International Journal of Scientific Research in Mathematical and Statistical Sciences
Volume-5,Issue-6,pp.01-12,December(2018)

Research Paper

E-ISSN: 2348-4519

A New Life Time Model: The Generalized Rayleigh-Truncated Negative Binomial Distribution

J. Gillariose[1*], L. Tomy[2]

[1]Department of Statistics, St.Thomas College, Pala, Kerala-686574, India
[2]Department of Statistics, Deva Matha College, Kuravilangad, Kerala-686633, India

Corresponding Author: jijugillariose@yahoo.com

Available online at: www.isroset.org

Received: 21/Oct/2018, Accepted: 14/Nov/2018, Online: 31/Dec/2018

***Abstract*—**In this paper, we introduced a generalized form of Rayleigh distribution. Shapes of probability density function and hazard rate function along with sub-models and statistical properties of the proposed distribution are provided. The usefulness of the new distribution is illustrated through real data set by comparing with other generalizations of the Rayleigh distribution. A reliability test plan for acceptance or rejection of a lot of products submitted for inspection with lifetimes governed by this distribution is developed and illustrated with real data set.

Keywords—Generalized family, Maximum likelihood estimation, Rayleigh distribution, Truncated negative binomial distribution

I. INTRODUCTION

The Rayleigh distribution is a special case of the Weibull distribution, which is considered to be a useful life distribution. This distribution is also known as the one parameter Burr type X distribution and it has been applied in communication theory to describe hourly median and instantaneous peak power of received radio signals. Additionally, it is a common choice in many areas including health, agriculture, biology, medicine, engineering and finance. Keeping these applications in mind, we consider this distribution in our study.

Indeed, from the practical point of view, the generalizations of existing distributions are used to model lifetime models. The generalized Rayleigh (GR) distribution has been derived in the literature because of limited mathematical structure of Rayleigh distribution. One of the attractive extensions of the Rayleigh distribution has been proposed by Vodǎ [1, 2]. Its probability density function (pdf) is given by

$$f(x,\theta,\lambda)=\frac{2\theta^{-2\lambda-2}x^{2\lambda+1}e^{-(x/\theta)^2}}{\Gamma(\lambda+1)}\;,\; x,\theta>0,\lambda>-1. \tag{1}$$

Here θ is the scale parameter, λ is the shape parameter, and $\Gamma(a)=\int_0^\infty t^{a-1}e^{-t}dt$ is the complete gamma function. Its survival function is

$$\overline{F}(x,\theta,\lambda)=1-\frac{\Gamma(\lambda+1,(x/\theta)^2)}{\Gamma(\lambda+1)}=\overline{\gamma}(\lambda+1,(x/\theta)^2)\;,\; x,\theta>0,\lambda>-1. \tag{2}$$

where $\Gamma(a,x)=\int_0^x t^{a-1}e^{-t}dt$ is the incomplete gamma function hence Karl Pearson's tables can be used for various values of θ and λ. Various statistical experts have mentioned extensions of GR distribution using different methods. Cordeiro et al. [3] introduced four-parameter beta-GR distribution, Gomes et al. [4] introduced the four-parameter Kumaraswamy-GR distribution, and MirMostafaee et al. [5] introduced Marshall-Olkin extended GR distribution respectively.

Adding parameters to an acknowledged distribution is a classical tool for obtaining more flexible distributions. In this work, we suggest a new generalization of the GR distribution by using the scheme introduced by Nadarajah et al. [6]. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with survival function $\overline{F}(x)$ and N be a truncated negative binomial random variable, independent of X_i's, with parameters $0<\alpha<1$ and $\beta>0$, such that

$$P(N=n)=\frac{\alpha^\beta}{1-\alpha^\beta}\binom{\beta+n-1}{\beta-1}(1-\alpha)^n; n=1,2,\ldots \tag{3}$$

If $U_N=min(X_1,X_2,\ldots X_N)$, then the survival function of U_N is

$$\overline{G}(x,\alpha,\beta)=\frac{\alpha^\beta}{1-\alpha^\beta}\left\{[F(x)+\alpha\overline{F}(x)]^{-\beta}-1\right\}x\in R,\alpha,\beta>0 \tag{4}$$

Similarly, if $\alpha > 1$ and N is a truncated negative binomial random variable with parameters $\frac{1}{\alpha}$ and $\beta > 0$, then $W_N = max(X_1, X_2, \ldots X_N)$ also has the same survival function given in (4). If $\alpha \to 1$ in (4), then $\overline{G}(x, \alpha, \beta) \to \overline{F}(x)$. If $\beta = 1$, then this family reduces to the Marshall-Olkin family of distributions [7]. Thus the family of distributions described in (4) is a generalization of the family of Marshall-Olkin distributions. The pdf of survival function given in equation (4) is

$$g(x, \alpha, \beta) = \frac{(1-\alpha)\beta\alpha^{\beta} f(x)}{(1-\alpha^{\beta})\{F(x)+\alpha\overline{F}(x)\}^{\beta+1}} \tag{5}$$

Jayakumar and Sankaran [8] defined a generalized uniform distribution using the approach of [6]. Babu [9] introduced Weibull truncated negative binomial distribution. Further, Jayakumar and Sankaran [10] introduced generalized exponential truncated negative binomial distribution and studied its properties. Also, Jose and Sivadas [11] used the family (4) to introduce the negative binomial Marshall-Olkin Rayleigh distribution.

The organization of the article is as follows. Section 2 deals with a new distribution and its sub-models. Section 3 provides statistical properties of the proposed distribution. In Section 4 we consider the estimation of parameters using maximum likelihood estimates and Section 5 gives a real-life application. In Section 6 we develop a sampling plan for accepting or rejecting a lot and minimum samples sizes and operating characteristic values are calculated and a numerical example is provided. The concluding remarks are given in Section 7.

II. GENERALIZED RAYLEIGH-TRUNCATED NEGATIVE BINOMIAL DISTRIBUTION

In this section, we introduce a new distribution called generalized Rayleigh-Truncated negative binomial (GR-TNB) distribution which contains some special sub-models and it seems to be quite flexible as an alternative model to be used in a variety of lifetime problems. By inserting (1) in (4) the survival function of GR-TNB distribution is given by

$$\overline{G}(x, \alpha, \beta, \lambda, \theta) = \frac{\alpha^{\beta}}{1-\alpha^{\beta}}\left\{[\gamma(\lambda+1, (x/\theta)^2) + \alpha\overline{\gamma}(\lambda+1, (x/\theta)^2)]^{-\beta} - 1\right\} \tag{6}$$

The corresponding pdf is given by

$$g(x, \alpha, \beta, \lambda, \theta) = \frac{(1-\alpha)\beta\alpha^{\beta} 2\theta^{-2(\lambda+1)} x^{2\lambda+1} e^{-(x/\theta)^2}}{\Gamma(\lambda+1)(1-\alpha^{\beta})\{\gamma(\lambda+1,(x/\theta)^2)+\alpha\overline{\gamma}(\lambda+1,(x/\theta)^2)\}^{\beta+1}} \tag{7}$$

In addition, the hazard rate function (hrf) of the GR-TNB distribution becomes

$$h(x, \alpha, \beta, \lambda, \theta) = \frac{(1-\alpha)\beta 2\theta^{-2\lambda-2} x^{2\lambda+1} e^{-(x/\theta)^2}[\gamma(\lambda+1,\frac{x^2}{\theta^2})+\alpha\overline{\gamma}(\lambda+1,\frac{x^2}{\theta^2})]^{-1}}{\Gamma(\lambda+1)(1-[\gamma(\lambda+1,(x/\theta)^2)+\alpha\overline{\gamma}(\lambda+1,(x/\theta)^2)]^{\beta})} \tag{8}$$

Notably, following distributions are special cases of the GR-TNB distribution.

- MOE GR Distribution : when β=1 in equation (7)
- MOE half normal : when $\beta = 1$ and $\lambda = \frac{1}{2}$ in equation (7)
- MOE Reyleigh : when $\beta = 1$ and $\lambda = 0$ in equation (7)

Figures 1 and 2 display some shapes of pdf and hrf of the GR-TNB model for selected parameter values, respectively. The hrf can be bathtub shaped, increasing and increasing-decreasing-increasing depending on the parameter values. The new model allows for great flexibility and hence it can be very useful in many practical situations for modeling positive data.

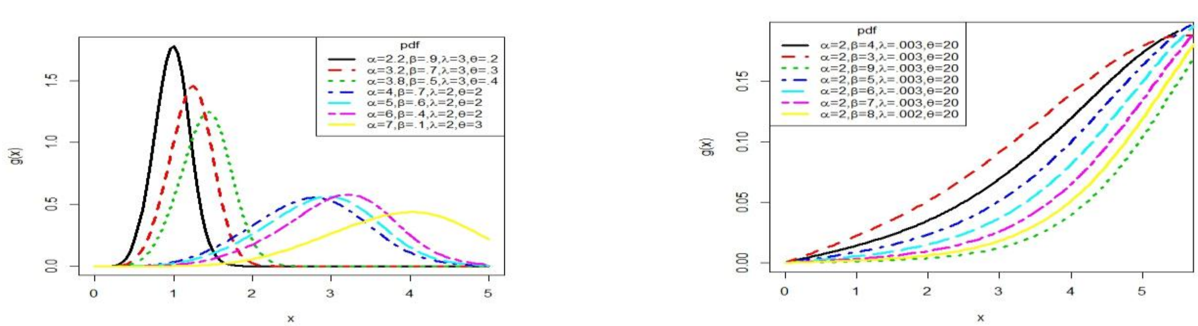

Figure 1: Graphs of pdf of the GR-TNB distribution for different values of α,β,λ and θ.

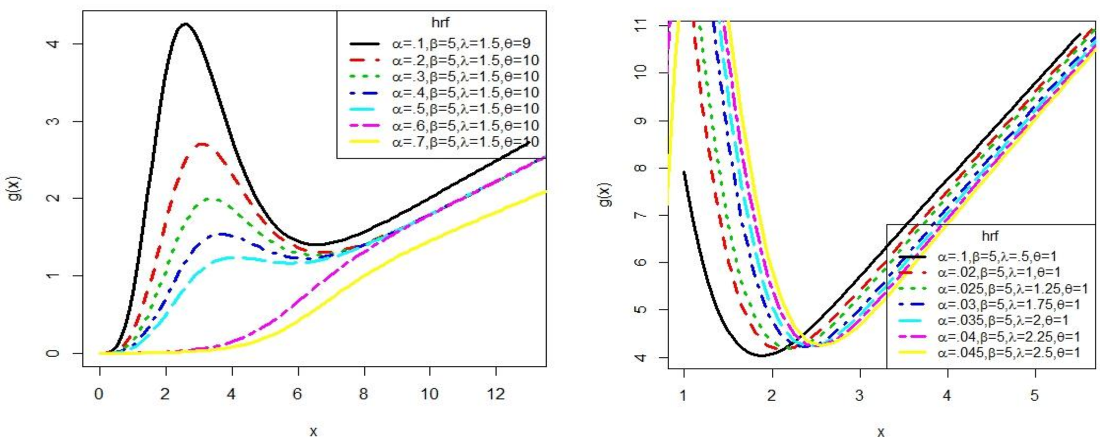

Figure 2: Graphs of hrf of the GR-TNB distribution for different values of α,β,λ and θ.

III. GENERAL PROPERTIES

A. Quantile Function

The quantile function of X follows GR-TNB distribution, it can be expressed as

$$Q(u) = \theta\sqrt{Q_G\left(\left[\frac{1}{1-\alpha}\left[\left\{\frac{(1-u)(1-\alpha^{\beta})}{\alpha^{\beta}}\right\}^{-\frac{1}{\beta}}+1\right]\right]\right)}$$

where u is generated from the Uniform(0, 1) distribution and $Q_G(.)$ is the (standardized) gamma quantile function (with shape parameter $\lambda+1$ and unit scale parameter) available in most of the statistical software packages.

B. Useful Expansions

We now give simple expansions for the pdf of the GR-TNB distribution.
We have

$$\Gamma(a,x) = x^a\sum_{i=0}^{\infty}\frac{(-x)^i}{i!(a+i)} \tag{9}$$

Thus the cumulative distribution function (cdf) of GR distribution comes as

$$\gamma(x,\theta,\lambda) = \frac{(x/\theta)^{2(\lambda+1)}}{\Gamma(\lambda+1)}\sum_0^{\infty}\frac{(-1)^i(x/\theta)^{2i}}{(\lambda+1+i)i!}$$

Also we have

$$\left(\sum_{i=0}^{\infty} a_i x^i\right)^n = \sum_{i=0}^{\infty} c_{i,n}x^i \tag{10}$$

where $n\varepsilon N$, $c_{0,n} = a_0^n$ and for $i = 1,2..$ the coefficients of $c_{i,n}$ are given by

$$c_{i,n} = (ia_0)^{-1}\sum_{p=0}^{\infty}(np-i+p)a_p c_{i-p,n}$$

Thus, from equations (9) and (10), we obtain

$$\gamma(x,\theta,\lambda)^j = \frac{(x/\theta)^{2j(\lambda+1)}}{\Gamma(\lambda+1)^j}\sum_{i=0}^{\infty} c_{i,j}x^{2i} \tag{11}$$

Consider the following series expansion

$$(1-z)^{-r} = \sum_{i=0}^{\infty}\binom{r+i-1}{i}z^i,\ |z|<0. \tag{12}$$

Since $1-\alpha<1$, there are two cases. If $0<\alpha<2$ then using expansions (11) and (12) in (7) we get the representation for the pdf of GR-TNB as

$$g(x) = \frac{2\alpha(1-\alpha)\beta\alpha^{\beta}e^{(x/\theta)^2}}{1-\alpha^{\beta}}\sum_{k=0}^{\infty}\sum_{j=1}^{k+1}\sum_{i=0}^{\infty} c_{i,j-1}m_{i,j}\theta^{-2j(\lambda+1)}x^{2j(\lambda+1)+2i-1} \tag{13}$$

Where $m_{i,j}=\frac{\binom{k}{j-1}\binom{\beta+k}{k}(-1)^{j-1}\bar{\alpha}^k}{\Gamma(\lambda+1)^j}$. If $\alpha>1/2$, then $|\bar{\alpha}|/\alpha<1$ so we can use following inequality

$$1-\bar{\alpha}\bar{\gamma}(x;\theta,\lambda)=\alpha(1+\frac{\bar{\alpha}}{\alpha}\bar{\gamma}(x;\theta,\lambda)) \tag{14}$$

Thus from (14) and (7) the pdf of GR-TNB as

$$g(x)=\frac{2(1-\alpha)\beta e^{-(x/\theta)^2}}{1-\alpha^\beta}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}c_{i,j-1}n_{i,j}\theta^{-2j(\lambda+1)}x^{2j(\lambda+1)+2i-1} \tag{15}$$

where $n_{i,j}=\frac{\binom{\beta+j-1}{j-1}(-1)^{j-1}\bar{\alpha}^{j-1}}{\alpha^j\Gamma(\lambda+1)^j}$.

C. Moments

The r^{th} ordinary moment of X is given by

$$\mu_r'=E(X^r)=\int_0^\infty x^r g(x)dx$$

Hence the r^{th} moment of GR-TNB distribution, when $0<\alpha<2$ is given by

$$\mu_r'=\frac{2\alpha(1-\alpha)\beta\alpha^\beta}{1-\alpha^\beta}\sum_{k=0}^{\infty}\sum_{j=1}^{k+1}\sum_{i=0}^{\infty}c_{i,j-1}m_{i,j}\theta^{-2j(\lambda+1)}\int_0^\infty e^{(x/\theta)^2}x^{2j(\lambda+1)+2i+r-1}$$

$$=\frac{2\alpha(1-\alpha)\beta\alpha^\beta}{1-\alpha^\beta}\sum_{k=0}^{\infty}\sum_{j=1}^{k+1}\sum_{i=0}^{\infty}c_{i,j-1}m_{i,j}\theta^{r+2i}\Gamma(r/2+(\lambda+1)j+i)$$

Similarly, when $\alpha>2$

$$\mu_r'=\frac{2(1-\alpha)\beta}{1-\alpha^\beta}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}c_{i,j-1}n_{i,j}\theta^{r+2i}\Gamma(r/2+(\lambda+1)j+i)$$

D. Bonferroni and Lorenz curves

The following equation can be used to determine Bonferroni and Lorenz curves

$$J(s)=\int_0^s tf(t)dt \tag{16}$$

They are defined by

$$B(p)=\frac{J(q)}{\mu_1'p}\ and\ L(p)=\frac{J(q)}{\mu_1'}$$

Where q=G^{-1}(p) can be calculated from quantile function for given p and $\mu_1'$is the mean.

If $0<\alpha<2$, from equation (13), it follow that

$$\int_0^s tf(t)dt=\frac{2\alpha(1-\alpha)\beta\alpha^\beta}{1-\alpha^\beta}\sum_{k=0}^{\infty}\sum_{j=1}^{k+1}\sum_{i=0}^{\infty}c_{i,j-1}m_{i,j}\theta^{1+2i}\gamma(1/2+(\lambda+1)j+i,(s/\theta)^2) \tag{17}$$

If $\alpha>1/2$, then from equation (15)

$$\int_0^s tf(t)dt=\frac{2(1-\alpha)\beta}{1-\alpha^\beta}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}c_{i,j-1}n_{i,j}\theta^{1+2i}\gamma(1/2+(\lambda+1)j+i),(s/\theta)^2) \tag{18}$$

IV. INFORMATION THEORY MEASURES

The concept of entropy plays a vital role in information theory. The entropy of a random variable is defined in terms of its probability distribution, and it has been used in various situations in science as a measure of variation of the uncertainty. Numerous measures of entropy have been studied and compared in the literature. Subsequently, Shannon (1948) formalized this idea by defining the entropy and mutual information concepts. The Kullback-Leibler's measure can be understood like a comparison criterion between two distributions. Rényi (1961) introduced a new measure of entropy called Rényi entropy is an extension of Shannon entropy. The Rényi entropy is defined as $I_R(\gamma)=\frac{1}{1-\delta}\log\int_R g^\delta(x)dx,\ \delta>0$ and $\delta\neq1$. Rényi entropy of order 1 is Shannon entropy. We consider first $g^\delta(x)$ given by,

$$g^\delta(x)=\left[\frac{2(1-\alpha)\beta\alpha^\beta}{\Gamma(\lambda+1)(1-\alpha^\beta)}\right]^\delta\frac{\theta^{-2\delta(\lambda+1)}x^{\delta(2\lambda+1)}e^{-\delta(x/\theta)^2}}{\{\gamma(\lambda+1,(x/\theta)^2)+\alpha\bar{\gamma}(\lambda+1,(x/\theta)^2)\}^{\delta(\beta+1)}}$$

If $0<\alpha<2$, then using the series expansion

$$g^\delta(x)=\left[\frac{2(1-\alpha)\beta\alpha^\beta}{(1-\alpha^\beta)}\right]^\delta\sum_{k=0}^{\infty}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty}c_{i,j}w_{i,j}\theta^{-2j(\lambda+1)-2\delta(\lambda+1)}e^{-\delta(x/\theta)^2}x^{2j(\lambda+1)+\delta(2\lambda+1)+2i}$$

where

$$w_{i,j} = \frac{\binom{\delta(\beta+1)+k-1}{k}\binom{k}{j}(-1)^{j}\bar{\alpha}^{k}}{\Gamma(\lambda+1)^{j+\delta}}$$

Corresponding Rényi entropy is

$$I_R(\delta) = \frac{1}{1-\delta}\log\left\{\left[\frac{2(1-\alpha)\beta\alpha^{\beta}}{(1-\alpha^{\beta})}\right]^{\delta}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty} c_{i,j}w_{i,j}\frac{\theta^{2i-\delta+1}\Gamma(j(\lambda+1)+i+\delta(2\lambda+1))}{2\delta^{j(\lambda+1)+i+\frac{\delta(2\lambda+1)}{2}-\frac{1}{2}}}\right\}$$

Similarly, suppose that $\alpha > \frac{1}{2}$. Using the series expansion

$$g^{\delta}(x) = \left[\frac{2(1-\alpha)\beta\alpha^{\beta}}{(1-\alpha^{\beta})}\right]^{\delta}\sum_{k=0}^{\infty}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty} c_{i,j}z_{i,j}\theta^{-2j(\lambda+1)-2\delta(\lambda+1)}e^{-\delta(x/\theta)^2}x^{2j(\lambda+1)+\delta(2\lambda+1)+2i}$$

where

$$z_{i,j} = \frac{\binom{\delta(\beta+1)+j-1}{j}(-1)^{j-1}\bar{\alpha}^{j-\delta\beta-\delta}}{\alpha^{j}\Gamma(\lambda+1)^{(j+\delta)}}$$

Thus we obtain that in the case $\alpha > 1/2$, the Rényi entropy is

$$I_R(\delta) = \frac{1}{1-\delta}\log\left\{\left[\frac{2(1-\alpha)\beta\alpha^{\beta}}{(1-\alpha^{\beta})}\right]^{\delta}\sum_{j=1}^{\infty}\sum_{i=0}^{\infty} c_{i,j}z_{i,j}\frac{\theta^{2i-\delta+1}\Gamma(j(\lambda+1)+i+\delta(2\lambda+1))}{2\delta^{j(\lambda+1)+i+\frac{\delta(2\lambda+1)}{2}-\frac{1}{2}}}\right\}$$

V. ESTIMATION

Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from the GR-TNB distribution with parameters α, β, λ and θ. Let $\Phi = (\alpha, \beta, \lambda, \theta)^T$ be the $p \times 1$ parameter vector. For determining the MLEs of α, β, λ and θ, we have the log-likelihood function

$$logl(\Phi) = nlog(1-\alpha) + nlog\beta + n\beta log\alpha + nlog2 - n(2\lambda+2)log\theta + (2\lambda+1)\sum_{i=1}^{n} log(x_i) - \sum_{i=1} (x_i/\theta)^2 - nlog\Gamma(\lambda+1) - nlog(1-\alpha^{\beta})$$
$$-(\beta+1)\sum_{i=1}^{n} log[\gamma(\lambda+1,(x_i/\theta)^2) + \alpha\bar{\gamma}(\lambda+1,(x_i/\theta)^2)]$$

The components of the score vector, $U(\Phi) = \frac{\partial \log l}{\partial \Phi} = \left(U_\alpha = \frac{\partial \log l}{\partial \alpha}, U_\beta = \frac{\partial \log l}{\partial \beta}, U_\lambda = \frac{\partial \log l}{\partial \lambda}, U_\theta = \frac{\partial \log l}{\partial \theta}\right)^T$ are given by

$$U_\alpha = n\left[\frac{\beta}{\alpha(1-\alpha^{\beta})} - \frac{1}{1-\alpha}\right] - \sum_{i=1}^{n}\frac{(\beta+1)\bar{\gamma}(\lambda+1,(x_i/\theta)^2)}{\gamma(\lambda+1,(x_i/\theta)^2)+\alpha\bar{\gamma}(\lambda+1,(x_i/\theta)^2)}$$

$$U_\beta = \frac{n}{\beta} + \frac{log\alpha(1-2\alpha^{\beta})}{1-\alpha^{\beta}} - \sum_{i=1}^{n} log[\gamma(\lambda+1,(x_i/\theta)^2) + \alpha\bar{\gamma}(\lambda+1,(x_i/\theta)^2)]$$

$$U_\theta = \sum_{i=1}^{n}\frac{x_i^2}{\theta^2} - \frac{n(2\lambda+2)}{\theta} - (\beta+1)\sum_{i=1}^{n}\frac{\bar{\alpha}\theta^{-2\lambda}x^{2(\lambda+1)}e^{-(x/\theta)^2}}{\Gamma(\lambda+1)[\gamma(\lambda+1,(x_i/\theta)^2)+\alpha\bar{\gamma}(\lambda+1,(x_i/\theta)^2)]}$$

$$U_\lambda = 2\sum_{i=1}^{n} logx_i - 2nlog\theta - n\Psi(\lambda+1) - (\beta+1)\sum_{i=1}^{n}\frac{\bar{\alpha}\times\bar{\gamma}'_{\lambda}(\lambda+1,(x_i/\theta)^2)}{[\gamma(\lambda+1,(x_i/\theta)^2)+\alpha\bar{\gamma}(\lambda+1,(x_i/\theta)^2)]}$$

where Ψ is a digamma function and

$$\bar{\gamma}'_{\lambda}(\lambda+1,(x_i/\theta)^2) = \frac{1}{\Gamma\lambda+1}\int_0^{\frac{x^2}{t^2}} e^{-t}t^{\lambda}log(t)dt - \Psi(\lambda+1)\gamma(\lambda+1,(x_i/\theta)^2).$$

Setting the nonlinear system of equations $U_\alpha = 0$, $U_\beta = 0$, $U_\theta = 0$ and $U_\lambda = 0$ solving them simultaneously yields the MLEs of $\hat{\Phi}$. These equations can be solved using statistical software.

VI. DATA ANALYSIS

In this section, we work with an application based on a real data set and found that GR-TNB distribution gives the best fit for the data. We consider a real data set reported by Badar and Priest [12], which represents the strength measured in GPa for single carbon fibers and impregnated at gauge lengths of 1, 10, 20 and 50 mm. Impregnated tows of 100 fibers were tested at gauge lengths of 20, 50, 150 and 300 mm. Here, we consider the data set of single fibers of 20 mm in the gauge with a sample of size 63.

The data are:
1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020.

We compare the results of GR-TNB distribution with following generalizations of Rayleigh distribution which are generalized by using different generators:

1. generalized beta Rayleigh(GBR) distribution [13]
2. Marshall-Olkin Rayleigh (MOR) distribution [7]
3. exponentiated Kumaraswamy Rayleigh (EKumR) distributions [14]
4. Weibull generalized Rayleigh (WGR) distribution [15]
5. log gamma Rayleigh (LGR) distribution [16]

For each distribution, we estimated the unknown parameters (by the maximum likelihood method), the values of the $-$log-likelihood ($-$logL), AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), the values of the Kolmogorov-Smirnov (K-S) statistic and the corresponding p-values.

All the computations were done through the use of R programming language. The result of comparison of the proposed distributions for these data is listed in Table 1. From these results we can observe that GR-TNB distribution provide smallest $-$logL, AIC, BIC and K-S statistics values and highest p-value as compare to other distributions. However, this strongly suggests that the proposed GR-TNB model yields a better fit to these data than the other distributions.

Plots of the histogram with fitted density functions and estimated cumulative distribution function for the data are displayed Figure 3 and Figure 4. The figure shows GR-TNB distribution acquires the satisfactory pattern of the histogram. Notice that the GR-TNB distribution is clearly a competitive model for all the other generalized Rayleigh distributions, since they have the same number of parameters. Therefore, the new model may be an interesting alternative to the other available generalized Rayleigh models in the literature.

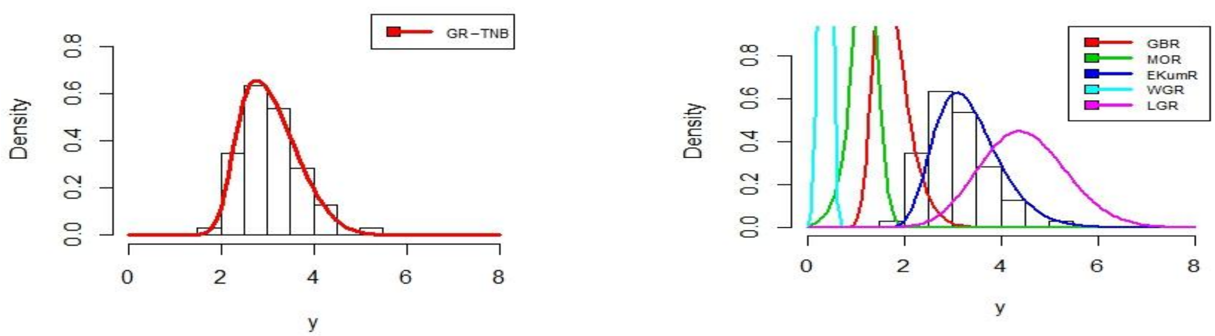

Figure 3: pdf for fitted distributions of the data set

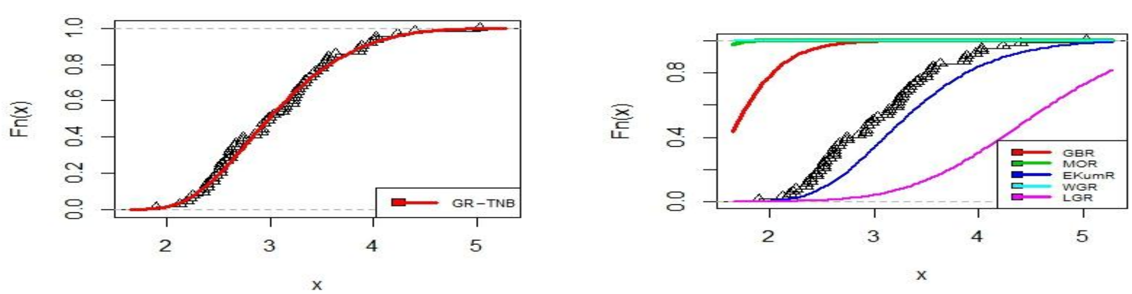

Figure 4: Estimated cumulative distribution function for the data set

Table 1: Estimated values, $-$logL, AIC, BIC, K-S statistics and p-value for data set.

Distribution	Estimates	−logL	AIC	BIC	K-S	p-value
GR-TNB($\alpha,\beta,\theta,\lambda$)	**$\hat{\alpha}$=0.01199792** **$\hat{\beta}$= 0.16877081** **$\hat{\theta}$= 1.15636093** **$\hat{\lambda}$= 8.80147227**	**55.97715**	**119.9543**	**128.5268**	**0.068448**	**0.9294**
GBR(a,b,c,θ)	$\hat{a}$=5.8445538 $\hat{b}$=0.4732349 $\hat{c}$= 4.5648350 $\hat{\theta}$= 0.5493788	56.21643	120.4329	129.0054	0.08484414	0.754889
MOR(a,θ)	$\hat{a}$=49.2637969 $\hat{\theta}$=0.4217449	61.92362	127.8472	132.1335	0.08946892	0.6942997
EKumR(a,b,c,θ)	a=12.1478406 $\hat{b}$=0.3468049 $\hat{c}$= 3.5844515 $\hat{\theta}$= 0.8429350	56.09576	120.1915	128.7641	0.07676342	0.8517467
WGR(a,b,β)	$\hat{a}$ = 2.5247099 $\hat{b}$= 2.0334893 $\hat{\beta}$= 0.1850751	61.95698	129.914	136.3434	0.0875886	0.7191889
LGR(a,b,θ)	$\hat{a}$= 6.4860608 $\hat{b}$= 0.6513691 $\hat{\theta}$= 1.0224604	57.6557	121.3114	127.7408	0.9986215	≈0

VII. ACCEPTANCE SAMPLING PLANS

Acceptance sampling is one the major component of statistical quality control which is used primarily for incoming or receiving inspection of any product or material. However, the sampling inspection has producer's risk and consumer's risk. In scientific sampling plans, these risks are quantified and the sampling criteria are adjusted to balance these risk in right economic factors involved. Recently Hybrid Group Acceptance Sampling Plan (HGASP) based on truncated life test discussed by [17].

A. Reliability Test Plan with GR-TNB Life Time

In this section, we develop reliability test plan with the life time governed by a distribution with scale parameter θ and with distribution function;

$$G(t,\alpha,\beta,\lambda,\theta)=1-\left[\frac{\alpha^{\beta}}{1-\alpha^{\beta}}\left\{[\gamma(\lambda+1,(t/\theta)^2)+\alpha\overline{\gamma}(\lambda+1,(t/\theta)^2)]^{-\beta}-1\right\}\right] \quad (19)$$

A common practice in life testing is to terminate the life test by a pre-determined time t and note the number of well-defined failures. One of the objectives of these experiments is to set a lower confidence limit on the average life. It is then desired to establish a specified average life with a given probability of at least p^*. The decision to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time t does not exceed a given number c called the acceptance number. The test may get terminated before the time t is reached when the number of failures exceeds c in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule, we are interested in obtaining the smallest sample sizes necessary to achieve the objective. Here we assume that α,β and γ are known while θ is unknown. So average life time depends only on θ. A sampling plan consists of

- the number of units n on test,

- the acceptance number c,
- the maximum test duration t, and
- the ratio $\frac{t}{\theta_0}$ where θ_0 is the specified average life.

The consumer's risk not to exceed $1-p^*$, so that p^* is a minimum confidence level with which a lot of true average life below θ_0 is rejected, by the sampling plan. For a fixed p^* our sampling plan is characterized by (n, c, $\frac{t}{\theta_0}$). Here we consider sufficiently large lots so that the binomial distribution can be applied. The problem is to determine for given values of p^*, $(0 < p^* < 1)$, θ_0 and c the smallest positive integer n such that

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \le 1-p^* \tag{20}$$

holds where $p = G(t,\alpha,\beta,\lambda,\theta_0)$ is given by (19) indicates the failure probabilities before time t which depends only on the ratio t/θ_0 it is sufficient to specify this ratio for designing the experiment. If the number of observed failures before t is less than or equal to c, from (20) we obtain:

$$G(t,\theta) \le G(t,\theta_0) \Leftrightarrow \theta \ge \theta_0 \tag{21}$$

The minimum values of n which satisfies the inequality (21) are for p^*=0.75, 0.90, 0.95, .99 and t = 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0 and $\alpha=\beta=\lambda=2$ obtained and displayed in Table 2.

Table 2: Minimum sample sizes necessary to assert the average life to exceed a given value t/θ_0 with probability p^* and the corresponding acceptance number c, $\alpha=\beta=\lambda=2$ using Binomial probabilities.

		t/θ_0										t/θ_0							
p^*	c	1.25	1.5	1.75	2	2.25	2.75	3		p^*	c	1.25	1.5	1.75	2	2.25	2.75	3	
.75	0	48	17	7	2	1	1	1		.90	0	80	28	12	3	2	1	1	0
	1	94	33	14	5	2	2	2			1	136	47	20	6	3	2	2	1
	2	137	48	21	7	4	3	3			2	186	64	28	9	4	4	3	2
	3	180	62	28	9	5	4	5			3	233	81	35	11	6	5	4	3
	4	220	76	34	11	6	5	6			4	279	97	42	13	7	6	5	4
	5	260	90	40	13	7	7	7			5	324	112	49	15	8	7	6	5
	6	300	104	46	15	9	8	8			6	368	128	56	18	9	8	8	6
	7	340	118	52	17	10	9	9			7	412	143	63	20	11	9	9	7
	8	380	133	59	19	11	10	10			8	454	158	70	22	12	10	10	8
	9	420	145	65	21	12	11	11			9	497	172	76	24	13	12	11	9
.95	0	104	36	15	4	2	2	1		.99	0	160	55	23	6	3	2	2	0
	1	165	57	25	7	3	3	2			1	231	95	34	10	4	3	3	1
	2	219	76	33	10	5	4	3			2	295	101	44	13	6	5	4	2
	3	270	93	41	12	6	5	5			3	350	120	52	15	7	6	5	3
	4	319	110	48	15	7	6	6			4	404	139	61	18	9	7	6	4
	5	367	127	56	17	9	7	7			5	457	157	69	20	10	8	7	5
	6	414	143	63	19	10	9	8			6	508	175	76	23	11	9	8	6
	7	459	159	70	21	11	10	9			7	558	192	84	25	13	11	9	7
	8	504	175	77	24	13	11	10			8	607	209	92	28	14	12	11	8
	9	549	190	84	26	14	12	11			9	655	226	99	30	15	13	12	9

If $p = G(t,\alpha,\beta,\lambda,\theta_0)$ is small and n is large (as is true in some cases of our present work), the binomial probability may be approximated by Poisson probability with parameter $\mu = np$ so that the left side of (20) can be written as

$$\sum_{i=0}^{c} \frac{e^{-\mu}\mu^x}{x} \le 1-p^* \tag{22}$$

where $p = G(t,\alpha,\beta,\lambda,\theta_0)$. The minimum values of n satisfying (22) are obtained for the same combination of p values as those used for (20). The results are given in Table 3.

The operating characteristic (OC) function of the sampling plan (n, c, t/θ_0) gives the probability L(p) of accepting the lot with:

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{23}$$

where $p = G(t,\alpha,\beta,\lambda,\theta)$ is considered as a function of the lot quality parameter θ. For given p^*, t/θ_0 the choice of c and n is made on the basis of OC. Values of the OC as a function of $W = \theta/\theta_0$ for a few sampling plans are given in Table 4. The OC curves for $c = 2$, $p^* = .75$ and different values of n as given in Table 4 are shown in Figure 5.

Table 3: Minimum sample sizes necessary to assert the average life to exceed a given value t/θ_0 with probability p^* and the corresponding acceptance number c, $\alpha = \beta = \lambda = 2$ using Poisson probabilities. probabilities.

	t/θ_0										t/θ_0								
p^*	c	1.25	1.5	1.75	2	2.25	2.75	3		p^*	c	1.25	1.5	1.75	2	2.25	2.75	3	
.75	0	49	18	8	3	2	2	2		.90	0	81	29	13	5	3	3	3	0
	1	95	34	15	6	4	3	3			1	137	48	22	8	5	5	4	1
	2	138	49	22	8	5	5	4			2	187	66	30	10	7	6	6	2
	3	180	63	29	10	6	6	6			3	235	83	37	13	8	8	7	3
	4	221	78	35	12	8	7	7			4	281	99	45	15	10	9	9	4
	5	261	92	41	14	9	8	8			5	326	114	52	18	11	10	10	5
	6	301	107	48	16	10	10	9			6	370	130	59	20	13	12	11	6
	7	341	119	54	19	12	11	10			7	414	145	65	22	15	13	12	7
	8	381	133	60	21	13	12	11			8	457	160	72	25	16	14	14	8
	9	419	147	66	23	14	13	13			9	499	175	79	27	17	15	15	9
.95	0	106	37	30	10	7	6	6		.99	0	162	57	26	9	7	6	5	0
	1	167	59	41	14	9	8	8			1	234	82	37	13	9	7	7	1
	2	222	78	52	18	11	10	10			2	296	104	47	16	10	9	9	2
	3	273	96	61	21	13	12	12			3	353	124	56	19	12	11	11	3
	4	322	113	70	24	15	14	13			4	408	143	64	22	14	13	12	4
	5	370	130	78	27	17	15	15			5	461	162	73	25	16	14	14	5
	6	417	146	87	30	18	17	16			6	512	179	81	28	17	16	15	6
	7	462	162	95	32	20	19	18			7	562	197	89	30	19	17	17	7
	8	508	178	103	35	22	20	19			8	612	214	96	33	21	19	18	8
	9	552	193	111	38	23	22	21			9	660	231	104	36	22	20	20	9

The producer's risk is the probability of rejecting lot when $\theta > \theta_0$. We can compute the producer's risk by first finding $p = F(t, \theta)$ and then using the binomial distribution function. For a given value of the producer's risk say 0.05, one may be interested in knowing what value of θ/θ_0 will ensure a producer's risk less than or equal to 0.05 if a sampling plan under discussion is adopted. It should be noted that the probability p may be obtained as function of θ/θ_0, as

$$p = G\left(\frac{t}{\theta_0}\frac{\theta_0}{\theta}\right) \tag{24}$$

The value θ/θ_0 is the smallest positive number for which the following inequality hold:

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq .95 \tag{25}$$

For a given sampling plan (n, c, t/θ_0) and specified confidence level p^*, the minimum values of θ/θ_0 satisfying the inequality (25) are given in Table 5.

B. Numerical Example

Consider the following ordered failure times of the release of a software given in terms of hours from the starting of the execution of the software denoting the times at which the failure of the software is experienced (Wood [18]). This data can be regarded as an ordered sample of size 10 with observations.

x_i , i = 1, . . . , 10 = 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218, 5823

Consider the specified average life to be 1000 hrs and the testing time be 2250 hrs, this leads to ratio of $t/\theta = 2.25$ with corresponding n and c as 10 and 5 obtained from Table 2 for $p^* = 0.99$. Therefore, the corresponding sampling plan for the above sample data is (n=10, c=5, $t/\theta_0 = 2.25$). Based on the observations, it is to decide whether the product is accepted or rejected. We accept the product only, if the number of failures after 2250 hrs is less than or equal to five. So we accept the product.

Table 4: Operating characteristic values of the sampling plan (n, c, t/θ_0 for given p^* and $\alpha = \beta = \lambda = 2$ under GR-TNB probabilities.

				$W = \theta/\theta_0$					
p^*	n	c	t/θ_0	.8	1	1.2	1.4	1.6	1.8
.75	48	2	1.25	0.000874	0.239616	0.767986	0.956849	0.992761	0.9987
	21	2	1.5	0.000899	0.238354	0.758215	0.951207	0.991042	0.9983
	11	2	1.75	0.000851	0.224325	0.744938	0.945785	0.989367	0.9978
	7	2	2	0.000573	0.171166	0.692567	0.929569	0.985298	0.9968
	5	2	2.25	0.000538	0.125061	0.621854	0.905043	0.979057	0.9952
	4	2	2.5	0.00047	0.082177	0.517006	0.859224	0.966524	0.992
	3	2	2.75	0.004868	0.142707	0.563329	0.869635	0.968604	0.9925
	3	2	3	0.000712	0.048586	0.336642	0.723364	0.918108	0.9781
	64	2	1.25	0.000031	0.098207	0.616272	0.913784	0.984165	0.9971

.90	28	2	1.5	0.000029	0.094699	0.598428	0.901776	0.980177	0.996
	14	2	1.75	0.000043	0.099003	0.602903	0.900198	0.978734	0.9955
	9	2	2	0.000016	0.00294	0.203976	0.659152	0.900198	0.9741
	6	2	2.25	0.000031	0.048796	0.473569	0.843475	0.962368	0.9911
	4	2	2.5	0.00047	0.082177	0.517006	0.859224	0.966524	0.992
	4	2	2.75	0.000016	0.014035	0.24718	0.676845	0.904131	0.9743
	3	2	3	0.000712	0.048586	0.336642	0.723364	0.918108	0.9781
.95	76	2	1.25	0.000002	0.047443	0.506506	0.873315	0.975135	0.9953
	33	2	1.5	0.000002	0.046238	0.48978	0.858061	0.969402	0.9937
	17	2	1.75	0.000002	0.040683	0.469109	0.843973	0.963956	0.992
	10	2	2	0.000003	0.033355	0.438649	0.82725	0.958093	0.9902
	6	2	2.25	0.000031	0.048796	0.473569	0.843475	0.962368	0.9911
	5	2	2.5	0.000007	0.017102	0.30969	0.741864	0.92965	0.982
	5	2	2.75	0.0004	0.00116	0.094586	0.486775	0.815862	0.9451
	3	2	3	0.000712	0.048586	0.336642	0.723364	0.918108	0.9781
.99	101	2	1.25	0.000001	0.005822	0.272528	0.743359	0.940267	0.9876
	44	2	1.5	0.000086	0.008536	0.293993	0.746194	0.937059	0.986
	22	2	1.75	0.000085	0.008306	0.288882	0.735645	0.930474	0.9835
	13	2	2	0.00005	0.005496	0.250193	0.702338	0.916734	0.979
	9	2	2.25	0.000016	0.002171	0.172882	0.62393	0.884727	0.9688
	6	2	2.5	0.000001	0.003233	0.173035	0.617564	0.881426	0.9674
	6	2	2.75	0.000001	0.000087	0.03321	0.330619	0.715158	0.9059
	4	2	3	0.000712	0.048586	0.336642	0.723364	0.918108	0.978116

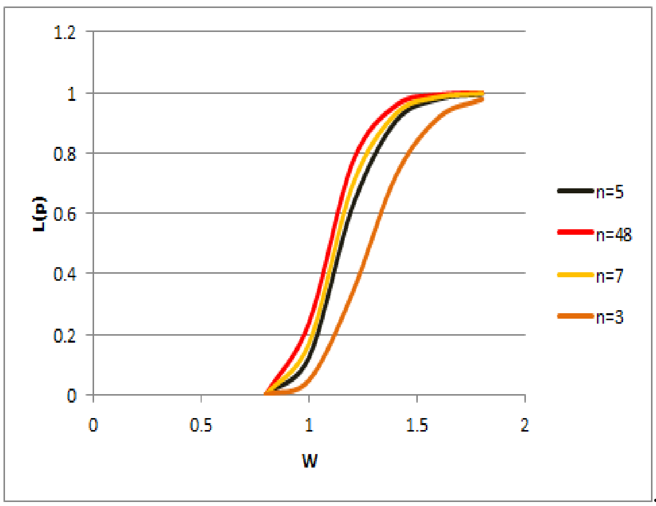

Figure 5: OC Curve

Table 5: Minimum ratio of true θ and required θ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = \beta = \lambda = 2$ under GR-TNB probabilities.

					t/θ_0				
p^*	c	1.25	1.5	1.75	2	2.25	2.5	2.75	3
.75		1.25	1.5	1.75	2	2.25	2.5	2.75	3
	0	3.34	3.21	3.61	3.23	3.63	3.33	3.66	3.99
	1	2.43	2.28	2.25	2.23	2.14	2.03	2.23	2.44
	2	2.11	2	1.93	1.95	1.94	1.98	2.17	2.1
	3	1.96	1.88	1.8	1.79	1.71	1.77	1.95	2.12
	4	1.88	1.79	1.75	1.7	1.7	1.66	1.83	1.99
	5	1.82	1.73	1.66	1.62	1.59	1.58	1.74	1.9
	6	1.77	1.68	1.63	1.58	1.59	1.61	1.77	1.66
	7	1.74	1.65	1.59	1.54	1.52	1.56	1.71	1.87
	8	1.73	1.64	1.56	1.52	1.52	1.52	1.68	1.83
	9	1.7	1.61	1.56	1.51	1.48	1.49	1.55	1.69
	10	1.68	1.6	1.54	1.49	1.45	1.46	1.6	1.75
	0	4.43	3.72	3.61	3.89	3.63	4.03	4.44	3.99
	1	2.67	2.49	2.4	2.38	2.38	2.38	2.61	2.44
	2	2.32	2.18	2.08	2.12	2.05	1.98	2.17	2.08

.90	3	2.11	2.01	1.99	1.95	1.83	1.91	2.1	1.91
	4	2.02	1.91	1.84	1.79	1.76	1.77	1.95	1.8
	5	1.94	1.84	1.8	1.7	1.71	1.68	1.84	1.72
	6	1.9	1.79	1.75	1.7	1.64	1.61	1.77	1.83
	7	1.84	1.75	1.7	1.65	1.62	1.63	1.8	1.76
	8	1.8	1.72	1.66	1.6	1.58	1.58	1.74	1.72
	9	1.79	1.68	1.63	1.58	1.56	1.55	1.7	1.69
	10	1.76	1.67	1.59	1.56	1.52	1.51	1.66	1.75
.95	0	4.07	3.88	3.87	3.7	3.86	3.79	4.17	3.75
	1	2.79	2.67	2.57	2.46	2.51	2.38	2.61	2.44
	2	2.43	2.28	2.2	2.19	2.05	2.16	2.2	2.08
	3	2.22	2.09	2.01	2	1.91	1.91	1.95	2.12
	4	2.07	1.98	1.92	1.86	1.83	1.77	1.83	1.99
	5	2.02	1.91	1.85	1.79	1.71	1.77	1.74	1.9
	6	1.94	1.86	1.85	1.73	1.7	1.71	1.77	1.83
	7	1.9	1.8	1.75	1.7	1.68	1.63	1.71	1.76
	8	1.86	1.77	1.7	1.67	1.61	1.65	1.68	1.72
	9	1.84	1.74	1.66	1.62	1.59	1.61	1.77	1.69
	10	1.8	1.71	1.85	1.56	1.52	1.51	1.66	1.75
.99	0	4.56	4.37	4.3	4.06	4.17	4.29	4.17	4.55
	1	3.15	2.92	2.82	2.78	2.67	2.64	2.61	2.85
	2	2.58	2.49	2.36	2.32	2.34	2.28	2.37	2.37
	3	2.37	2.25	2.14	2.08	2.05	2.03	2.1	2.12
	4	2.22	2.13	2.03	2.00	1.94	1.95	1.95	1.99
	5	2.14	2.03	1.94	1.9	1.83	1.84	1.84	1.9
	6	2.07	1.94	1.88	1.820	1.78	1.77	1.77	1.83
	7	2.02	1.9	1.82	1.76	1.76	1.75	1.8	1.76
	8	1.96	1.87	1.79	1.76	1.74	1.69	1.74	1.83
	9	1.92	1.83	1.75	1.7	1.66	1.65	1.7	1.78
	10	1.9	1.79	1.75	1.67	1.62	1.66	1.66	1.75

VIII. CONCLUSIONS

In this note, we propose a four-parameter distribution namely, the Generalized Rayleigh-Truncated Negative Binomial (GR-TNB) distribution. The GR-TNB distribution provides a flexible mechanism for statistical analysis of positive data. This distribution is motivated by the wide use of the Rayleigh distribution in practice, and also for the fact that generalization of Marsall-Olkin extended distribution can be constructed by adding a parameter (sayβ). We have studied some properties of proposed distribution include: shapes of pdf and hrf. We have provided a mathematical treatment of the distribution and estimation of the model parameters are computed by the method of maximum likelihood. The real data application of the GR-TNB distribution show that it could provide a better fit than other generalizations of Rayleigh distribution used in lifetime data analysis. Also reliability test plan is derived on the basis that the life distribution of the test item follows GR-TNB distribution. We provide the minimum sample size needed to decide for acceptance or rejection of a lot. Some useful tables are provided and applied to a real data to establish the test plan.

ACKNOWLEDGEMENTS

The first author is grateful to the Department of Science and Technology (DST), Govt. of India for the financial support under the INSPIRE Fellowship.

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AUTHORS PROFILE

Miss.Jiju Gillariose is a Research Scholar in the Department of Statistics at St.Thomas College, Pala, Kerala, India affiliated to MG University, Kottayam.

Dr.Lishamol Tomy completed MSc. and Ph.D. in Statistics from MG University,Kottayam. She is currently working as Assistant Professor in Department of Statistics, Deva Matha College, Kuravilangad, Kerala, India**.** She has more than 10 years of Research and Teaching experience in the field of Statistics and has published more than 30 research papers in national and international journals.