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Unsteady MHD free convection flow between two heated vertical parallel plates in the presence of a uniform magnetic field

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Abstract—The problem of unsteady viscous incompressible MHD free convection flow of an electrically conducting fluid between two heated vertical parallel plates in the presence of a uniform magnetic field applied transversely to the flow is considered. The induce field along the lines of motion varies transversely to the flow and the fluid temperature changing with time. An analytical solution for velocity, induced field and the temperature distributions are obtained for small and large Magnetic Reynolds numbers. The skin-friction at the two plates is obtained. Velocity distribution, induced field and skin-friction are plotted graphically against the distance from the plates. It has been observed that with the increase in the Magnetic Reynolds number, at constant Hartmann number, leads to an increase in the skin-friction gradually. But with the increase in Hartmann number, at constant Magnetic Reynolds number, the skin-friction decreases.

Keywords—*Fully developed flow; Conducting fluid; induced field; Skin-friction; Magnetic Reynolds number.*

I. INTRODUCTION

The phenomenon of magnetohydrodynamic flow has been a subject of growing interest in view of its possible applications in many branches of science and technology and also industry. The subject of magneto hydrodynamics has attracted the attention of a large number of scholars due to its diverse applications in several problems of technological importance.

Skin friction measurement techniques commonly used are point measurement systems wherein the average (averaged over the measurement volume) skin friction at a point is determined. The measurement method may be a direct method, indirect measurement or empirical determination methods. There are excellent reviews on these methods (Hanratty and Campbell [1], Winter [2], Schetz [3] and Naughton and Sheplak [4]) those discuss the advantages and disadvantages of each of the methods.

Free convection flow past different types of vertical bodies is studied because of their wide applications. Free convection flow of fluids past a semi-infinite isothermal vertical plate was first investigated by Pohlhausen [5] who solved the problem by momentum integral method.

An exact solution to MHD Stokes problem for an infinite vertical plate with variable temperature has been studied by Soundalgekar [6]. He neglected the induced magnetic field and observed that in air, an increase in time t, leads to an increase in the skin-friction but in water, it decreases. An increase in the Hartmann number *M*, leads to an increase in the skin-friction.

In a fluid, the variation of temperature causes variation of density. This in turn changes force of buoyancy which governs the fluid motion. This type of unsteady fluid motion under the action of uniform magnetic field applied externally reduces the heat transfer and the skin friction considerably. This process of reduction of heat transfer and skin friction of the fluid motion has various engineering applications such as nuclear reactor, power transformation etc. Several authors' studies this type of MHD free convection laminar flow. Das and Sanyal [7] and Borkakati et. al [8] investigated the fully developed flow of a viscous incompressible conducting fluid in presence of a uniform magnetic field. Recently Gourla et. al [9] discussed an unsteady free convection flow through the vertical parallel plates in the presence of uniform magnetic field.

Ostrach [10, 11] considered the combined natural and forced convection flow of a viscous incompressible fluid between two vertical parallel plates. Grief et.al [12] and Gupta et.al [13] studied the incompressible free convection flow through a porous medium. Soundalgekar and Ramana Murty [14] discussed the heat transfer in MHD flow with pressure gradient, suction and injection. He observed that an increase in the magnetic field parameter leads to an increase in velocity, skin-friction, rate of heat transfer and a fall in temperature. Also an increase in suction leads to a fall in the value of the skin-friction and the rate of heat transfer, opposite to the case of injection. Soundalgekar and Bhatt. [15] considered the laminar convection flow through a porous medium between two vertical plates.

Kim [16] investigate the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. He observed that for a constant plate moving velocity with the given magnetic and permeability parameters, and Prandtl and Grashof numbers, the effect of increasing values of suction velocity parameter results in an increasing surface skin friction. It is also observed that the surface skin friction decreases by increasing the plate moving velocity.

The problem of combined heat and mass transfer of an electrically conducting fluid in MHD free convection adjacent to a vertical surface with Ohmic heating and viscous dissipation is analyzed by Chen [17]. He presented the results for the velocity, temperature, and concentration distributions, as well as the local skin-friction coefficient, local Nusselt number, and the local Sherwood number.

Dahake S.P. and Dubewar A.V.[18] have studied effects of radiation on magnetohydrodynamic convection flow past an impulsively started vertical plate submersed in a porous medium with suction.

Recently A Study of Unsteady MHD Vertical Flow of an Incompressible, Viscous, Electrically conducting Fluid bounded by Two Non-Conducting Plates in Presence of a Uniform inclined Magnetic Field investigated by Goswami, Singha and Deka [19]

Very recently Sharma and Dubewar [20] discussed MHD Flow between Two Parallel Plates under the Influence of Inclined Magnetic Field by Finite Difference Method. They considered, when the upper plate is moving with constant velocity under the influence of inclined magnetic field, the lower plate is held stationary.

In this paper, we have investigated the fully developed free convection laminar flow of an incompressible viscous electrically conducting fluid between two vertical parallel plates in the presence of a uniform magnetic field applied transversely to the flow. This induces a field along the lines of motion which varies transversely to the flow. The temperature of the fluid is assumed to be changing with time. The analytical solutions for velocity, induced magnetic field and the temperature distributions are obtained for small and large Magnetic Reynolds number R_m .The skin-friction at the two plates are obtained for different magnetic field parameters and are plotted graphically. . It has been observed that with the increase in R_m , the Magnetic Reynolds number, at constant M , the Hartmann number, leads to an increase in the skin-friction gradually. But with the increase in M , at constant R_m , the skin-friction decreases.

II. FORMULATION OF THE PROBLEM

We are considering an unsteady laminar convective flow of a viscous incompressible electrically conducting fluid between two vertical parallel plates. Let X -axis be taken along vertically upward direction through the central line of the channel and Y -axis is taken perpendicular to the X -axis. The plates of the channel are at $y = \pm h$. A uniform magnetic field B_0 is applied parallel to Y -axis which in turn induces a field along X -axis that varies along Y -axis. The velocity and magnetic field distributions are $\vec{V} = [u(y), 0.0]$ and $\vec{B} = [B(y), B_0, 0]$ respectively. Here B_0 and $B(y)$ are applied and induced magnetic field respectively.

In order to derive the governing equations of the problem the following assumptions are made.

(i) The fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected.

(ii) Hall effect and Polarization effect are negligible.

(iii) Initially (i.e. at time $t = 0$) the plates and the fluid are at zero temperature (i.e. $T = 0$) and there is no flow within the channel.

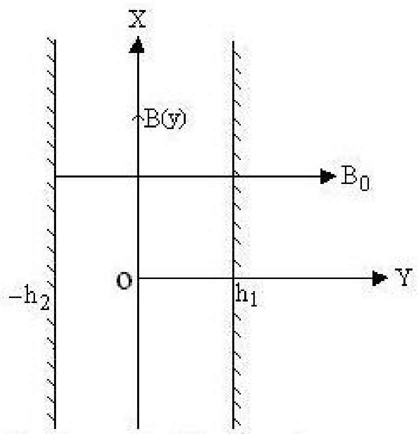


Fig. 1 Geometrical Configuration

(iv) At time $t > 0$, the temperature of the plate $y = \pm h$ change according to $T = T_0(1 - e^{-nt})$ where T_0 is a constant temperature and $n \geq 0$ is a real number denoting the decay factor.

(v) The plates are considered to be infinite and all the physical quantities are functions of y and t only.

III. GOVERNING EQUATIONS

Under the above assumptions the governing equations are

$$\nabla \cdot \vec{V} = 0$$
(1)

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\left(\frac{1}{\rho}\right)\nabla p + \nu \nabla^2 \vec{V} + \frac{1}{\rho}\left(\vec{J} \times \vec{B}\right) + \vec{Z}$$
.
(2)

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{V} \times \vec{B}) - \left(\frac{1}{\sigma \mu_e}\right)\nabla^2 \vec{B} = 0 .$$
(3)

$$\rho C_p \left(\frac{\partial T}{\partial t}\right) = \frac{d}{dy}\left(k \frac{dT}{dy}\right).$$
(4)

where the third term in the right hand side of equation (2) is the magnetic body force and $\vec{J}$ is the current density due to the magnetic field defined by

$$\vec{J} = \frac{(\nabla \times \vec{B})}{\mu_e} \quad (5)$$

$\vec{Z}$ is the force due to buoyancy, $\vec{Z} = \beta g (T_0 - T)$
(6)

Where

k = Thermal conductivity,

σ = Electrical conductivity,

ρ = fluid density,

μ_e = Permeability of the medium,

μ = co-efficient of viscosity,

$\upsilon_m = \frac{1}{\sigma \mu_e}$, magnetic diffusitivity,

$\upsilon = \frac{\mu}{\rho}$, kinematic viscosity,

Using the velocity and magnetic field distribution as stated above, the equations (1) to (4) are as follows:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{B_0}{(\rho \mu_e)} \frac{\partial B}{\partial y} + \beta g (T_0 - T) \quad (7)$$

$$\frac{\partial B}{\partial t} + B_0 \frac{\partial u}{\partial y} + \left(\frac{1}{\sigma \mu_e}\right)\frac{\partial^2 B}{\partial y^2} = 0 \quad (8)$$

$$\frac{\partial T}{\partial t} = \left(\frac{k}{\rho C_p}\right)\frac{\partial^2 T}{\partial y^2}$$

$$\Rightarrow \frac{\partial T}{\partial t} = \alpha_1 \frac{\partial^2 T}{\partial y^2} \quad (9)$$

Here, $\alpha_1 = \frac{k}{\rho C_p}$.

The boundary conditions are

$$\left.\begin{array}{l} \mathrm{t} = 0 : \mathrm{u} = 0, \mathrm{B} = 0, \mathrm{T} = 0 \text{ at } \mathrm{y} = \pm \mathrm{h} \\ \mathrm{t} > 0 : \mathrm{u} = 0, \mathrm{B} = 0, \mathrm{T} = T_0 (1 - e^{-nt}) \text{ at } \mathrm{y} = \pm \mathrm{h} \end{array}\right\}$$
(10)

Considering the non-dimensional terms

$$t^* = \nu t / h^2 , \; b = \frac{B}{B_0} , \; y^* = \frac{y}{h}, \; u^* = \frac{\nu u}{\beta g T_0 h^2},$$
$$\overline{T} = \frac{(T_0 - T)}{T_0}, \quad (11)$$

We get From (7)

$$\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial t^*}\frac{\partial t^*}{\partial t} = \nu \frac{\partial}{\partial y^*}\left(\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right)\frac{\partial y^*}{\partial y} +$$
$$\beta g T_0 \overline{T} + \frac{B_0}{\rho \mu_e}\left(\frac{\partial B}{\partial b}\frac{\partial b}{\partial y^*}\frac{\partial y^*}{\partial y}\right)$$

this reduces to

$$\left(\beta g T_0\right)\frac{\partial u^*}{\partial t^*}=\left(\frac{B_0{}^2}{\rho\mu_e h}\right)\frac{\partial b}{\partial y^*}+\beta g T_0\overline{T}+\left(\beta h T_0\right)\frac{\partial^2 u^*}{\partial y^{*2}}$$

finally takes the form

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\overline{T}+\left(\frac{M^2}{R_a R_m}\right)\frac{\partial b}{\partial y}$$

(12)

From (8)

$$\frac{\partial B}{\partial b}\frac{\partial b}{\partial t^*}\frac{\partial t^*}{\partial t}+B_0\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}+$$

$$\left(\frac{1}{\sigma\mu_e}\right)\frac{\partial}{\partial y^*}\left(\frac{\partial B}{\partial b}\frac{\partial b}{\partial y^*}\frac{\partial y^*}{\partial y}\right)\frac{\partial y^*}{\partial y}=0$$

$$\Rightarrow\frac{\partial b}{\partial t}+\left(\frac{R_a}{P_r}\right)\frac{\partial u}{\partial y}+\left(\frac{1}{R_m P_r}\right)\frac{\partial^2 b}{\partial y^2}=0$$

(13)

From (9)

$$\frac{\partial T}{\partial\overline{T}}\frac{\partial\overline{T}}{\partial t^*}\frac{\partial t^*}{\partial t}=\alpha_1\frac{\partial}{\partial y^*}\left(\frac{\partial T}{\partial\overline{T}}\frac{\partial\overline{T}}{\partial y^*}\frac{\partial y^*}{\partial y}\right)\frac{\partial y^*}{\partial y}$$

$$\Rightarrow\frac{\partial\overline{T}}{\partial t^*}=\frac{\alpha_1}{\nu}\frac{\partial^2\overline{T}}{\partial y^{*2}}$$

$$\Rightarrow\frac{\partial\overline{T}}{\partial t}=\frac{1}{P_r}\frac{\partial^2\overline{T}}{\partial y^2} \qquad (14)$$

Where

M is the Hartmann number, $M=\sqrt{\dfrac{B_0{}^2 h^2\sigma}{\rho\nu}}$,

P_r is the Prandtl number, $P_r=\dfrac{\nu}{\alpha_1}$,

R_e is the Reynolds number, $R_e=\dfrac{u_0 h}{\nu}$,

R_a is the Rayleigh number, $R_a=\dfrac{\beta g h^3 T_0}{\nu\alpha_1}$,

R_m is the Magnetic Reynolds number, $R_m=\alpha_1\mu_e\sigma$,

α_1 is the thermal diffusitivity, $\alpha_1=\dfrac{k}{\rho C_p}$,

k is the thermal conductivity,

ν_e is the magnetic diffusivity, $\nu_e=\dfrac{1}{\sigma\mu_e}$,

ν is the kinetic viscosity, $\nu=\dfrac{\mu}{\rho}$,

σ is the electrical conductivity,

ρ is the fluid density,

μ_e is the permeability of the medium and

μ is the co-efficient of viscosity.

The boundary condition (10) reduces to

$$\left.\begin{array}{l} t=0: u=0, b=0, \overline{T}=1 \text{ at } y=\pm h \\ t>0: u=1, b=0, \overline{T}=e^{-nt} \text{ at } y=\pm h \end{array}\right\}$$

(15)

IV SOLUTIONS

To solve the equation (12) to equation (14) subject to the boundary condition (15), we apply the transformation of variables

$u=f(y)e^{-nt}$, $b=g(y)e^{-nt}$ and $\overline{T}=\phi(y)e^{-nt}$ 16)

Substituting (16) in equations (12-14), we have

From (12)

$$-nf(y)e^{-nt}=e^{-nt}\frac{d^2 f}{dy^2}+\phi(y)e^{-nt}+\left(\frac{M^2}{R_a R_m}\right)\frac{dg}{dy}e^{-nt}$$

$$\Rightarrow\frac{d^2 f}{dy^2}+nf+\phi+\left(\frac{M^2}{R_a R_m}\right)\frac{dg}{dy}=0$$

(17)

From equation (13)

$$-ne^{-nt}g(y)+\left(\frac{R_a}{P_r}\right)\frac{df}{dy}e^{-nt}+\left(\frac{1}{R_mP_r}\right)\frac{d^2g}{dy^2}e^{-nt}=0$$

$$\Rightarrow\frac{d^2g}{dy^2}-(nR_mP_r)g+(R_mR_a)\frac{df}{dy}=0 \qquad (18)$$

From equation (14)

$$-ne^{-nt}\phi(y)=\frac{1}{P_r}\frac{d^2\phi}{dy^2}e^{-nt}$$

$$\Rightarrow\frac{d^2\phi}{dy^2}+(nP_r)\phi=0 \qquad (19)$$

The corresponding boundary conditions are:

For t = 0: f=0, g=0, $\phi=1$ at $y=\pm1$
(20)

The solutions of equations (17-19) subject to the boundary conditions (20) are

$$\phi(y)=\frac{\cos[ay]}{\cos[a]} \qquad (21)$$

$$f(y)=K_1(\cosh[\alpha y]-\sinh[\alpha y])+K_2(\cosh[\alpha y]+\sinh[\alpha y])+K_3(\cosh[\beta y]-\sinh[\beta y])$$
$$+K_4(\cosh[\beta y]+\sinh[\beta y])+C_4\cos[ay]$$
(22)

$$g(y)=\frac{1}{\alpha}\begin{pmatrix}(\sinh[\alpha]+\sinh[\alpha y])A_1C_6+(\cosh[\alpha]- \\ \cosh[\alpha y])A_2C_6\end{pmatrix}+\frac{1}{\beta}\begin{pmatrix}(\sinh[\beta]+\sinh[\beta y])A_3C_7+(\cosh[\beta]- \\ \cosh[\beta y])A_4C_7\end{pmatrix}$$
$$-\frac{1}{a}(\sin[a]+\sin[ay])C_8 \qquad (23)$$

Where $a=\sqrt{nP_r}$,

$$K_1=\frac{\cos[a]C_4C_{15}+\cosh[\beta]C_{16}}{2\cosh[\beta]C_{14}-2\cosh[\alpha]C_{15}}+\frac{\sinh[\beta](C_{13}C_{15}\cosh[\alpha]-C_{13}C_{14}\cosh[\beta]+C_{11}C_{16}\cosh[\alpha])}{(C_{10}\sinh[\beta]-C_{12}\sinh[\alpha])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$
$$+\frac{\sinh[\beta](\cos[a]C_4(C_{11}C_{14}-C_9C_{15})-C_9C_{16}\cosh[\beta])}{(C_{10}\sinh[\beta]-C_{12}\sinh[\alpha])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$$K_2=\frac{\cos[a]C_4C_{15}+\cosh[\beta]C_{16}}{2\cosh[\beta]C_{14}-2\cosh[\alpha]C_{15}}-\frac{\sinh[\beta](C_{13}C_{15}\cosh[\alpha]-C_{13}C_{14}\cosh[\beta]+C_{11}C_{16}\cosh[\alpha])}{(C_{10}\sinh[\beta]-C_{12}\sinh[\alpha])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$
$$-\frac{\sinh[\beta](\cos[a]C_4(C_{11}C_{14}-C_9C_{15})-C_9C_{16}\cosh[\beta])}{(C_{10}\sinh[\beta]-C_{12}\sinh[\alpha])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$$K_3=\frac{\cos[a]C_4C_{14}+\cosh[\alpha]C_{16}}{2\cosh[\alpha]C_{15}-2\cosh[\beta]C_{14}}+\frac{\sinh[\alpha](C_{13}C_{15}\cosh[\alpha]-C_{13}C_{14}\cosh[\beta]+C_{11}C_{16}\cosh[\alpha])}{(C_{12}\sinh[\alpha]-C_{10}\sinh[\beta])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$
$$+\frac{\sinh[\alpha](\cos[a]C_4(C_{11}C_{14}-C_9C_{15})-C_9C_{16}\cosh[\beta])}{(C_{12}\sinh[\alpha]-C_{10}\sinh[\beta])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$$K_4=\frac{\sinh[\alpha](C_{13}C_{14}\cosh[\beta]-C_{13}C_{15}\cosh[\alpha])}{(C_{12}\sinh[\alpha]-C_{10}\sinh[\beta])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$$+\frac{C_4\cos[a]((C_{10}\sinh[\beta]-C_{11}\sinh[\alpha]-C_{12}\sinh[\alpha])C_{14}+K_5)}{(C_{12}\sinh[\alpha]-C_{10}\sinh[\beta])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$$+\frac{C_9\sinh[\alpha](\cosh[\beta]C_4(C_{11}C_{14}-C_9C_{15})-C_9C_{16}\cosh[\beta])}{(C_{10}\sinh[\beta]-C_{12}\sinh[\alpha])(2C_{14}\cosh[\beta]-2C_{15}\cosh[\alpha])}$$

$K_5=C_9C_{15}\sinh[\alpha]$, $K_6=\frac{1}{2}(C_{11}+C_{12})$,

$$\alpha=\frac{\sqrt{-C_1-\sqrt{C_1^{\,2}+4C_2}}}{\sqrt{2}},$$

$$\beta=\frac{\sqrt{-C_1+\sqrt{C_1^{\,2}+4C_2}}}{\sqrt{2}},$$

$C_1=(1-R_mP_r)n-M^2$, $C_2=n^2R_mP_r$,
$C_3=(a^2+nR_mP_r)\sec[a]$,

$$C_4 = \frac{C_3}{a^4 - a^2 C_1 - C_2}\ ,\ C_5 = -\frac{R_m R_a}{M^2}\ ,$$

$$C_6 = (\alpha^2 + n)C_5\ ,$$

$$C_7 = (\beta^2 + n)C_5\ ,$$

$$C_8 = (a^2 C_4 - nC_4 - \sec[a])C_5\ ,$$

$$C_9 = (C_6 + R_m R_a)\alpha \sinh[\alpha]\ ,$$

$$C_{10} = (C_6 + R_m R_a)\alpha \cosh[\alpha]\ ,$$

$$C_{11} = (C_7 + R_m R_a)\beta \sinh[\beta]\ ,$$

$$C_{12} = (C_7 + R_m R_a)\beta \cosh[\beta]\ ,$$

$$C_{13} = (C_8 - R_m R_a C_4)a \sin[a]\ ,$$

$$C_{14} = a\beta \sinh[\beta]C_6\ ,$$

$$C_{15} = a\alpha \sinh[\beta]C_7\ ,\ C_{16} = \alpha\beta \sin[a]C_8\ ,$$

$$A_1 = \frac{\cos[a]C_4 C_{15} + \cosh[\beta]C_{16}}{\cosh[\beta]C_{14} - \cosh[\alpha]C_{15}}\ ,$$

$$A_2 = \frac{\sinh[\beta]\left(C_{13}C_{15}\cosh[\alpha] - C_{13}C_{14}\cosh[\beta] + C_{11}C_{16}\cosh[\alpha]\right)}{\left(C_{10}\sinh[\beta] - C_{12}\sinh[\alpha]\right)\left(C_{14}\cosh[\beta] - C_{15}\cosh[\alpha]\right)}$$

$$+\frac{\sinh[\beta]\left(\cos[a]C_4(C_{11}C_{14} - C_9 C_{15}) - C_9 C_{16}\cosh[\beta]\right)}{\left(C_{10}\sinh[\beta] - C_{12}\sinh[\alpha]\right)\left(C_{14}\cosh[\beta] - C_{15}\cosh[\alpha]\right)}\ ,$$

$$A_3 = \frac{C_4 C_{14}\cos[a] + C_{16}\cosh[\beta]}{-C_{14}\cosh[\beta] + C_{15}\cosh[\alpha]}\ ,$$

$$A_4 = \frac{\sinh[\alpha]\left(C_{13}C_{15}\cosh[\alpha] - C_{13}C_{14}\cosh[\beta] + C_{11}C_{16}\cosh[\alpha]\right)}{\left(-C_{10}\sinh[\beta] + C_{12}\sinh[\alpha]\right)\left(C_{14}\cosh[\beta] - C_{15}\cosh[\alpha]\right)}$$

$$+\frac{\sinh[\alpha]\left(\cos[a]C_4(C_{11}C_{14} - C_9 C_{15}) - C_9 C_{16}\cosh[\beta]\right)}{\left(-C_{10}\sinh[\beta] + C_{12}\sinh[\alpha]\right)\left(C_{14}\cosh[\beta] - C_{15}\cosh[\alpha]\right)}\ ,$$

V SKIN FRICTION

The skin friction at the plates $y = \pm 1$, is defined as

$$\tau = -\left[\mu \frac{du}{dy}\right]_{\pm 1} \tag{24}$$

Substituting the non-dimensional quantities (11), we get

$$\tau = -\left[\mu \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}\right]_{y=\pm 1}$$

$$\Rightarrow \tau = -\left(\frac{\mu\beta g T_0 h}{\nu}\right)\left[\frac{\partial u^*}{\partial y^*}\right]_{y=\pm 1}$$

removing the asterisks, we get

$$\tau = -\left(\frac{\mu\beta g T_0 h}{\nu}\right)\left[\frac{\partial u}{\partial y}\right]_{y=\pm 1} \tag{25}$$

using relation (16), we get

$$\tau = -\left(\frac{\mu\beta g T_0 h}{\nu}\right)\left[\frac{df}{dy}e^{-nt}\right]_{y=\pm 1} \tag{26}$$

VI RESULTS AND DISCUSSION

Figure (2-5) has been obtained by plotting the velocity distribution f against y at different Magnetic Reynolds number R_m and Hartmann number M when n=1.0, $P_r = 0.71$, $R_a = 1.0$, $R_e = 1.0$.

Figure (6-9) has been obtained by plotting the induced magnetic field g against y at different values of Magnetic Reynolds number R_m and Hartmann number M when n=1.0, $P_r = 0.71$, $R_a = 1.0$, $R_e = 1.0$.

Figure (10-11) has been obtained by plotting the skin friction against y by considering the same above fluid parameters.

Figure (12-13) has been obtained by plotting the temperature distribution ϕ against y at different Prandtl number P_r and n=1.0 .

For computational process MATHEMATICA V5.1 is used. All these plotting has been done by using MATLAB 6.0.0.88

(i) When R_m is small, the variation of velocity increases very slowly for all values of M .

(ii) When R_m is high, the rate of fluctuation of the velocity and induced field is faster when M is big (=5.5) [figures (5) and (9)].

(iii) Velocities at the central plane of the channel are maximum and gradually decline towards the plates for all values of R_m and M ,[figure-(2-5)].

(iv) At high values of R_m (=50 to 1000), the fluid velocity and the induced field increases steadily and distinctly with the increase of R_m when M is high (=5.5) while they remains almost same when M is small (=1.5).

(v) With the increase of R_m at constant M, the skin-friction gradually increases but it is decreases with the increase of M at constant R_m, [figure-(10-11)].

(vi) In figure (12) we have observed that the temperature increases steadily and distinctly with the increases of P_r.

(vii) In figure (13) we have noticed that at high values of P_r (=5 to 12), the temperature first decreases then increases slowly.

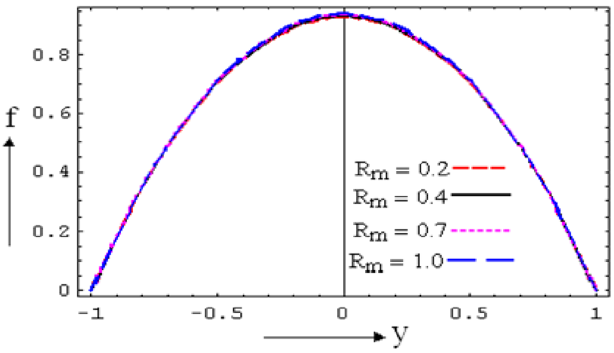
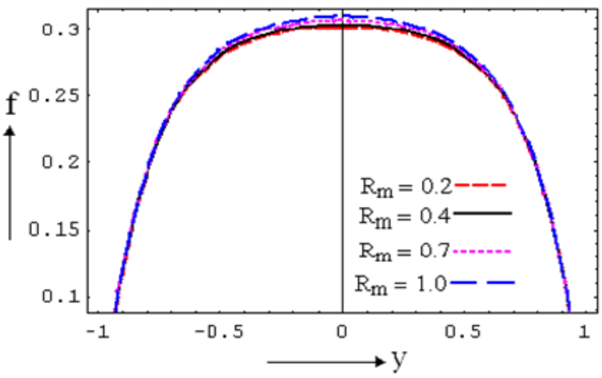


Fig.2: Velocity profiles at small R_m for $M = 1.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$



. *Fig.3 :* Velocity profiles at small R_m for $M = 5.5$; $R_a = 1.0$; $n = 1.0$; $P_r = 0.71$; $R_e = 1.0$

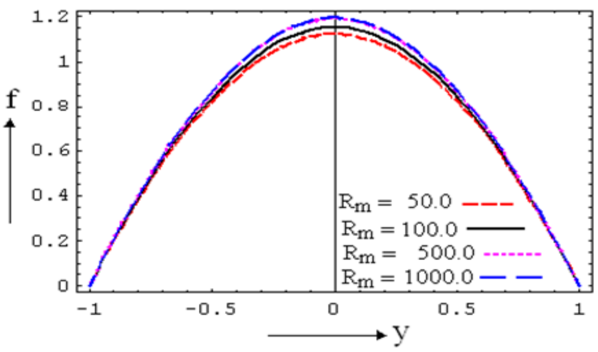


Fig.4 : Velocity profiles at large R_m for $M = 1.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

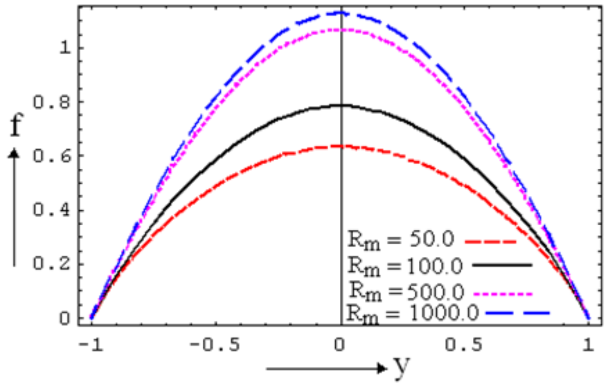


Fig.5 : Velocity profiles at large R_m for $M = 5.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

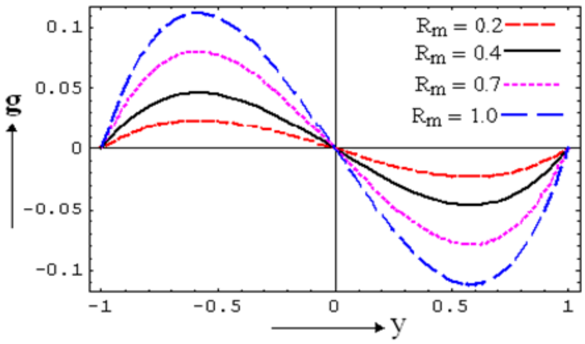


Fig.6 : Induced magnetic field profiles at small R_m for $M = 1.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

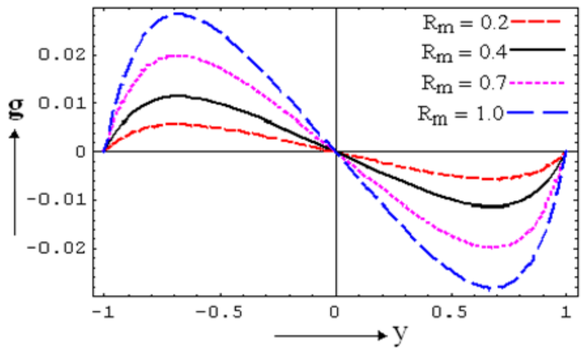


Fig.7 : Induced magnetic field profiles at small R_m for $M = 5.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

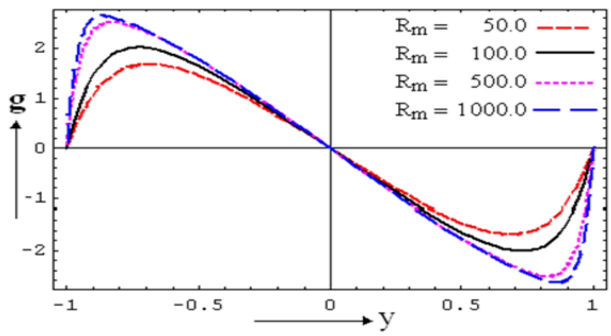


Fig.8 : Induced magnetic field profiles at large R_m for $M = 1.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

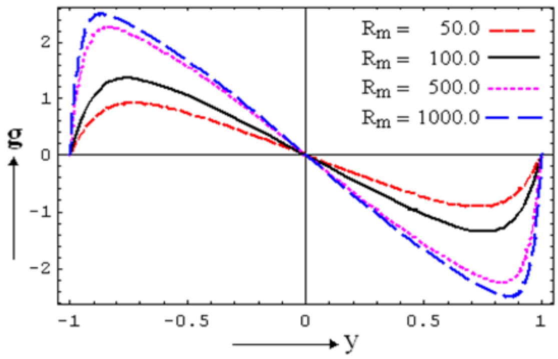


Fig.9 : Induced magnetic field profiles at large R_m for $M = 5.5$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

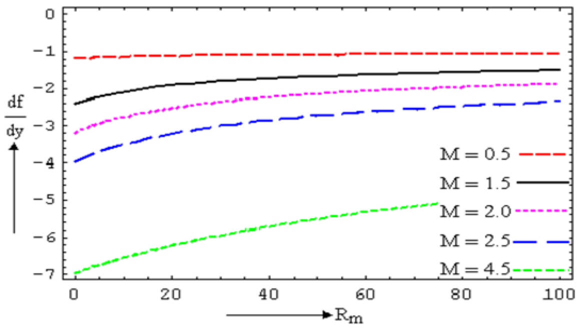


Fig.10 : Variation of friction factor (df/dy) at $y = +1$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

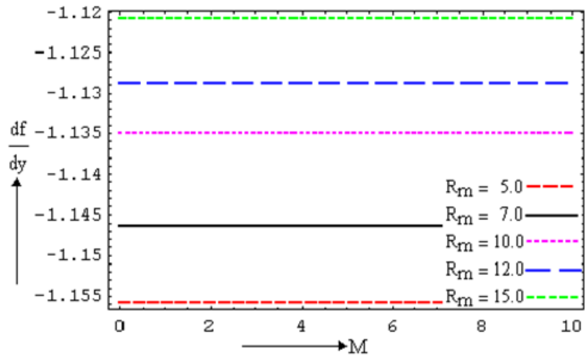


Fig.11 : Variation of friction factor (df/dy) at $y = +1$, $R_a = 1.0$, $n = 1.0$, $P_r = 0.71$, $R_e = 1.0$

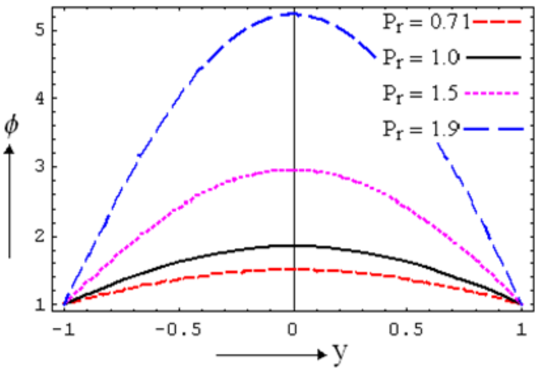


Fig.12 : Temperature profiles at small P_r for $n = 1.0$

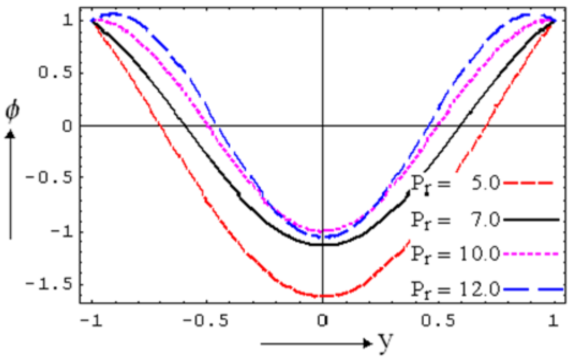


Fig.13 : Temperature profiles at large P_r for $n = 1.0$

VI. CONCLUSION AND FUTURE SCOPE

In this article we have studied about free convection laminar flow of an incompressible viscous electrically conducting fluid between two vertical parallel plates in the presence of a uniform magnetic field applied transversely to the flow. Also analytical methods is applied to compute for velocity, induced magnetic field and the temperature distributions for small and large Magnetic Reynolds number. The skin-friction at the two plates are obtained for different magnetic field parameters and the results are plotted graphically. The fluid properties density, viscosity, thermal and electrical conductivity are supposed to be constant in our discussion. But in actual practice, especially the fluid viscosity and density vary with temperature. Hence there are opportunities to extend the problem by considering fluid density and viscosity as variable with temperature. The thermal diffusion effect is not considered in this paper, which may be included to make it more practical.

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