Application of TOPSIS Method for Decision Making

R. M. Zulqarnain 1*, M. Saeed 2, N. Ahmad 2, F. Dayan 2, B. Ahmad 4

1School of Mathematics, Northwest University, Xian 710127, China.
2School of Science, Department of Mathematics, University of Management and Technology Lahore, Punjab, Pakistan.
3Department of Mathematics and Statistics, The University of Lahore, Pakistan.
*Corresponding author: ranazulqarnain7777@gmail.com, Tel: +86-13028563376

Abstract—In this paper, we discuss the order preference by similarity ideal solution (TOPSIS) method with basic concepts and determine the TOPSIS algorithm. Secondly, we construct a graphical model for the TOPSIS method by using the TOPSIS algorithm. Finally, we use the developed method for decision making in our daily life. In this work, we use the TOPSIS method for the selection of a car by using hypothetical data and examined that the civic is the best automotive car according to given parameters.

Keywords— Multiple Criteria Decision Making (MCDM), TOPSIS, Positive Ideal Solution (PIS), Negative Ideal Solution (NIS)

I. INTRODUCTION

Decision Making is the best procedure to choose a superlative alternative from all feasible alternatives. Almost in all other issues, the overall number of criteria because decision making the general alternatives is pervasive. Such criteria normally contrast one another so there might be no way out satisfying all criteria simultaneously. To deal with such problems the decision-makers want to solve the MCDM problem. There are different methods to solve MCDM problems. One of them presented by Hwang and Yoon in [1] is known as a TOPSIS to solve the MCDM problem with many alternatives. The core concept of this technique is that the chosen alternative should have the smallest geometrical distance from the PIS and the largest geometrical distance from NIS [2].

Nowadays this technique used in different fields of life such as energy [3–7] medicine [2,8–10] engineering and manufacturing systems [11–16] safety and environmental fields [17–22] chemical engineering [5,23,24] and water resources studies [5,19,23,25]. Chen & Hwang extend the idea of the TOPSIS method and presented a new model for TOPSIS[26]. Zulqarnain et al. developed the graphical model of the TOPSIS method and used for the selection of a medical clinic for the diagnosis of disease [27]. Moreover, to solve uncertain data Chen extended the TOPSIS for Group Decision Making in the fuzzy atmosphere [28] and used the newly proposed method for decision making. The importance weights of multi-criteria and alternative rating w.r.t. these criteria were treated as linguistic variables, evaluated by a group of decision-makers. To facilitate the decision making in a fuzzy environment many researchers extended the TOPSIS technique reported in the literature [3,4,18,19,25,29–35,6,8,11–15,17]. The author’s developed the idea of generalized interval-valued fuzzy soft matrices (IVFSM) in [36]. Zulqarnain et al [37] used the trapezoidal fuzzy numbers by Sánchez’s approach for disease identification. The usage of interval numbers is too a significant enhancement of [38–40]. The extension of TOPSIS under fuzzy data has been used to express the prospect of achievement for pancreatic transplantation [8]. A decision-making method on IVFSM introduced in [41] and the authors provided the application of IVFSM [42] and comparative study with a fuzzy soft matrix in [43].

Mahmoodzadeh et al. developed a technique for the project assortment by combining fuzzy AHP and TOPSIS methods and used the upgraded technique to calculate the weights of each criterion at first and then the TOPSIS algorithm was engaged for ranking the projects to be selected [44]. The authors faced some difficulties to determine the accurate value of the elements of the decision matrix, such as their values were considered as intervals, to overcome these difficulties they extended the TOPSIS method with interval data in [38]. Several approaches have been established for MCDM problems, in [45] the authors provided a proper guideline of how and which method could be used for MCDM problems according to the situation.

The authors extended the TOPSIS to Atanassov intuitionistic fuzzy set and proposed the algorithm of extended TOPSIS for multi-attribute group decision-making problem in [46]. The idea of multiple attribute intuitionistic fuzzy group decision-making algorithm was introduced in [46]. Many researchers worked on the TOPSIS method and used in medical diagnosis and for decision making in different fields of life reported in the literature [47–50].
The following paper is organized as follows. In section II, we study and discuss some basic concepts of the TOPSIS method and present the classical TOPSIS algorithm. We proposed the graphical model for the TOPSIS method by using the TOPSIS algorithm. In section III, we use the TOPSIS method for decision-making and choose the best car by using hypothetical data. Lastly, the conclusion is made in section IV.

THE EFFICIENCY OF TOPSIS [51]

First, it is important to discuss the efficiency of the TOPSIS method. The calculation time of the TOPSIS method rises slightly when the number of criteria increases to 16. However, the point to be noted is that the time does not exceed 10 seconds see [51].

The TOPSIS is examined by changing the number of criteria and users, and it was determined that the efficiency is high when the number of users is under 320 and the number of criteria is not more than 16.

TOPSIS Method

Hwang and Yoon [1] developed a technique to resolve MCDM known as the TOPSIS method. To support the shortest Euclidean distance, they proposed the PIS and NIS and each criterion needs to be maximized or minimized. They claimed that the TOPSIS method helps rank alternatives closeness which based on optimum ideal solution and obtained the maximum level from available alternatives. The best alternative has rank one and the worst alternative approaches rank zero. For every alternative, there is an intermediate ranking between the best answer extremes. An identical set of choice criteria permits correct weighting of relative disease and therefore the optimum disease is alarming which needs attention. Here are presented the steps for the TOPSIS technique. TOPSIS views an MCDM problem with m-alternatives as a geometric system with m points in the n-dimensional space [52]. The core concept of this technique is that the chosen alternative should have the smallest geometrical distance from the PIS and the largest geometrical distance from the NIS [53]. To apply TOPSIS [54], a common assumption is that criteria should be either monotonically increasing or decreasing so that PIS and NIS can be easily identified.

II. CLASSICAL TOPSIS ALGORITHM

Step 1: Establishment of DM

Construct the decision matrix as follows

\[
DM = \begin{bmatrix}
A_1 & c_{11} & c_{12} & \cdots & c_{1q} \\
A_2 & c_{21} & c_{22} & \cdots & c_{2q} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_p & c_{p1} & c_{p2} & \cdots & c_{pq}
\end{bmatrix}
\]

Where \( l \) is the alternative index \((l = 1, 2, \ldots, q)\); \( n \) is the number of potential sites and \( m \) is the criteria index \((m = 1, 2, \ldots, p)\).

The elements \( R_1, R_2, \ldots, R_q \) of the DM define the criteria while \( A_1, A_2, \ldots, A_p \) defining the alternatives.

Step 2: Calculation of the Normalized Decision Matrix (NDM)

To represent the relative performance of the alternatives the NDM constructed as follows.

\[
NDM = L_{nm} = \frac{c_{nm}}{\sqrt{\sum_{l=1}^{q} c_{nm}^2}}
\]

Step 3: Determination of the Weighted Normalized Decision Matrix (WNDM)

By multiplying every element of each column of NDM got a weighted decision matrix.

\[
V = V_{lm} = W_m \times L_{nm}
\]

Step 4: Identification of the PIS and NIS

The PIS \((PIS)\) and the NIS \((NIS)\) are defined concerning the weighted decision matrix as follows

\[
NIS = I^- = \{V_1, V_2, \ldots, V_q\}, \text{ where:}
\]

\[
V_m = \{ (\min(V_{lm}) \text{ if } m \in J) ; (\max(V_{lm}) \text{ if } m \in J') \}
\]

Where \( J' \) is associated with the non-beneficial attributes and \( J \) is associated with beneficial attributes.

Step 5: Separation Distance from PIS and NIS of each alternative

\[
S_{l}^+ = \sqrt{\sum_{m=1}^{n} (V_{lm}^+ - V_{lm})^2} ; l = 1, 2, \ldots, q
\]

\[
S_{l}^- = \sqrt{\sum_{m=1}^{n} (V_{lm}^- - V_{lm})^2} ; l = 1, 2, \ldots, q
\]

Where, \( l \) = Alternative index, \( m \) = Criteria index.

Step 6: Relative Closeness to the Ideal Solution.

The relative closeness of the ideal solution is computed as

\[
C_i = \frac{S_{i}^-}{(S_{i}^+ + S_{i}^-)} , 0 \leq C_i \leq 1
\]

Step 7: Ranking of Preference Order

The ranking is done based on the values of \( C_i \), the higher value of the relative closeness has a high rank and hence the better performance of the alternative. Rank the preference in descending order to compare the better performances of alternative.
III. APPLICATION OF TOPSIS METHOD

A person wants to choose the car for his family. For the selection of best automotive car, he hires a team of three experts (decision-makers) such as \( l = 3 \) represented by \( D = \{ D_1, D_2, D_3 \} \). Firstly, the experts select four best automotive cars showroom in the city as follows \( A = \{ \text{Civic, Corolla, Swift, Hyundai} \} \) and decide the four evaluation criteria represented by \( C = \{ \text{Style, Safety, Fuel Efficiency, Expanses} \} \) for selection of one of the best automotive car out of four cars.

C = \{ \begin{align*} & \text{benefit criteria} \\
& \text{Cost criteria} \end{align*} \} \\
J_1 = \{ X_1: \text{Style} \} \\
J_2 = \{ X_2: \text{Safety} \} \\
J_3 = \{ X_3: \text{Fuel Efficiency} \} \\
J_4 = \{ X_4: \text{Expenses} \}

SOLUTION BY TOPSIS

TOPSIS method will be illustrated with the help of a car selection problem. Here the set of alternatives is \( A = \{\text{Civic, Corolla, Swift, Hyundai}\} \) and the set of evaluation criteria is \( C = \{ \text{Style (St), Safety (S}, \text{ Fuel Efficiency (FE), Expenses (Exp)} \} \) for selection of one of the best automotive car out of four cars.

Step 1: Construction of a Decision Matrix

The decision matrix is given in the following table.

<table>
<thead>
<tr>
<th>Cars</th>
<th>St</th>
<th>Sa</th>
<th>FE</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Corolla</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Swift</td>
<td>9</td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Hyundai</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Step 2: Normalization

By using the following formula, we get

Step 3: Computation of the weight matrix

The weights assigned by the experts (decision makers) to the criteria are given by the matrix \( W = [w_1, w_2, w_3, w_4] \) where \( w_1 = 0.1 \), \( w_2 = 0.4 \), \( w_3 = 0.3 \), \( w_4 = 0.2 \).

Step 4: Computation of WNDM \( \hat{R} = [r_{ij}]_{4 \times 5} \)

To get WNDM, multiplying each column of NDM in Table 3 by weights \( w_j \), of weight vector computed in the step 3.

Step 5: The calculation of PIS and NIS

To find the PIS \( A^* \)

<table>
<thead>
<tr>
<th>Cars</th>
<th>St</th>
<th>Sa</th>
<th>FE</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic</td>
<td>0.046</td>
<td>0.244</td>
<td>0.162</td>
<td>0.106</td>
</tr>
<tr>
<td>Corolla</td>
<td>0.053</td>
<td>0.192</td>
<td>0.144</td>
<td>0.092</td>
</tr>
<tr>
<td>Swift</td>
<td>0.059</td>
<td>0.164</td>
<td>0.144</td>
<td>0.118</td>
</tr>
<tr>
<td>Hyundai</td>
<td>0.040</td>
<td>0.192</td>
<td>0.144</td>
<td>0.080</td>
</tr>
</tbody>
</table>

Therefore \( A^* = \{ 0.059, 0.244, 0.162, 0.080 \} \)

To find the NIS \( A^- \)
Table 6: Negative Ideal Solution

<table>
<thead>
<tr>
<th>Benefit Criteria ∈ J⁺</th>
<th>Cost Criteria ∈ J⁻</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>St</td>
</tr>
<tr>
<td>Civic</td>
<td>0.046</td>
</tr>
<tr>
<td>Corolla</td>
<td>0.053</td>
</tr>
<tr>
<td>Swift</td>
<td>0.059</td>
</tr>
<tr>
<td>Hyundai</td>
<td>0.040= v₁^*</td>
</tr>
</tbody>
</table>

Therefore A⁻ = {0.040, 0.164, 0.144, 0.118}

Step 6: Determine the separation measures for each alternative
Calculating separation from PIS A⁺

Table 7: Calculation of S⁺

<table>
<thead>
<tr>
<th>St</th>
<th>Sa</th>
<th>FE</th>
<th>Exp</th>
<th>Σᵢ₌₁(vᵢ⁺ – vᵢ⁻)²</th>
<th>S⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic</td>
<td>(0.046-0.059)²</td>
<td>(0.244-0.244)²</td>
<td>(0)²</td>
<td>(0.026)²</td>
<td>0.00845</td>
</tr>
<tr>
<td>Corolla</td>
<td>(0.053-0.059)²</td>
<td>(0.192-0.244)²</td>
<td>(-0.018)²</td>
<td>(0.012)²</td>
<td>0.003208</td>
</tr>
<tr>
<td>Swift</td>
<td>(0.053-0.059)²</td>
<td>(0.164-0.244)²</td>
<td>(-0.018)²</td>
<td>(0.038)²</td>
<td>0.008186</td>
</tr>
<tr>
<td>Hyundai</td>
<td>(0.053-0.059)²</td>
<td>(0.192-0.044)²</td>
<td>(-0.018)²</td>
<td>(0)²</td>
<td>0.003389</td>
</tr>
</tbody>
</table>

Calculating separation from NIS A⁻

Table 8: Calculation of S⁻

<table>
<thead>
<tr>
<th>St</th>
<th>Sa</th>
<th>FE</th>
<th>Exp</th>
<th>Σᵢ₌₁(vᵢ⁻ – vᵢ⁺)²</th>
<th>S⁻</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic</td>
<td>(0.046-0.040)²</td>
<td>(0.244-0.164)²</td>
<td>(0.018)²</td>
<td>(-0.012)²</td>
<td>0.006904</td>
</tr>
<tr>
<td>Corolla</td>
<td>(0.053-0.040)²</td>
<td>(0.192-0.164)²</td>
<td>(0)²</td>
<td>(-0.026)²</td>
<td>0.001629</td>
</tr>
<tr>
<td>Swift</td>
<td>(0.053-0.040)²</td>
<td>(0.164-0.164)²</td>
<td>(0)²</td>
<td>(0)²</td>
<td>0.000361</td>
</tr>
<tr>
<td>Hyundai</td>
<td>(0.053-0.040)²</td>
<td>(0.192-0.164)²</td>
<td>(0)²</td>
<td>(-0.038)²</td>
<td>0.002228</td>
</tr>
</tbody>
</table>

Step 7: Computation of RCC to the ideal solution Cᵢ⁺
RCC to the ideal solution Cᵢ⁺ is computed as follows

Cᵢ⁺ = S⁻ / (Σᵢ₌₁ S⁻ + Σᵢ₌₁ S⁺) = 0.74 (Best)

Similarly, we can get

C₂⁺ = 0.41
C₃⁺ = 0.17
C₄⁺ = 0.45

Hence “Civic” is the best automotive car with the above evaluation criteria.

IV. CONCLUSION

In this paper, we discuss the TOPSIS method in detail and constructed a graphical model for the TOPSIS method. We used the TOPSIS method for the selection of the best automotive car by using hypothetical data and examined that Civic is the best car according to the above selected parameters.

REFERENCES

3734–3743, 2011.


