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A New Operation on Sinusoidal Fuzzy Number

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***Abstract*—** Fuzzy numbers exists in many fields such as Management, agriculture, Medicine, Engineering etc. In this paper a new operation on sinusoidal fuzzy number has been defined and introduced. In addition to it, this paper also deals with the study of new basic alpha cut operations on addition, subtraction and multiplication of sinusoidal fuzzy number with the suitable numerical examples.

Mathematics Subject Classification—03E72
Keywords—*Fuzzy arithmetic, Sinusoidal Fuzzy number.*

I. INTRODUCTION

Lotfi. A. Zadeh (1965) & Dieter Klaua (1965) has introduced fuzzy sets [4]. In general, fuzzy set theory has been described in the interval [0,1] with the membership function. It was specifically designed to represent mathematical uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. Fuzzy systems are suitable for approximate reasoning, especially for the system with a mathematical model that is difficult to derive, in 1951, Dwyer suggested interval arithmetic by means of Zadeh's Extension principle [4]. D.Dubois and H. Prade in 1978 has defined any of the fuzzy number as a fuzzy subset of a real line [3]. In 1982, Dubois and Prade proposed a more efficient procedure. Fuzzy numbers are an extension of real numbers, [4,5,7,8]. As there exists various fuzzy numbers, triangular fuzzy number are the most commonly used membership function (Dubois and Prade) [3],1980.

In real life situations we come across so many functions which are random in nature. Signal generators are one among them. Signal generators are widely utilized in various experimental courses [6,14,16,17,22]. Since random signal is not possible in all cases, uncertainty of some element is always associated with it. In order to overcome this a new fuzzy number has been introduced in this paper named Sinusoidal Fuzzy number. This paper constitutes the basic operations of sinusoidal fuzzy number based on alpha cut sets. It has been illustrated with some numerical examples.

II. PRELIMINARIES [19,20]

Definition 2.1: Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A: X \to [0,1]$ and A*(x)* is interpreted as the degree of membership of element x in fuzzy A for each $x \in X$ .The value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A

Definition 2.2: A Fuzzy number "A" is a convex normalized fuzzy set on the real line R such that:

- There exists at least one $x_o \in R$ with $\mu_A(x_o) = 1$
- $\mu_A(x)$ is piecewise continuous

Definition 2.3: A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2, and a_3 are real numbers and its membership function is given below.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & for\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

III. SINUSOIDAL FUZZY NUMBER (SFN)

A fuzzy number $\tilde{S}$ is a Sinusoidal fuzzy number denoted by $\tilde{S}=(s_1,s_2,s_3,s_4,s_5)$ where s_1,s_2,s_3,s_4,s_5 are real numbers and its membership function $\mu_S(x)$ is given below

$$\mu_{\tilde{S}}(x) = \begin{cases} 0 & \text{for } x < s_1 \\ \left(\dfrac{x - s_1}{s_2 - s_1}\right) & \text{for } s_1 \le x \le s_2 \\ \left(\dfrac{s_3 - x}{s_3 - s_2}\right) & \text{for } s_2 \le x \le s_3 \\ \left(\dfrac{x - s_3}{s_3 - s_4}\right) & \text{for } s_3 \le x \le s_4 \\ \left(\dfrac{x - s_5}{s_5 - s_4}\right) & \text{for } s_4 \le x \le s_5 \\ 0 & \text{for } x > s_5 \end{cases}$$

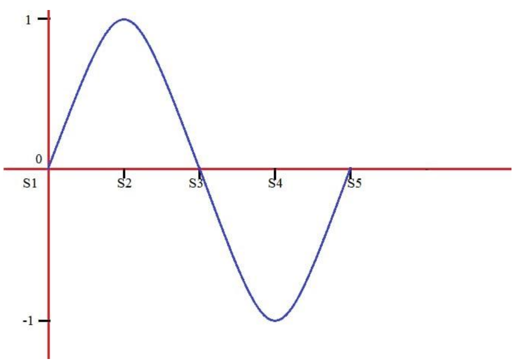

Figure-3.1 Graphical representation of Sinusoidal Fuzzy Number

Remark:
Membership function $\mu_{\tilde{S}}(x)$ are continuous function

3.1 Positive Sinusoidal fuzzy number:
A positive sinusoidal fuzzy number $\tilde{S}$ is denoted as $\tilde{S}$ = (s_1,s_2,s_3,s_4,s_5) where all $s_i > 0$ for all i = 1,2,3,4,5

3.2 Negative Sinusoidal fuzzy number:
A negative sinusoidal fuzzy number $\tilde{S}$ is denoted as $\tilde{S}$ = (s_1,s_2,s_3,s_4,s_5) where all $s_i < 0$ for all i = 1,2,3,4,5

Example: $\tilde{S}$= (-6, -5, -4, -3, -2)

Note:
A negative sinusoidal fuzzy number can be written as the negative multiplication of a positive sinusoidal fuzzy number.

Example:
$\tilde{S}$= (-2, -4, -6, -8, -10) then -$\tilde{S}$ = (2, 4, 6, 8, 10)

Definition 3.3:
Let $\tilde{S}$ = (s_1, s_2, s_3, s_4, s_5) and $\tilde{T}$ = (t_1, t_2, t_3, t_4, t_5) be two sinusoidal fuzzy number. If S is identically equal to T only if $s_1 = t_1, s_2 = t_2, s_3 = t_3, s_4 = t_4, s_5 = t_5$.

IV. ALPHA CUT OPERATIONS [1,2,9,10]

If $\tilde{S}$ = (s_1, s_2, s_3, s_4, s_5) is a sinusoidal fuzzy number, we get a crisp interval by α cut operations interval. S_α shall be obtained as follows for all $\alpha \in [-1,1]$

$$S_\alpha = \begin{cases} [s_1 + \alpha(s_2 - s_1), s_3 - \alpha(s_3 - s_2)] & ,\alpha \in [0,1] \\ [s_3 - \alpha(s_4 - s_3), s_5 + \alpha(s_5 - s_4)] & ,\alpha \in [-1,0] \end{cases}$$

4.1 Operations of Sinusoidal fuzzy number:
Following are the three operations that can be performed on Sinusoidal fuzzy number. Suppose $\tilde{S}$= (s_1, s_2, s_3, s_4, s_5) and $\tilde{T}$ = (t_1, t_2, t_3, t_4, t_5) are Sinusoidal fuzzy numbers then

(i) Addition: $\tilde{S}$ +$\tilde{T}$ =
$(s_1 + t_1, s_2 + t_2, s_3 + t_3, s_4 + t_4, s_5 + t_5)$
(ii) Subtraction: $\tilde{S}$ - $\tilde{T}$ =
$(s_1 - t_5, s_2 - t_4, s_3 - t_3, s_4 - t_2, s_5 - t_1)$
(iii) Multiplication: $\tilde{S}$*$\tilde{T}$ =
$(s_1 * t_1, s_2 * t_2, s_3 * t_3, s_4 * t_4, s_5 * t_5)$

Example 4.1.1:
If $\tilde{S}$= {1,2,3,4,5} and $\tilde{T}$ = {2,4,6,8,10}
Then $\tilde{S}$ + $\tilde{T}$ = {3,6,9,12,15}

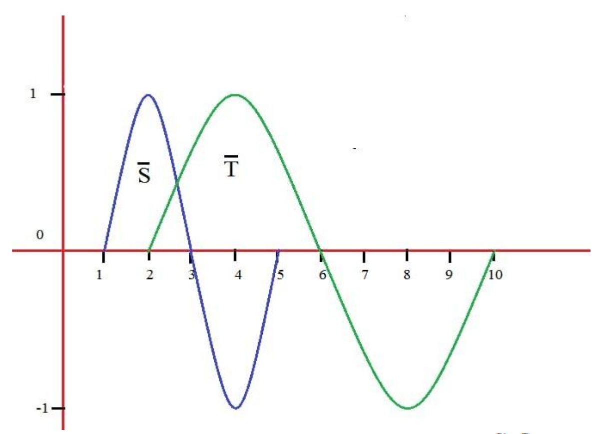

Figure 4.1.1: Sinusoidal Fuzzy number $\tilde{S}$, $\tilde{T}$

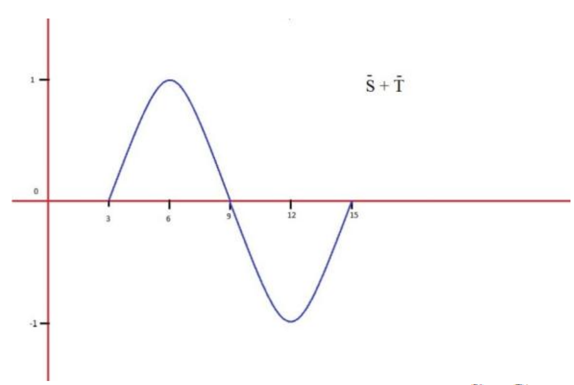

Figure 4.1.2 Sinusoidal Fuzzy number $\tilde{S}$ + $\tilde{T}$

Example 4.1.2:
If $\tilde{S}$= {1,2,3,4,5} and $\tilde{T}$ = {2,4,6,8,10}

Then $\tilde{S} - \tilde{T}$ = { -9, -6, -3,0,3}

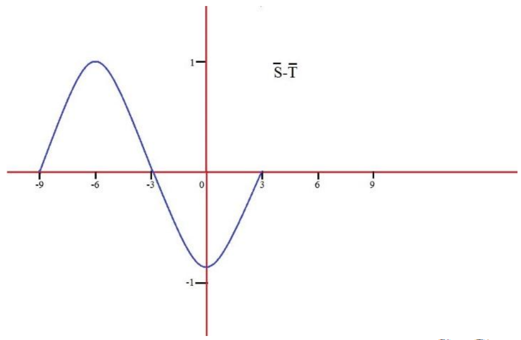

Figure 4.3 Sinusoidal Fuzzy number $\tilde{S}$ - $\tilde{T}$

Example 4.1.3:
If $\tilde{S}$= {1,2,3,4,5} and $\tilde{T}$ = {2,4,6,8,10}
Then $\tilde{S}*\tilde{T}$ = {2,8,18,32,50}

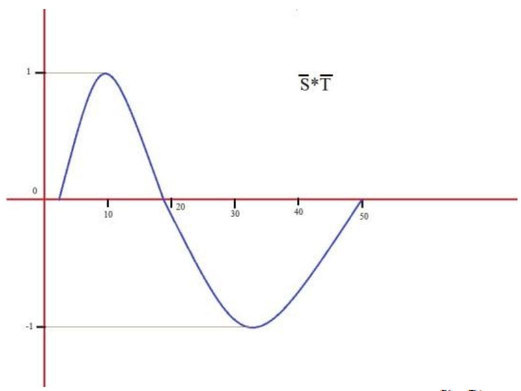

Figure 4.4 Sinusoidal Fuzzy number $\tilde{S}*\tilde{T}$

4.2 Alpha cut operations for Addition, Subtraction & Multiplication:

Let $\tilde{S}$ = (s_1,s_2,s_3,s_4,s_5) and $\tilde{T}$ = (t_1,t_2,t_3,t_4,t_5) be two sinusoidal fuzzy numbers then the for all α ϵ[-1,1] , α cut operations are defined are as follows

$$S_\alpha = \begin{cases} [s_1 + \alpha(s_2 - s_1), s_3 - \alpha(s_3 - s_2)], & \alpha \in [0,1] \\ [s_3 - \alpha(s_4 - s_3), s_5 + \alpha(s_5 - s_4)], & \alpha \in [-1,0] \end{cases}$$

And

$$T_\alpha = \begin{cases} [t_1 + \alpha(t_2 - t_1), t_3 - \alpha(t_3 - t_2)], & \alpha \in [0,1] \\ [t_3 - \alpha(t_4 - t_3), t_5 + \alpha(t_5 - t_4)], & \alpha \in [-1,0] \end{cases}$$

$S_\alpha + T_\alpha =$

$$\begin{cases} [(s_1 + \alpha(s_2 - s_1) + t_1 + \alpha(t_2 - t_1)), (s_3 - \alpha(s_3 - s_2) + t_3 - \alpha(t_3 - t_2))], \alpha \in [0,1] \\ [(s_3 - \alpha(s_4 - s_3) + t_3 - \alpha(t_4 - t_3)), (s_5 + \alpha(s_5 - s_4) + t_5 + \alpha(t_5 - t_4))], \alpha \in [-1,0] \end{cases}$$

$S_\alpha - T_\alpha =$

$$\begin{cases} [(s_1 + \alpha(s_2 - s_1) - (t_5 + \alpha(t_5 - t_4)), (s_3 - \alpha(s_3 - s_2) - (t_3 - \alpha(t_4 - t_3))], \alpha \in [0,1] \\ [(s_3 - \alpha(s_4 - s_3) - (t_3 - \alpha(t_3 - t_2)), (s_5 + \alpha(s_5 - s_4) - (t_1 + \alpha(t_2 - t_1))], \alpha \in [-1,0] \end{cases}$$

$S_\alpha * T_\alpha =$

$$\begin{cases} [(s_1 + \alpha(s_2 - s_1) * (t_1 + \alpha(t_2 - t_1)), (s_3 - \alpha(s_3 - s_2) * (t_3 - \alpha(t_3 - t_2))], \alpha \in [0,1] \\ [(s_3 - \alpha(s_4 - s_3) * (t_3 - \alpha(t_4 - t_3)), (s_5 + \alpha(s_5 - s_4) * (t_5 + \alpha(t_5 - t_4))], \alpha \in [-1,0] \end{cases}$$

Example 4.2.1:
Let $\tilde{S}$ = {-3,2,4,6,8} and $\tilde{T}$ = { -1,0,6,9,12}

$$S_\alpha = \begin{cases} [-3 + 5\alpha, 4 - 2\alpha], & \alpha \in [0,1] \\ [4 - 2\alpha, 8 + 2\alpha], & \alpha \in [-1,0] \end{cases}$$

$$T_\alpha = \begin{cases} [-1 + \alpha, 6 - 6\alpha], & \alpha \in [0,1] \\ [6 - 3\alpha, 12 + 3\alpha], & \alpha \in [-1,0] \end{cases}$$

Hence

$$S_\alpha + T_\alpha = \begin{cases} [-4 + 6\alpha, 10 - 8\alpha], & \alpha \in [0,1] \\ [10 - 5\alpha, 20 + 5\alpha], & \alpha \in [-1,0] \end{cases}$$

When $\alpha = 1$ $S_1 + T_1 = [2,2]$

When $\alpha = 0$ $S_0 + T_0 = [-4,10]$

When $\alpha = -1$ $S_{-1} + T_{-1} = [15,15]$

When $\alpha = 0$ $S_0 + T_0 = [10,20]$

Hence $S_\alpha + T_\alpha$ = [-4,2,10,15,20] where all the points coincide with the sum of the two Sinusoidal Fuzzy numbers.

Therefore, the addition of two α cuts lies within the interval.

Example 4.2.2:
Let $\tilde{S}$= {-3,2,4,6,8} and $\tilde{T}$ = { -1,0,6,9,12}

$$S_\alpha - T_\alpha = \begin{cases} [-15 + 2\alpha, -2 + \alpha], & \alpha \in [0,1] \\ [-2 + 4\alpha, 9 + \alpha], & \alpha \in [-1,0] \end{cases}$$

When $\alpha = 1$ $S_1 + T_1 = [-13,-1]$ When $\alpha = 0$ $S_0 + T_0 = [-15,-2]$
When $\alpha = -1$ $S_{-1} + T_{-1} = [-6,8]$ When $\alpha = 0$ $S_0 + T_0 = [-2,9]$

Hence $S_\alpha - T_\alpha$= [-15, -13, -6, -2, -1,8,9] where all the points coincide with the difference of the two Sinusoidal Fuzzy numbers.

Therefore, the subtraction of two α cuts lies within the interval.

Example 4.2.3:
Let $\tilde{S}$ = {-3,2,4,6,8} and $\tilde{T}$ = { -1,0,6,9,12}

$$S_\alpha * T_\alpha = \begin{cases} [(-3+5\alpha)(-1+\alpha), (4-2\alpha)(6-6\alpha)] & ,\alpha \in [0,1] \\ [(4-2\alpha)(6-3\alpha), (8+2\alpha)(12+3\alpha)] & ,\alpha \in [-1,0] \end{cases}$$

$$S_\alpha * T_\alpha = \begin{cases} [(5\alpha^2-8\alpha+3), (12\alpha^2-36\alpha+24)] & ,\alpha \in [0,1] \\ [(6\alpha^2-24\alpha+24), (6\alpha^2+48\alpha+72)] & ,\alpha \in [-1,0] \end{cases}$$

When $\alpha = 1$ $S_1 + T_1 = [0,4]$ when $\alpha = 0$ $S_0 + T_0 = [3,24]$
When $\alpha = -1$ $S_{-1} + T_{-1} = [54,30]$ when $\alpha = 0$ $S_0 + T_0 = [24,72]$

Hence $S_\alpha * T_\alpha$ = [0,3,4,24,30,54,72] where all the points coincide with the product of the two Sinusoidal Fuzzy numbers.
Therefore, the multiplication of two α cuts lies within the interval.

4.3 Symmetric Image:

If $\tilde{S} = (s_1,s_2,s_3,s_4,s_5)$ is the Sinusoidal Fuzzy number then

$-\tilde{S} = (-s_5,-s_4,-s_3,-s_2,-s_1)$ which is the symmetric image of S is

also a Sinusoidal Fuzzy number

Example:4.3
If $\tilde{S}$ = {2,4,6,8,10}
Then $-\tilde{S}$ = {-10, -8, -6, -4, -2} which is again a Sinusoidal Fuzzy number.

V. CONCLUSION

In this paper Sinusoidal Fuzzy number has been newly introduced and its alpha cut operations of arithmetic interval function using addition, subtraction and multiplication has been elaborated and illustrated with numerical examples. This sinusoidal Fuzzy number is very much helpful in signal processing and in the generation of the random signals, furthermore it helps in the generation of square wave and sine wave signals too.

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