New Modification of Third-Order Iterative Method with Optimal Fourth-Order Convergence for Solving Nonlinear Equations

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Abstract— Behl’s method is a third-order iterative method involving three functional evaluations for solving nonlinear equation. In this paper, we presented a new iterative method free second derivative with three real parameters by using second-order Taylor expansion. In order to avoid the second derivative, it is approximated by using equality of Chun-Kim’s method and Newton-Steffensen’s method. The result of study shows that the proposed method converges quartically and requires three evaluation of functions per iteration with efficiency index equal to 1.587401. Numerical simulation is presented to examine the performance of the proposed method by using several real test functions. The final results show that the proposed method is more efficient and perform better than some other kind of methods.

Keywords— Behl’s method, Chun-Kim’s method, Third-order iterative method, Newton-Steffensen’s method, Numerical simulation

I. INTRODUCTION

Nonlinear equation is mathematical model in many problems of sciences, engineering and technology. For the most actual cases, an engineer frequently encounter the problem because of the complicated nonlinear equations cannot be solve using an analytical procedure. So, finding a solution of nonlinear equation is one of the important topics that is often discussed in numerical analysis [1].

The problem of nonlinear equation is how to solve the equation as form

\[ f(x) = 0, \quad (1) \]

where \( f : D \subset R \rightarrow R \) is the scalar function in the open interval \( D \).

The alternative solution to find a root of (1) is calculated by arithmetics processing that is known as an iterative method.

Generally, the various classical iterative method are constructed by using Taylor series expansion.

Let \( \alpha \) be a solution of nonlinear equation (1) and \( x_n \) is an approximation solution at \( n \)-th iteration, then Taylor series expansion for \( f(x) \) about \( x_n \) as following

\[ f(x) = f(x_n) + f'(x_n)(x-x_n) + \frac{f''(x_n)}{2!}(x-x_n)^2 + \cdots, \quad (2) \]

The classical iterative method that known widely as basic approximation to solve (1) is written as form

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad (3) \]

The method of (2) converges quadratically with efficiency index equal to \( 2^{1/2} \approx 1.414213 \).

The Newton method in (2) is a second-order iterative method which is constructed by using first-order Taylor expansion. Other second-order iterative methods are Schroder’s method [2] and Steffensen’s method [3].

Then from the second-order Taylor expansion, we get an iterative method having cubically convergence. It is known as Chebyshev’s, Halley’s and Euler’s (Irrational Halley) method that are written as

\[ x_{n+1} = x_n - \left( 1 + \frac{L_f}{2} \right) \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots, \quad (4) \]

\[ x_{n+1} = x_n - \left( 1 + \frac{2}{2-L_f} \right) \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots, \quad (5) \]

and

\[ x_{n+1} = x_n - \left( 1 + \frac{2}{1+\sqrt{1-2L_f}} \right) \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots, \quad (6) \]

where

\[ L_f = \frac{f(x_n)f''(x_n)}{f'(x_n)^2}. \]
Recently, some researchers have been using several technical approximation to construct either the family of second-order or the family of third-order iterative methods, such as: geometrical intepretation [4, 5, 6, 7, 8, 9, 10, 11], circle of curvature [12], Homotopy perturbation method [13, 10], Adomian decomposition method [13, 14, 15, 16], modified Adomian decomposition method [17], and variant of Newton’s method [18, 19, 20, 21].

Now, Behl [6] use an exponential function to construct a third-order iterative method in form

\[ x_{n+1} = x_n - \left( \frac{4-L_f}{4-3L_f} \right) f(x_n). \]  

(7)

All of the third-order iterative method above involves three evaluation of functions. Based on Kung-Traub [22] conjectured that a multipoint iterative method without memory based on \( n \) functional evaluations will be to achieve an optimal order of convergence \( 2^{n-1} \). So, the methods don’t have an optimal order of convergence.

The aim of this paper is to develop third-order iterative method in (7) by using second-order Taylor expansion with three real parameters. The proposed method has second derivative, then we reduced its second derivative by using approximation of equality of two third-order iterative method [23, 24, 25].

This paper is organized as follows. In Section 2, we provide the short reviewing for some definitions and the describing of the proposed iterative method. Section 3 describes the analytical method used to find the order of convergence. The examining performance of the proposed method by using numerical simulation is shown in Section 4. Finally, A conclusion is presented in Section 5.

II. PRELIMINARIS AND THE PROPOSED METHOD

In this section, we give some basic definitions which will be used in this paper. Furthermore, we construct a fourth-order iterative method with optimal order of convergence.

Definition 1. A sequence of iterates \( \{ x_n : n \geq 0 \} \) is said to converge with order \( p \geq 1 \) to a point \( \alpha \) if

\[ \lim_{n \to \infty} \left| \frac{x_{n+1} - \alpha}{x_n - \alpha} \right|^p = c, \]  

(3)

and the error equation is

\[ e_{n+1} = ce_n^p + O(e_n^{p+1}), \]  

(4)

where \( e_n = x_n - \alpha \) and \( c \) is the asymptotic constant error.

For some \( c > 0 \). If \( p = 1 \), the equation is said to converge linearly to \( \alpha \). In that case, we require \( c < 1 \); the constant \( c \) is called the rate of linear convergence of \( x_n \) to \( \alpha \).

Definition 2. Let \( \alpha \) be a root of the function \( f \) and suppose that \( x_{n-1}, x_n, x_{n+1} \) are closer to the root \( \alpha \) then the computational order of convergence (COC) \( p \) can be approximated using the formula

\[ p = \frac{\ln |x_{n+1} - \alpha|/|x_n - \alpha|}{\ln |x_n - \alpha|/|x_{n-1} - \alpha|}. \]  

(5)

Definition 3. Let \( d \) be the number of new pieces of information required by a method. A piece of information typically is any evaluation of a function or one of its derivatives. The efficiency of the method is measured by the concept of efficiency index [11] and is defined by

\[ E = p/d, \]  

(6)

where \( p \) is the order of the method.

To develop the two-point iterative method (7) with optimal order of convergence, we begin deriving the third-order iterative method [6].

Consider an exponentially fitted straight line that is written as

\[ y(x) = e^{p(x-x_0)} \left( A(x-x_n) + B \right), \]  

(7)

with its first derivative and second derivative

\[ y'(x) = p e^{p(x-x_0)} \left( A(x-x_n) + B \right) + e^{p(x-x_0)} A, \]  

(8)

\[ y''(x) = p^2 e^{p(x-x_0)} \left( A(x-x_n) + B \right) + 2 p e^{p(x-x_0)} A. \]  

(9)

Substitute \( x = x_n \) into (7) and (8) and suppose \( y(x_n) = f(x_n) \) , \( y'(x_n) = f'(x_n) \), we get

\[ A = f(x_n) - p f(x_n), \]  

(10)

Suppose that the straight line (7) through the x-axis at \( x = x_{n+1} \), then \( y(x_{n+1}) = 0 \). So, we have

\[ x_{n+1} = x_n - \frac{B}{A} , \]  

(11)

or

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n) - p f(x_n)}, n \geq 0. \]  

(12)

Iterative method (12) is the well-known one parameter family of Newton’s method.

Again, from (9) we find

\[ p^2 f(x_n) - 2 p f'(x_n) + f''(x_n) = 0, \]  

(13)

when \( |p| \neq 1 \), then \( p^2 \) can be neglected. So, we get \( p \) in form
\[ p = \frac{f''(x_n)}{2f'(x_n)}. \]  

Based on (13), we can write (13) as
\[
p\left(pf'(x_n) - 2f'(x_n)\right) + f''(x_n) = 0.
\]

From (15), we find implicit form of \( p \) that is written as
\[
p = \frac{f''(x_n)}{2f'(x_n) - p*f'(x_n)},
\]
where \( p^* \) is defined by (14).

Substitute (14) into (16), we get a new form of \( p \),
\[
p = \frac{2f'(x_n)f''(x_n)}{4f'(x_n) - f''(x_n)f''(x_n)}.
\]

Based on (12) and (17), we get
\[
x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \left( \frac{4f'(x_n)^2 - f'(x_n)f''(x_n)}{4f'(x_n)^2 - 3f'(x_n)f''(x_n)} \right).
\]

Equation (18) is third-order convergence including three functional evaluations.

To improve (18), we consider third-order iterative method (18) with two real parameter that is written as
\[
x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \left( \frac{4f'(x_n)^2 - f'(x_n)f''(x_n)}{4f'(x_n)^2 - 3f'(x_n)f''(x_n)} \right).
\]

Furthermore, we construct a iterative method by using Taylor expansion. So, we consider second order Taylor series expansion \( f(x) \) about \( x_n \) as:
\[
f(x) = f(x_n) + (x-x_n)f'(x_n) + \frac{(x-x_n)^2}{2!}f''(x_n),
\]
If \( x_{n+1} \) is an approximation root at \( (n+1) \) iteration, then (20) can be written as
\[
f(x_{n+1}) \approx f(x_n) + (x_{n+1} - x_n)f'(x_n) + \frac{(x_{n+1} - x_n)^2}{2!}f''(x_n).
\]

Let \( x_{n+1} \) is closely to \( \alpha \), then \( f(x_{n+1}) \approx 0 \). So, from (21) we get
\[
x_{n+1} - x_n = \frac{f(x_n)}{f'(x_n)} - \frac{(x_{n+1} - x_n)^2}{2} \frac{f''(x_n)}{f'(x_n)}.
\]

where \( x_{n+1} \) is third-order iterative method that is defined by (19).

Substitute (19) into (22), we find a new iterative method with second derivative as following
\[
x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)} \left( \frac{f''(x_n)f'(x_n)^2}{2f'(x_n)^3} \right) \left( \frac{-4f'(x_n)^2 + \lambda f''(x_n)f''(x_n)}{(-4f'(x_n)^2 + 3\beta f''(x_n)f''(x_n))^2} \right).
\]

Equation (23) still contains second derivative of \( f(x_n) \). Therefore, we reduce it by using equality of Newton Steffensen’s method [3] with one real parameter and Halley’s method [4] that are written in form
\[
x_{n+1} = x_n - \frac{f'(x_n)^2}{f''(x_n)} \left( f(x_n) - \theta f(y_n) \right),
\]
and
\[
x_{n+1} = x_n - \frac{2f'(x_n)f'(x_n)}{2f'(x_n)^2 - f''(x_n)f''(x_n)},
\]
where \( y_n \) is defined by (2), respectively.

Using (24) and (25), we have a new approximation for \( f''(x_n) \) in form
\[
f''(x_n) \approx \frac{2\theta f'(x_n)^2 f(y_n)}{f'(x_n)^2},
\]
Based on (26), substitute it into (23) and simplify it, then we get a new iterative method with three real parameters
\[
y_n = x_n - \frac{f'(x_n)}{f'(x_n)},
\]
\[
x_{n+1} = x_n - \frac{f'(x_n)}{f'(x_n)} \left( -2f(x_n) + \theta f(y_n) \right) \left( f'(x_n) - \frac{3\beta f'(y_n)}{f(y_n)} \right)^2.
\]

Equation (28) is a two point iterative methods involving three evaluation of functions namely \( f(x_n), f'(x_n) \) dan \( f(y_n) \).

III. ORDER OF CONVERGENCE

Based on the Kung and Traub [22], the convergence order of a two point iterative method will be optimal if the method converges quartically.

Theorem following will describe the convergence order of the method in (28).

**Theorem 1.** Let \( f : D \subset \Re \rightarrow \Re \) be a differentiable function in open interval \( D \). Then assume that is a simple root of \( f(x) = 0 \). Suppose \( x_0 \) is a given value that is sufficiently close to \( \alpha \), then iterative method (18) has fourth-order of
convergence for $\theta = -1$, $\beta = 0$ and $\lambda = 2$ that satisfy the following error
\[
e_{n+1} = (-c_2c_3 + 4c_3^3)e_n^4 + O(e_n^5).
\]

(29)

where $c_k = \frac{f^{(k)}(\alpha)}{k! f'(\alpha)}, k = 2, 3, 4, 5, \ldots$

**Proof**: Let $\alpha$ is an exact root of $f(x)$ then $f(\alpha) = 0$. Furthermore, assume $f'(\alpha) \neq 0$ and $x_n = e_n + \alpha$, then Taylor series expansion for $f(x_n)$ and $f'(x_n)$ about $\alpha$, we have
\[
f(x_n) = f'(\alpha)(e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + O(e_n^5)),
\]

(30)

and
\[
f'(x_n) = f'(\alpha)
\left(1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_4e_n^4 + O(e_n^5)\right).
\]

(31)

Use (30) and (31), we find
\[
\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + (2c_2^2 - 2c_3)e_n^3 + (-4c_2^3 + 7c_2c_3 - 3c_4)e_n^4
\\
+ O(e_n^5).
\]

(32)

From (27) and $x_n = e_n + \alpha$, we get
\[
y_n = \alpha + c_2e_n^2 + (-2c_2^2 + 2c_3)e_n^3 + \cdots + O(e_n^5).
\]

(33)

Furthermore, using Taylor series expansion $f(x)$ about $\alpha$ at $x = y_n$, we get
\[
f(y_n) = f'(\alpha)
\left(c_2e_n^2 + (2c_2 - 2c_3)e_n^3
\\
+ (3c_4 + 5c_2^2 - 7c_2c_3)e_n^4 + O(e_n^5)\right).
\]

(34)

By using (30), (31), and (34) we get
\[
\theta f(y_n)(-2f(x_n) + \theta f(y_n))^2 = f'(\alpha)
\left(4\theta c_2e_n^2 + O(e_n^5)\right),
\]

(34)

and
\[
f'(x_n)(-2f(x_n) + 3\beta f(x_n))^2 = f'(\alpha)
\left(4\theta - 4 - 3\beta c_2^2e_n^4
\\
+ (20 - 12\beta + 9\theta^2\beta^2)c_2 + 4(5 - 6\theta^2\beta)c_3\right)e_n^4 + O(e_n^5).
\]

(35)

Equation (34) is divided by (35), and is found
\[
\frac{\theta f(y_n)(-2f(x_n) + \theta f(y_n))^2}{f'(x_n)(-2f(x_n) + 3\beta f(x_n))^2} = \frac{\theta c_2e_n^2}{c_2}
\\
+ \left(-4\theta^2 + 3\beta - \lambda\right)c_2^2 + 20\theta c_3\right)e_n^4 + \cdots + O(e_n^5). (36)
\]

Furthermore, by using (23) and (28), we get:
\[
\frac{f(x_n)}{f'(x_n)} = \frac{-\theta f(y_n)(-2f(x_n) + \theta f(y_n))^2}{f'(x_n)(-2f(x_n) + 3\beta f(x_n))^2} = e_n - (\theta + 1)c_2e_n^2
\\
+ (-2(\theta + 1)c_3(\theta^2 + \theta(\lambda - 3\beta)c_2^2)e_n^3
\\
+ \cdots + O(e_n^5).
\]

(37)

Substitute (37) into (28) and take $x_{n+1} = e_{n+1} + \alpha$ and $x_n = e_n + \alpha$, then we get convergence order of (28) in form
\[
e_{n+1} = - (\theta + 1)c_2e_n^2 + (\theta^2 + \theta(\lambda - 3\beta)c_2^2 + 2(\theta + 1)c_3)\right)e_n^3
\\
+ \left(4 + 13\theta + 7(\lambda - 3\beta)\theta^2 + \frac{1}{4}(27\beta^2 - 12\beta + \lambda)\theta^4\right)c_2
\\
+ (-7 - 14\theta + 4(\lambda - 3\beta)\theta^2)c_2c_3 + 3(\theta + \theta)c_4\right)e_n^4
\\
+ O(e_n^5).
\]

(38)

Equation (38) give an information that the convergence order of (28) will increase if we take $\theta = -1$. So, by substituting the value of $\theta$ into (38), we get
\[
e_{n+1} = (2 + 3\beta - \lambda)c_2^3 + \left((7 + 12\beta - 4\lambda)c_2
\\
+ \frac{1}{4}(\lambda^2 + 4(3\beta + 7)\lambda - (36 + 48\beta + 27\beta^2)c_3\right)e_n^4
\\
+ O(e_n^5).
\]

(39)

Base on (39), the convergence order of the proposed method is at least three. So, (33) still involve two real parameters, the we get increase again by using relationship $\lambda = 3\beta + 2$, $\beta \in \mathbb{R}$. Substitute it into (39), then we get
\[
e_{n+1} = (4 + 3\beta)c_2^3 - c_2c_3\right)e_n^4 + O(e_n^5).
\]

(40)

The proof is completed. $\square$

Equation (40) is the convergence order of (27)–(28) for $\theta = -1$, $\lambda = 3\beta + 2$, and $\beta \in \mathbb{R}$.

\[
x_{n+1} = x_n - \left(1 - \frac{f(y_n)(2f(x_n) + (3\beta + 2)f(y_n))^2}{f(x_n)(2f(x_n) + 3\beta f(y_n))^2}\right)\frac{f(x_n)}{f'(x_n)}.
\]

(41)

Furthermore by taking any value of $\beta$, it will appear some four-iterative methods.

For $\beta = 0$,
\[
x_{n+1} = x_n - \left(1 - \frac{f(y_n)(f(x_n) + f(y_n))^2}{f(x_n) f(x_n)^2}\right)\frac{f(x_n)}{f'(x_n)}.
\]

(42)
For $\beta = -\frac{2}{3}$,
\[
x_{n+1} = x_n - \frac{f(y_n)f(x_n)}{(f'(y_n) - f'(x_n))^2} f'(x_n).
\]

### IV. NUMERICAL SIMULATION

In this section, numerical examples are presented to illustrate the efficiency of the proposed method in (27) – (28) by using several test functions. The zeros approximation of the test functions was displayed round up to 16th decimal places.

To show the performance of Eq. (27) - (28) for $\theta = 1, \beta = 0$ and $\lambda = 2$ (MBM4), we compare it with Newton’s method (NM)[11], classical Chebyshev-Halley’s method with $\beta = 1/2$ (HM)[3], Newton-Steffensen’s method (NSM)[20], third-order iterative method (BM3)[6].

<table>
<thead>
<tr>
<th>Iterative Method</th>
<th>Order of Convergence</th>
<th>Functional evaluation</th>
<th>Efficiency index</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM</td>
<td>2</td>
<td>2</td>
<td>1.41421356</td>
</tr>
</tbody>
</table>

Table 1 Comparison of efficiency index

All computations are performed by using Maple 13.0 with 850 digits floating point arithmetics for the following several test functions.

\[
| x_{n+1} - x_n | \leq \varepsilon
\]

dengan $\varepsilon = 10^{-95}$.

The used real functions are as following:

\[
f_1(x) = xe^{-x} - 0.1, \quad \alpha = 0.111832559158,
\]

\[
f_2(x) = e^x - 4, \quad \alpha = 4.306584728220,
\]

\[
f_3(x) = \cos(x) - x, \quad \alpha = 0.739085133215,
\]

\[
f_4(x) = x^3 + 4x^2 - 10, \quad \alpha = 1.365230013414,
\]

\[
f_5(x) = e^{-x^2} - 1 - \cos(x+1) + x^3 + 1, \quad \alpha = -1.0000000000000000,
\]

\[
f_6(x) = \sin^2(x) - x^3 + 1, \quad \alpha = 1.4044916482153412.
\]

The number of iteration and COC for compared iterative methods shown at the Table 1 following.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$x_0$</th>
<th>NM</th>
<th>HM</th>
<th>NSM</th>
<th>BM3</th>
<th>MBM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$</td>
<td>0.2</td>
<td>0.3</td>
<td>8 (2.0000)</td>
<td>5 (3.000)</td>
<td>5 (3.000)</td>
<td>5(3.000)</td>
</tr>
<tr>
<td>$f_2(x)$</td>
<td>4.0</td>
<td>4.5</td>
<td>8 (2.0000)</td>
<td>5 (3.000)</td>
<td>5 (3.000)</td>
<td>5(3.000)</td>
</tr>
<tr>
<td>$f_3(x)$</td>
<td>1.0</td>
<td>2.0</td>
<td>8 (2.0000)</td>
<td>5 (3.000)</td>
<td>5 (3.000)</td>
<td>5(3.000)</td>
</tr>
<tr>
<td>$f_4(x)$</td>
<td>2.0</td>
<td>2.0</td>
<td>8 (2.0000)</td>
<td>5 (3.000)</td>
<td>5 (3.000)</td>
<td>5(3.000)</td>
</tr>
<tr>
<td>$f_5(x)$</td>
<td>1.2</td>
<td>2.0</td>
<td>8 (2.0000)</td>
<td>5 (3.000)</td>
<td>5 (3.000)</td>
<td>5(3.000)</td>
</tr>
</tbody>
</table>

Based on Table 1, the proposed method converges quartically for all test function. This gives an information that the convergence order of the method is four. Beside in, for all compared iteration method, the proposed method has the least number of iteration.

Furthermore, to find an accuracy for compared iterative methods, we use absolute value of $f(x_n)$ and of relatil error to measure the accuration with Total Number of Functional Evaluation (TNFE) = 12 which given at Table 3 and 4.

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$x_0$</th>
<th>NM</th>
<th>HM</th>
<th>NSM</th>
<th>BM3</th>
<th>MBM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$</td>
<td>0.2</td>
<td>0.3</td>
<td>3.0850 (e–56)</td>
<td>2.7757 (e–55)</td>
<td>1.2725 (e–45)</td>
<td>1.6466 (e–93)</td>
</tr>
<tr>
<td>$f_2(x)$</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0253 (e–33)</td>
<td>2.1103 (e–53)</td>
<td>5.5769 (e–42)</td>
<td>1.3675 (e–100)</td>
</tr>
</tbody>
</table>

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Table 3 and 4 shown that the proposed method (MBM4) has the lowest absolute value of $f(x_n)$ and relatif error for all tested real functions. It is clearly that the accuracy of the iterative method in (27) – (28) is better than others.

V. CONCLUSION AND FUTURE SCOPE

This research work have developed a new fourth-order convergence method for solving nonlinear equation $\theta = 1$, $\lambda = 3\beta + 2$, $\beta \in \mathbb{R}$. The method requires two evaluation of functions and one its first derivative per iteration with the efficiency index equal to $4^{1/3} \approx 1.5874$. The numerical results show that the proposed method has better performance as compared with the other methods. Therefore, the results of this study provide a new contribution in computational sciences area.

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