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Hamiltonian Decomposition of Special Class of Ladder Graphs

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Abstract— A path in a graph G that contains every vertex of G is called Hamiltonian path. A cycle in a graph G that contains every vertex of G is called a Hamiltonian cycle of G. A Hamiltonian graph is a graph that contains a Hamiltonian cycle. A graph is uniquely Hamiltonian if it contains exactly one Hamiltonian cycle. A decomposition of a graph G into Hamiltonian cycles will be called Hamiltonian decomposition of G. In this paper, we decompose some families of graphs such as ladder, triangular ladder, diagonal ladder, and circular ladder using the concept of Hamiltonian decomposition. Also using the concept of Hamiltonian decomposition of ladder graphs we decompose the diagonal ladder graph.

Keywords— *Hamiltonian cycle, Hamiltonian graph, uniquely Hamiltonian, Hamiltonian decomposition, Ladder Graph*

I. INTRODUCTION

Graph theory is proved to be tremendously useful in modeling the essential features of systems with finite components. Graphical models are used to represent telephone network, railway network, communication problems, traffic network etc. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. The concept of decomposition of graphs in Hamiltonian cycles, Hamiltonian path decomposition of regular graphs was introduced by Klas Markstrom[1]. Decomposition of complete graphs into Hamilton cycles was discussed in [2] and J.C. Bermond [3] deals about Hamiltonian Decompositions of Graphs, Directed Graphs and Hypergraphs.

A path in a graph G that contains every vertex of G is called **Hamiltonian path**. A cycle in a graph G that contains every vertex of G is called a **Hamiltonian cycle** of G. A **Hamiltonian graph** is a graph that contains a Hamiltonian cycle. A graph is **uniquely Hamiltonian** if it contains exactly one Hamiltonian cycle. A decomposition of a graph G into Hamiltonian cycles will be called **Hamiltonian decomposition** of G.

The **Ladder graph L_n,** $n \geq 2$ is defined by P_n x P_2, where P_n is a path with n vertices and x denotes the Cartesian product and P_2 is a path with two vertices.

A **Triangular Ladder graph TL_n,** $n \geq 2$ is a graph obtained from the Ladder graph $L_n = P_n$ x P_2 by adding the edges u_iv_{i+1} for $1 \leq i \leq n-1$.

The **Diagonal Ladder graph** is a Ladder graph with additional edges u_iv_{i+1} and v_iu_{i+1}, denoted by **DL_n**.

The **Circular Ladder graph CL_n**, $n \geq 3$ is defined by $CL_n = C_n$ x K_2, where C_n is a cycle with n vertices and x denotes the cartesian product and K_2 is a complete graph with two vertices.

Section I contains the introduction of Hamiltonian decomposition of graphs and related topics , Section II contains Hamiltonian decomposition of ladder graphs, Section III contains Hamiltonian decomposition of triangular ladder graphs, Section IV contains Hamiltonian decomposition of diagonal ladder graphs , Section V contains Hamiltonian decomposition of circular ladder graphs , Section VI concludes research work.

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II. HAMILTONIAN DECOMPOSITION OF LADDER GRAPH L_n

Theorem: 2.1

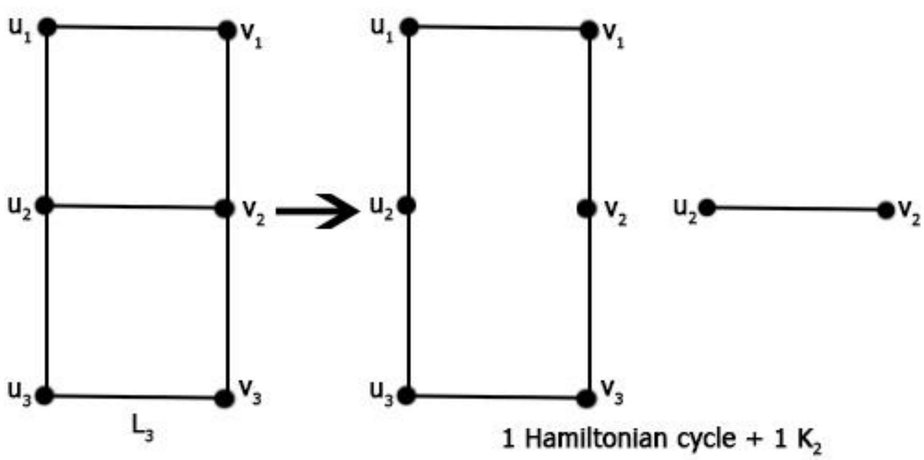
The Ladder graph L_n, $n \geq 2$ is uniquely Hamiltonian and the rest of the edges form n-2 copies of K_2.

Proof:

Let L_n, $n \geq 2$ be the Ladder graph with vertex set $V:\{u_1,u_2,u_3,...,u_n,v_1,v_2,v_3,...,v_n\}$. By definition, clearly we get two paths Q_1 and Q_2 of length n, Q_1: $u_1u_2u_3...u_n$ and Q_2: $v_1v_2v_3...v_n$.

Form a spanning cycle: connect the vertex u_1 of Q_1 to v_1 of Q_2 and connect the vertex u_n of Q_1 to v_n of Q_2 at the same time. This is the only Hamiltonian cycle. Because connecting any vertex u_i, $2 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n-1$ of Q_2 makes a cycle which is not a spanning cycle. Therefore, L_n, $n \geq 2$ is uniquely Hamiltonian. The remaining edges joining the vertices u_i, $2 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n-1$ of Q_2 makes n-2 copies of K_2.

Example:2.2



III. HAMILTONIAN DECOMPOSITION OF TRIANGULAR LADDER GRAPH TL_n

Theorem: 3.1

The Triangular Ladder graph TL_n, $n \geq 2$ is uniquely Hamiltonian and the rest of the edges form a path P_{2n-2}.

Proof:

Let TL_n, $n \geq 2$ be the Triangular Ladder graph with vertex set $V:\{u_1,u_2,u_3,...,u_n,v_1,v_2,v_3,...,v_n\}$. By definition, clearly we get two paths Q_1 and Q_2 of length n, Q_1: $u_1u_2u_3...u_n$ and Q_2: $v_1v_2v_3...v_n$.

Form a spanning cycle: connect the vertex u_1 of Q_1 to v_1 of Q_2 and connect the vertex u_n of Q_1 to v_n of Q_2 at the same time. This is the only Hamiltonian cycle. Because connecting any vertex u_i, $2 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n-1$ of Q_2 or connecting any vertex u_i, $1 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n$ of Q_2 makes a cycle which is not a spanning cycle. Therefore, TL_n, $n \geq 2$ is uniquely Hamiltonian. The remaining edges joining the vertices u_i, $2 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n-1$ of Q_2 or connecting any vertex u_i, $1 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n$ of Q_2 makes a path P_{2n-2}.

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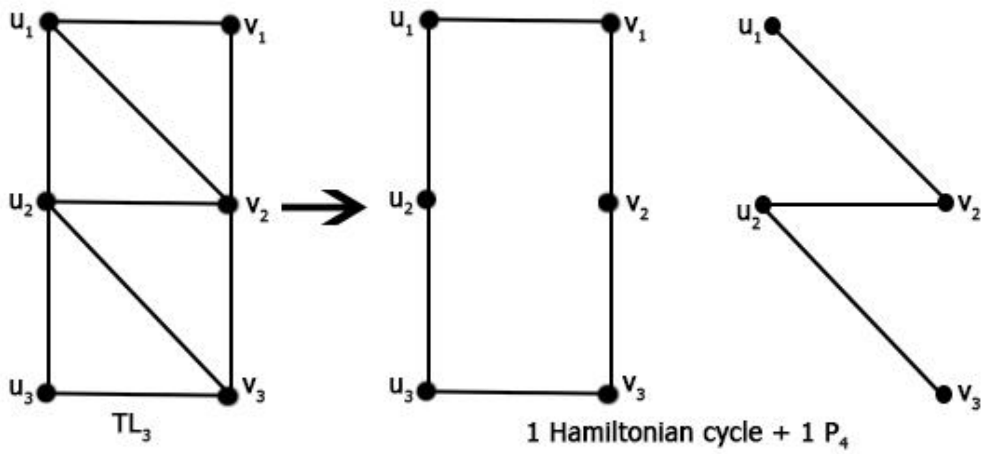
Example: 3.2



IV. HAMILTONIAN DECOMPOSITION OF DIAGONAL LADDER GRAPH DL_n

Theorem: 4.1

The Diagonal Ladder graph DL_n, $n \geq 2$ can be decomposed into $\begin{cases} 3 \text{ Hamiltonian cycles if } n = 2. \\ 2 \text{ Hamiltonian cycles and } n-2 \text{ copies of } K_2 \text{ if } n \geq 3. \end{cases}$

Proof:

Let DL_n, $n \geq 2$ be the Diagonal Ladder graph with vertex set $V:\{u_1,u_2,u_3,...,u_n,v_1,v_2,v_3,...,v_n\}$.

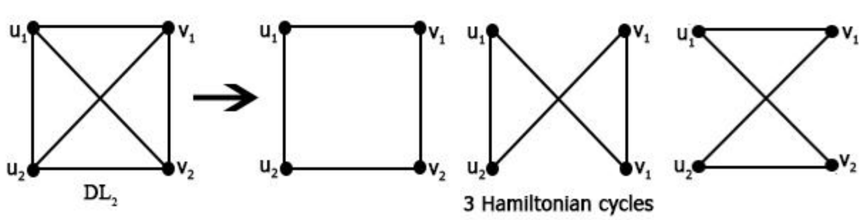
Case 1: $n = 2$

Let DL_2 be the Diagonal Ladder graph with vertex set V: $\{u_1,u_2,v_1,v_2\}$. By definition, DL_2 is obtained from L_2 which is uniquely Hamiltonian, so that DL_2 has a Hamiltonian cycle. Also DL_2 has additional edges u_1v_2 and u_2v_1. Connecting these two edges in two ways, we get two Hamiltonian cycles. Therefore, DL_2 has 3 Hamiltonian cycles.

Case 2: $n \geq 3$

By definition, DL_n is obtained from L_n which is uniquely Hamiltonian, so that DL_n has a Hamiltonian cycle. Now, we construct two paths Q_1 and Q_2 with the additional edges u_iv_{i+1} and $u_{i+1}v_i$ of length n, $Q_1:u_1v_2u_3v_4...u_{n-3}v_{n-2}u_{n-1}v_n$ and $Q_2:v_1u_2v_3u_4...v_{n-3}u_{n-2}v_{n-1}u_n$. Form a spanning cycle: connect the vertex u_1 of Q_1 to v_1 of Q_2 and connect the vertex v_n of Q_1 to u_n of Q_2 at the same time. Therefore, DL_n, $n \geq 3$ contains 2 Hamiltonian Cycles. Also the remaining edges joining the vertices u_i, $2 \leq i \leq n-1$ of Q_1 to v_i, $2 \leq i \leq n-1$ of Q_2 makes n-2 copies of K_2.

Example: 4.2



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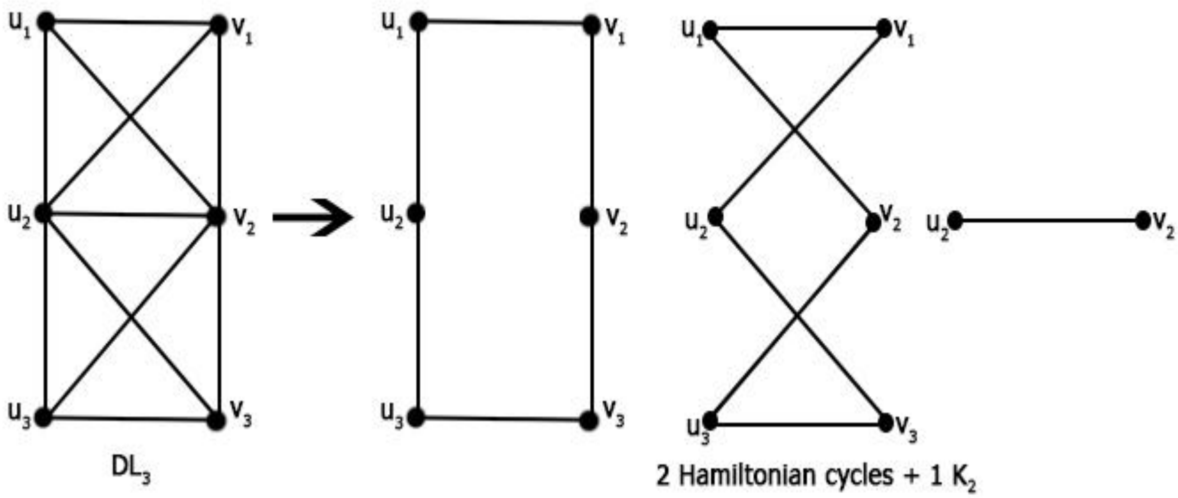

V. HAMILTONIAN DECOMPOSITION OF CIRCULAR LADDER GRAPH CL_n

Theorem: 5.1

The Circular Ladder graph CL_n, $n \geq 3$ can be decomposed into $\begin{cases} n \text{ Hamiltonian cycles if } n \text{ is odd} \\ n+2 \text{ Hamiltonian cycles if } n \text{ is even} \end{cases}$

and the rest of the edges form the 1-factorization.

Proof:

Let CL_n, $n \geq 3$ be the Circular Ladder graph with vertex set $V:\{u_1,u_2,u_3,...,u_n,v_1,v_2,v_3,...,v_n\}$. We consider two cases to prove this theorem.

Case 1: n is odd

We construct n paths in the inner cycle

Q_1: $u_1u_2u_3...u_{n-2}u_{n-1}u_n$,

Q_2: $u_2u_3u_4...u_{n-1}u_nu_1$,

. . .

. . .

. . .

Q_n: $u_nu_1u_2...\ u_{n-3}u_{n-2}u_{n-1}$, $n \geq 3$.

Also, we construct n paths in the outer cycle

R_1: $v_1v_2v_3...v_{n-2}v_{n-1}v_n$,

R_2: $v_2v_3v_4...v_{n-1}v_nv_1$,

. . .

. . .

. . .

R_n: $v_nv_1v_2...\ v_{n-3}v_{n-2}v_{n-1}$, $n \geq 3$.

Now, we connect the end vertices of each paths Q_i and R_i, $1 \leq i \leq n$, $n \geq 3$, we get n spanning cycles.

Case 2: n is even

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We construct n paths in the inner cycle

Q_1: $u_1u_2u_3...u_{n-2}u_{n-1}u_n$,

Q_2: $u_2u_3u_4...u_{n-1}u_nu_1$,

. . .

. . .

. . .

Q_n: $u_nu_1u_2...\ u_{n-3}u_{n-2}u_{n-1}$, $n \geq 4$.

Also, we construct n paths in the outer cycle,

R_1: $v_1v_2v_3...v_{n-2}v_{n-1}v_n$,

R_2: $v_2v_3v_4...v_{n-1}v_nv_1$,

. . .

. . .

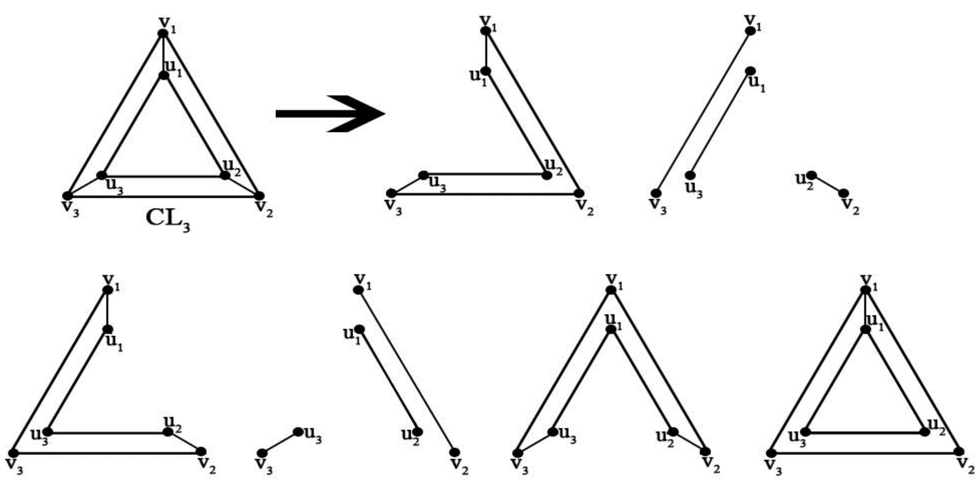
. . .

R_n: $v_nv_1v_2...\ v_{n-3}v_{n-2}v_{n-1}$, $n \geq 4$.

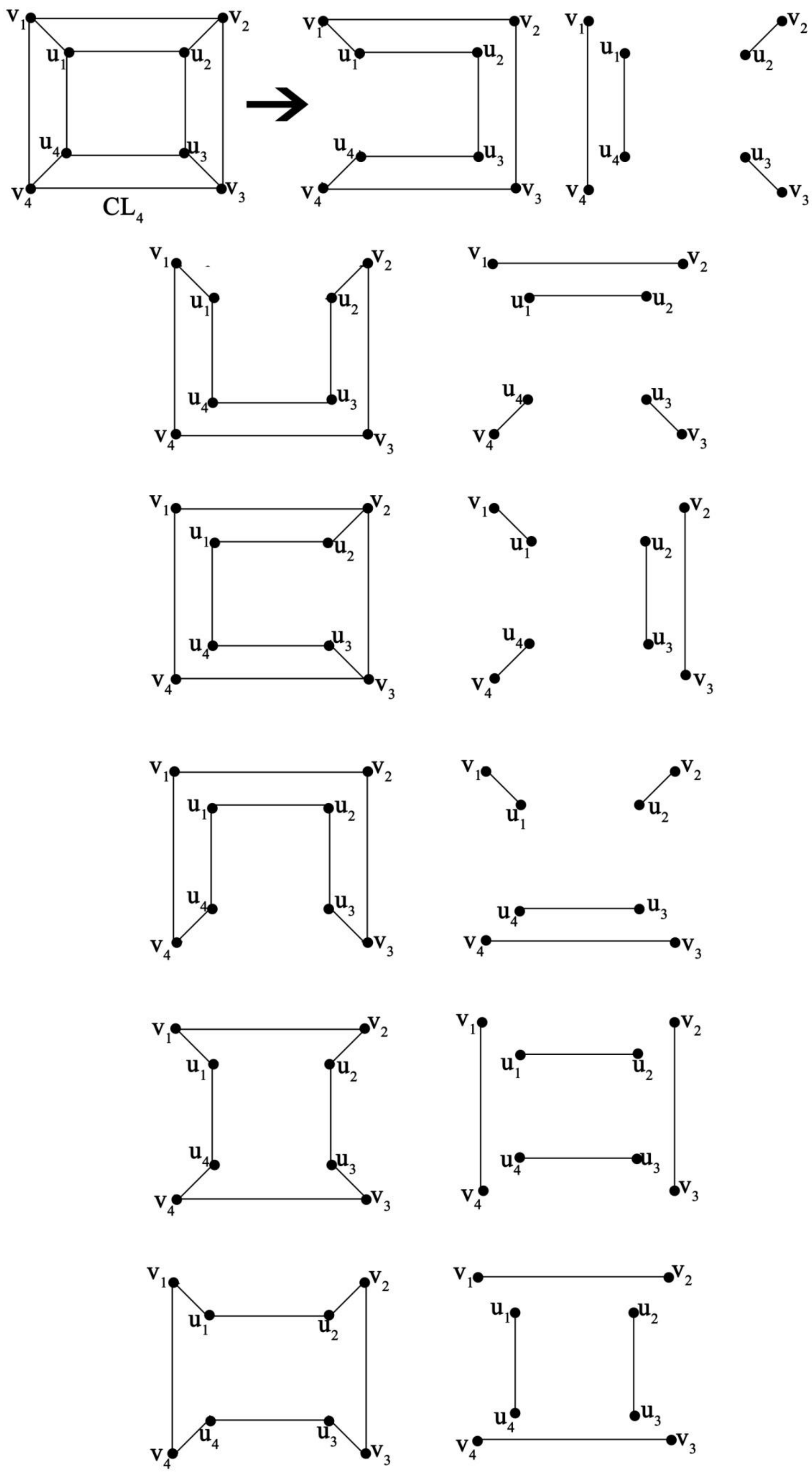
Now, we connect the end vertices of each paths Q_i and R_i, $1 \leq i \leq n$, $n \geq 4$, we get n spanning cycles. Also, we construct two more spanning cycles, C_1: $u_1v_1v_2u_2...u_{n-1}v_{n-1}v_nu_nu_1$ and C_2: $u_2v_2v_3u_3...u_nv_nv_1u_1u_2$. Therefore, if n is even, we get n+2 spanning cycles.

The remaining edges of Circular Ladder graph CL_n, $n \geq 3$ in both cases form the 1-factorization.

Example: 5.2



3 Hamiltonian cycles + 1 factorization



6 Hamiltonian cycles + 1 factorization

V. CONCLUSION

We investigate Hamiltonian decomposition of some families of graphs such as ladder, triangular ladder, circular ladder graphs using the concept of Hamiltonian decomposition of graphs. Also, we investigate Hamiltonian decomposition of diagonal ladder graphs using the concept of Hamiltonian decomposition of ladder graphs.

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