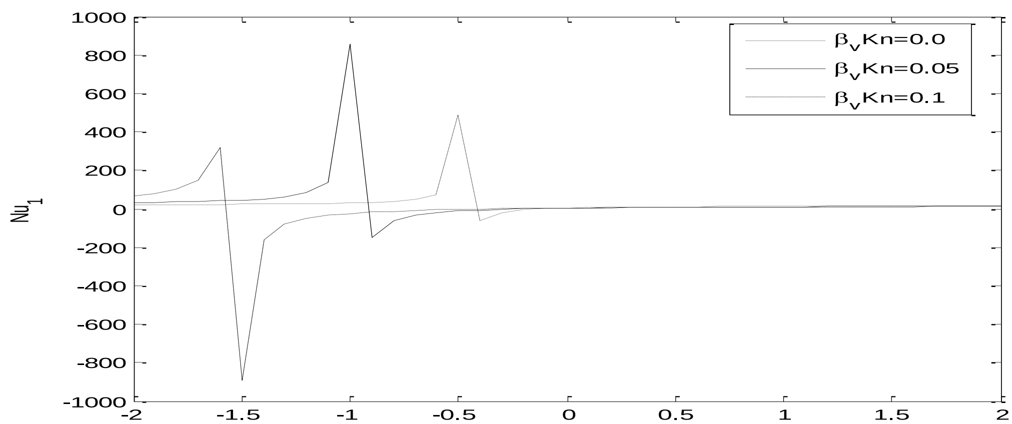


Figure 9 Variation of Nu_1 with Br for different values of $\beta_v Kn$ for $A = 0.5$, $ln = 1.64$

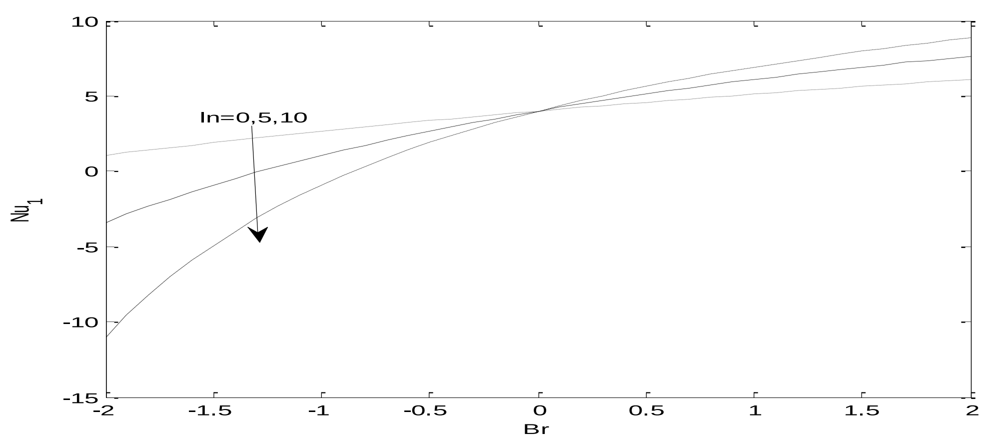


Figure 10 Variation of Nu_1 for different values of ln for $A = -0.5$, $\beta_v Kn = 0.05$

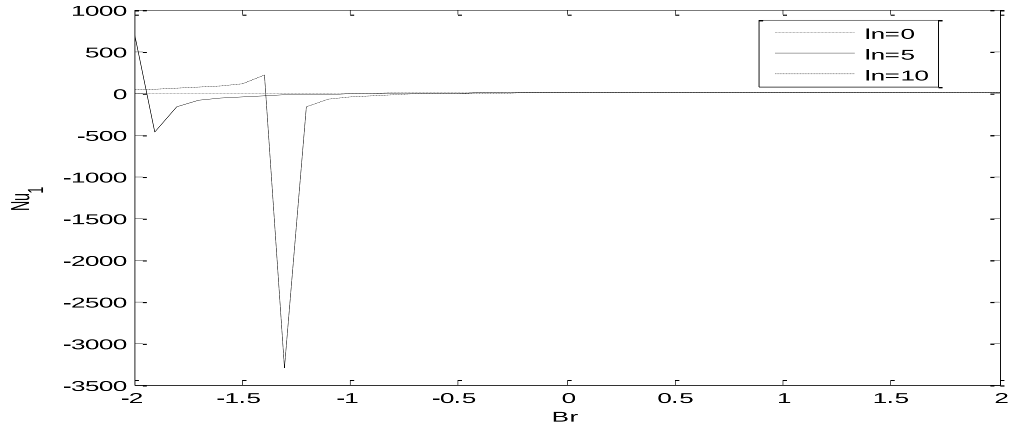


Figure 11 Variation of Nu_1 for different values of ln for $A = 0.0$, $\beta_v Kn = 0.05$

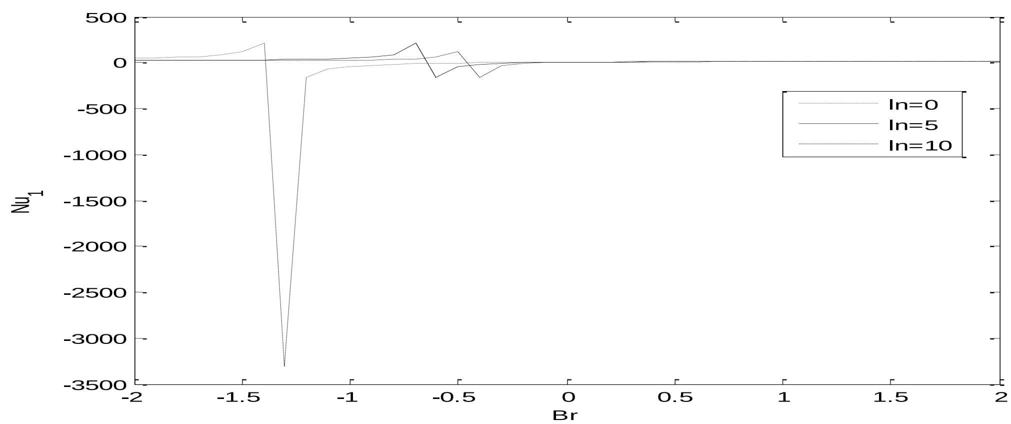


Figure 12 Variation of Nu_1 for different values of ln for $A = 0.5$, $\beta_v Kn = 0.05$

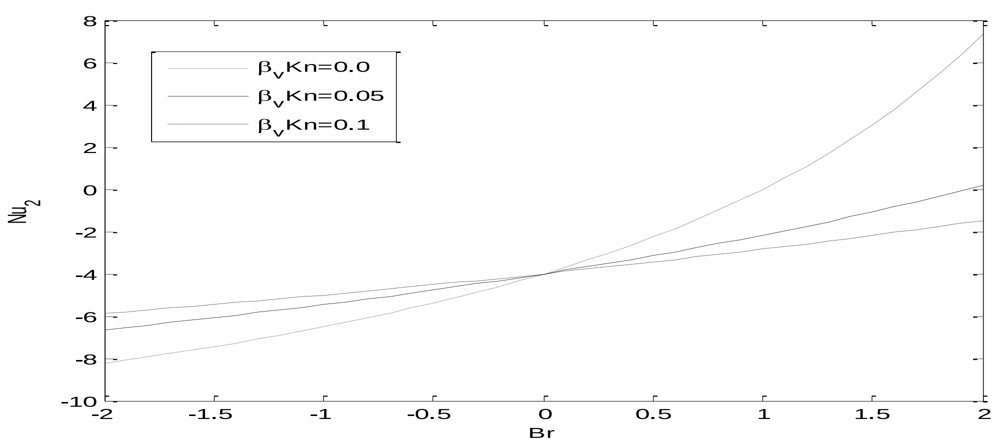


Figure 13 Variation of Nu_2 for different values of $\beta_v Kn$ for $A = -0.5$, $ln = 1.64$

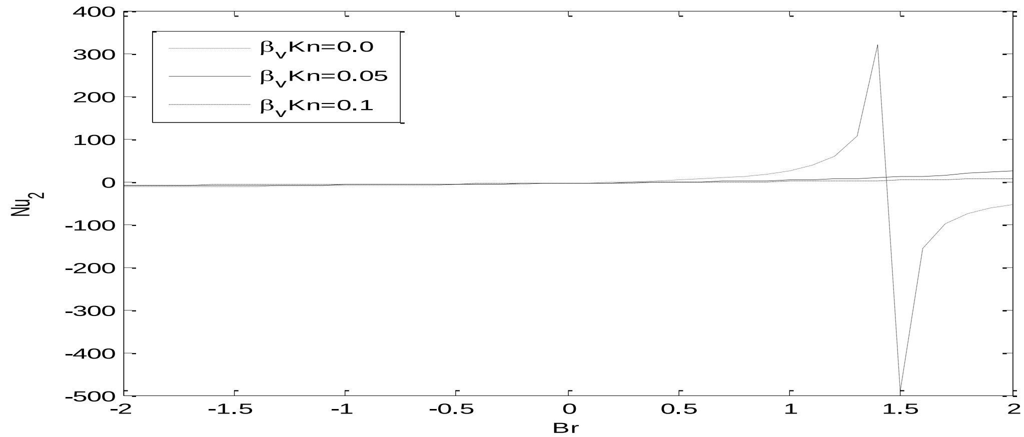


Figure 14 Variation of Nu_2 for different values of $\beta_v Kn$ for $A = 0.0$, $ln = 1.64$

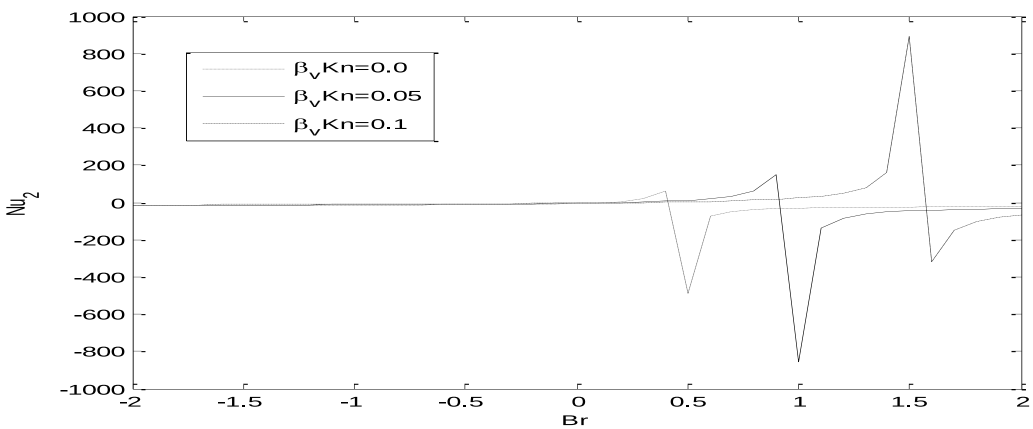


Figure 15 Variation of $Nu_2$for different values of $\beta_v Kn$ for $A = 0.5$, $ln = 1.64$

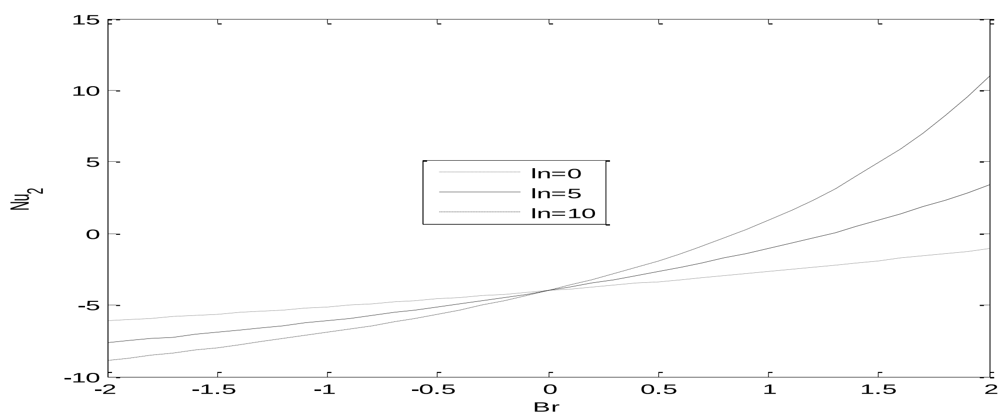


Figure 16 Variation of Nu_2 for different values of ln for $A = -0.5$, $\beta_v Kn = 0.05$

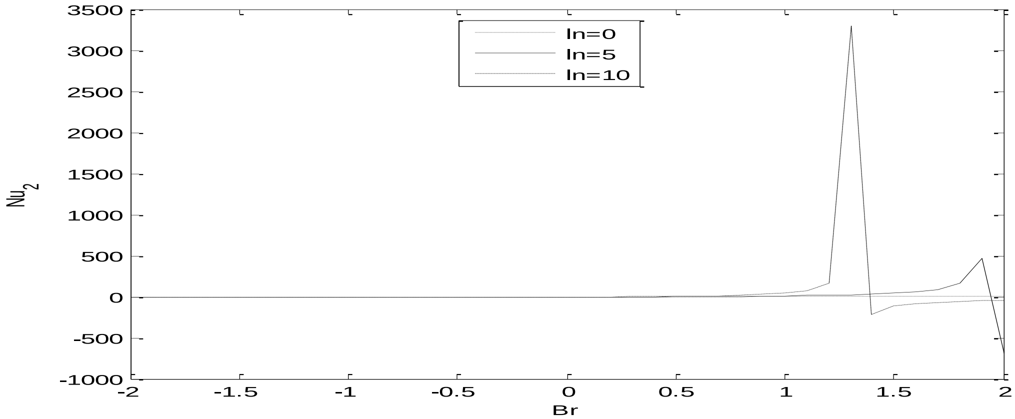


Figure 17 Variation of Nu_2 with Br for different values of ln for $A = 0.0$, $\beta_v Kn = 0.05$

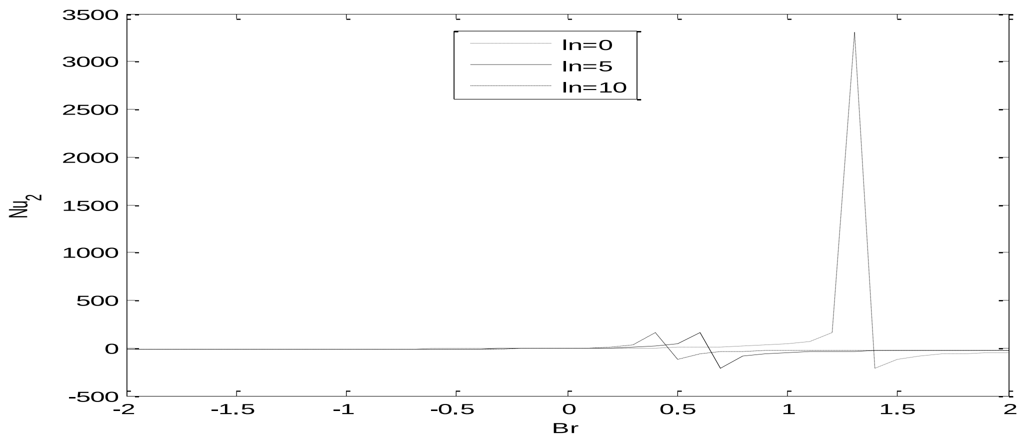


Figure 18 Variation of Nu_2 with Br for different values of ln for $A = 0.5$, $\beta_v Kn = 0.05$.

Funding

This research received no grant from any funding public agency, commercial, institution or any organization.

Appendix A

Constants used in the present work are as follows:

$$a_0 = \frac{4}{(1+4\beta_v Knln)}, a_1 = \frac{P}{2}, a_2 = \frac{-p(1+8\beta_v Kn)}{32}, a_3 = -p(1+12\beta_v Kn)$$

$$a_4 = \frac{(1+16\beta_v Knln)}{16}, a_5 = \frac{\beta_v Knln}{192}, a_6 = a_0\beta_v Knln.$$

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