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Bianchi Type - III Charged Fluid Universe in Brans-Dicke Theory of Gravitation

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Abstract – We investigate the spatially homogeneous Bianchi Type-III space time with electromagnetic field tensor and relativistic charged perfect fluid in Brans-Dicke (B-D) theory of gravity. Solutions have been obtained by using a general approach of solving the partial differential equations. It is observed that the convergent and isotropic solution of the metric function can be derived with the components of the vector potentials.

Keywords- *Bianchi type-III universe, Brans-Dicke theory of gravitation, electromagnetic field, perfect fluid, vector potentials*

I. INTRODUCTION

In recent years there has been a lot of interest in several alternative theories of gravitation; out of which the most important among them is scalar-tensor theory of gravitation formulated by Brans-Dicke [1]. This theory of gravity is one of the most competent theory due to its vast cosmological implications [2].In this theory, the scalar field has the dimensions of universe of the gravitational constant and its role is confined to its effect on gravitational field equations. This theory of gravity is mediated by a scalar field ϕ in addition to the usual metric tensor field g_{ij} present in Einstein's theory. Among the various modifications of general relativity, the B-D theory of gravity is well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor.

In recent years, the study of Bianchi type models in the context of B-D theory has attracted many authors Pawar *et.al* [3], Sharif *et.al* [4], Kandalkar *et.al* [5], Raut *et.al* [6], Katore *et.al* [7]. A detailed discussion of B-D cosmology is given by Singh *et al*.[8]. Lorenz-Petzold [9] studied exact Bianchi type–III solutions in the presence of electromagnetic field. Bianchi type-I space–time in scalar-tensor theory have been investigated by Kumar *et al*.[10]. Adhav *et al*.[11] studied LRS Bianchi type-II cosmological model with anisotropic dark energy, Katore *et al*.[12,13] explored Bianchi type-V and plane symmetric space-time filled with dark energy models in B-D theory. Bianchi type - III dark energy model in scalar tensor theory of gravitation explained by Naidu *et al*.[14]. Adhav *et al*. [15] explored Bianchi type-III cosmological model with negative constant deceleration parameter in B-D theory of gravity in presence of perfect fluid. Shamir *et al*. [16] have studied anisotropic dark energy Bianchi type-III cosmological models in B-D theory of gravity.

The Brans-Dicke field equations are given by

$$G_j^{\mu} = \frac{-8\pi}{\phi}\left(T_j^{\mu}\right) - \frac{\omega}{\phi^2}\left(g^{\mu i}\phi_{,i} - \frac{1}{2}g_j^{\mu}\phi_{,k}\phi^{,k}\right) - \frac{1}{\phi}\left(g^{\mu i}\phi_{i;j} - g_j^{\mu}\phi_{;k}^{;k}\right),$$

where ω is a dimensionless coupling constant. The function ϕ is known as B-D scalar field. Karade and Solanke [17] investigated Bianchi type-III universe field with the perfect fluid and scalar field coupled with electromagnetic fields in $f(R,T)$ theory of gravity. Recently Bhoyar *et al*.[18] discussed the Bianchi type-III and Kantowski Sachs cosmological model containing magnetic field with variable cosmological constant.

This motivates us to investigate Bianchi type-III charged fluid universe in B-D Theory of gravitation.

The paper is organized as follows:

Section II, deals with the derivation and solutions of the field equations. A brief summary is given is section III.

II. THE METRIC AND FIELD EQUATIONS

Here, we consider a spatially homogeneous Bianchi Type-III space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + c^2dz^2, \quad (1)$$

where A, B and C are functions of t and m is constant.

For the charged fluid, the field equations of B-D theory assume that

$$G_j^\mu = \frac{-8\pi}{\phi}\left(T_j^\mu + E_j^\mu\right) - \frac{\omega}{\phi^2}\left(g^{\mu i}\phi_{,i} - \frac{1}{2}g_j^\mu\phi_{,k}\phi^{,k}\right) - \frac{1}{\phi}\left(g^{\mu i}\phi_{i;j} - g_j^\mu\phi_{;k}^{;k}\right), \quad (2)$$

where G_j^μ is Einstein tensor, E_j^μ is energy momentum tensor for electromagnetic field, T_j^μ is energy momentum tensor for perfect fluid with conservation equation.

$$\phi_{;k}^{;k} = \frac{1}{\sqrt{-g}}\left[\sqrt{-g}\phi^k\right]_{,k}$$

and other symbols and notations have their conventional meanings.

Electromagnetic field

The energy momentum tensor for electromagnetic field is given by

$$E_{ij} = \frac{1}{4}F_{ab}F^{ab}g_{ij} - F_{ai}F_{bj}g^{ab}, \quad (3)$$

Here the electromagnetic field tensor F_{ij} has the expression

$$F_{ij} = \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i}, \quad (4)$$

where V_i is a four potential vector.

To achieve the compatibility with space time (1), we assume electromagnetic vector potential as

$$V_i = [\alpha(x)v_1(t), v_2(t), v_3(t), v_4(t)], \quad (5)$$

Noting (4) and (5) we can deduce easily the following

$$F_{14} = \alpha\dot{v}_1,\ F_{24} = \dot{v}_2, F_{34} = \dot{v}_3, F_{43} = -\dot{v}_3, \quad (6)$$

From equations (4), (5) and (6), we can deduce

$$F_{ab}F^{ab} = -2\left[\frac{\alpha^2\dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2e^{-2mx}} + \frac{\dot{v}_3^2}{C^2}\right], \quad (7)$$

Using (3) we can deduce the components of energy momentum tensors

$$E_1^1 = \frac{1}{2}\frac{\alpha^2\dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2^2}{B^2e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3^2}{C^2}, \quad (8a)$$

$$E_2^1 = \frac{\alpha\dot{v}_1\dot{v}_2}{A^2}, \quad (8b)$$

$$E_3^1 = \frac{\alpha\dot{v}_1\dot{v}_3}{A^2}, \quad (8c)$$

$$E_2^2 = -\frac{1}{2}\frac{\alpha^2\dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3^2}{C^2}, \quad (8d)$$

$$E_3^2 = \frac{\dot{v}_2\dot{v}_3}{B^2e^{-2mx}}, \quad (8e)$$

$$E_1^3 = \frac{\alpha\dot{v}_1\dot{v}_3}{C^2}, \quad (8f)$$

$$E_3^3 = -\frac{1}{2}\frac{\alpha^2\dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2^2}{B^2e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2}, \quad (8g)$$

$$E_4^4 = \frac{1}{2}\frac{\alpha^2\dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2}, \quad (8h)$$

The stress energy tensor of a perfect fluid with density ρ, pressure p and four velocity u_i is given by

$$T_j^i = (\rho + p)u^iu_j - p\delta_i^j, \quad (9)$$

where $g_{ij}u^iu^j = 1$

For co-moving coordinate system, we have

$u_x = 0, u_y = 0, u_z = 0, u_t \neq 0,$

Accordingly (9) provides

$$T_1^1 = (\rho + p)u^1u_1 - p\delta_1^1 = -p,$$

$$T_2^2 = (\rho + p)u^2u_2 - p\delta_2^2 = -p,$$

$$T_3^3 = (\rho + p)u^3u_3 - p\delta_3^3 = -p,$$

$$T_4^4 = (\rho + p)u^4 u_4 - p\delta_4^4 = \rho,$$

$$T_1^1 + E_1^1 = \frac{1}{2}\frac{\alpha^2 \dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2{}^2}{B^2 e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3{}^2}{C^2} - p, \qquad (9a)$$

$$T_2^1 + E_2^1 = \frac{\alpha \dot{v}_1 \dot{v}_2}{A^2}, \qquad (9b)$$

$$T_3^1 + E_3^1 = \frac{\alpha \dot{v}_1 \dot{v}_3}{A^2}, \qquad (9c)$$

$$T_2^2 + E_2^2 = -\frac{1}{2}\frac{\alpha^2 \dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3^2}{C^2} - p, \qquad (9d)$$

$$T_3^2 + E_3^2 = \frac{\dot{v}_2 \dot{v}_3}{B^2 e^{-2mx}}, \qquad (9e)$$

$$T_3^3 + E_3^3 = -\frac{1}{2}\frac{\alpha^2 \dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} - p, \qquad (9f)$$

$$T_4^4 + E_4^4 = \frac{1}{2}\frac{\alpha^2 \dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} + \rho, \qquad (9g)$$

Conservation Law is

$$\frac{\partial}{\partial x^{ij}}\left(\sqrt{-g}F^{ij}\right) = 0, \qquad (10)$$

This equation with different combination of i and j, gives following equations

$$\left[\frac{\dot{v}_1}{v_1}\right]^{\cdot} + \frac{\dot{v}_1{}^2}{v_1{}^2} + \frac{\dot{v}_1}{v_1}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right] = 0, \qquad (10a)$$

$$\left[\frac{\dot{v}_2}{v_2}\right]^{\cdot} + \frac{\dot{v}_2^2}{v_2^2} + \frac{\dot{v}_2}{v_2}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0, \qquad (10b)$$

$$\left[\frac{\dot{v}_3}{v_3}\right]^{\cdot} + \frac{\dot{v}_3^2}{v_3^2} + \frac{\dot{v}_3}{v_3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] = 0, \qquad (10c)$$

$$\phi_{;k}^{;k} = -\ddot{\phi} - \dot{\phi}\left[\frac{\dot{A}}{\mathrm{A}} + \frac{\dot{B}}{\mathrm{B}} + \frac{\dot{C}}{C}\right], \qquad (10d)$$

From the vanishing components of Einstein tensor, using equations (2) and (4), we deduce

$$\frac{\dot{v}_1 \dot{v}_2}{v_1 v_2} = \frac{\dot{v}_1 \dot{v}_3}{v_1 v_3} = \frac{\dot{v}_2 \dot{v}_3}{v_2 v_3} = 0, \qquad (11)$$

$$\frac{\dot{v}_1}{v_1} = \frac{\dot{v}_2}{v_2} = \frac{\dot{v}_3}{v_3} = \frac{\dot{D}}{D}, \qquad (12)$$

where D is an unknown function of t

Integrating this with respect to t, we get

$$v_1 = k_1 D \quad , \; v_2 = k_2 D \quad , \quad v_3 = k_3 D \qquad (13)$$

where k_1, k_2 and k_3 are constants

Inserting (12) in (11), we get

$$\left(\frac{\dot{D}}{D}\right)^2 = 0, \qquad (14)$$

With the aid of equation (12), we can write the equation (10) as,

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{B}}{\mathrm{B}} + \frac{\dot{C}}{C} - \frac{\dot{A}}{\mathrm{A}}\right) = 0, \qquad (15a)$$

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right) = 0, \qquad (15b)$$

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \qquad (15c)$$

From equations (15a), (15b)and (15c) ,we have

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C}, \qquad (16)$$

Integrating with respect to t, we get

$$A = k_4 B, \quad B = k_5 C, \quad C = k_6 A, \qquad (17)$$

where k_4, k_5 and k_6 are constants.

We attempt to express the component of T_j^i in terms of T_4^4 for this consider the expression

$$\frac{\alpha^2 \dot{v}_1{}^2}{A^2} + \frac{\dot{v}_2{}^2}{B^2 e^{-2mx}} + \frac{\dot{v}_3^2}{C^2} = \left[\frac{\alpha^2 v_1{}^2}{A^2} + \frac{v_2{}^2}{B^2 e^{-2mx}} + \frac{v_3{}^2}{C^2}\right]\left(\frac{\dot{D}}{D}\right)^2 = 0$$

$$T_4{}^4 = \frac{1}{2}\frac{\alpha^2 \dot{v}_1{}^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2{}^2}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} + \rho = \rho, \qquad (18a)$$

$$T_1^1 = -T_4^4 + \rho - p, \qquad (18b)$$

$$T_2^2 = -T_4^{\ 4} + \rho - p, \qquad (18c)$$

$$T_3^3 = -T_4^4 + \rho - P, \qquad (18d)$$

Now, considering the non-vanishing component of Einstein tensor, from equation (2), we derive

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \frac{-8\pi}{\phi}\left[-T_4^4 + \rho - p\right] - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \qquad (19a)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\frac{8\pi}{\phi}\left[-T_4^4 + \rho - p\right] - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right), \qquad (19b)$$

$$-\frac{m^2}{A^2} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi}{\phi}\left[-T_4^4 + \rho - p\right] - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \qquad (19c)$$

$$-\frac{m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = -\frac{8\pi}{\phi}[\rho] + \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \qquad (19d)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0, \qquad (19e)$$

Integrating (19e) with respect to t, we get

$$A = k_7 B, \qquad (20)$$

where k_7 is constant.

From equations (19a) and (19b), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = 0, \qquad (20a)$$

From equations (19b) and (19c), we get

$$\frac{m^2}{A^2} + \frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\left[\frac{\dot{C}}{C} - \frac{\ddot{B}}{B}\right] + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0, \qquad (20b)$$

Using equations (19c) and (19a), we obtain

$$-\frac{m^2}{A^2} + \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\left[\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right] + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right] = 0, \qquad (20c)$$

Eliminating $\frac{m^2}{A^2}$ between (20b) & (20c), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \qquad (20d)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] = 0, \qquad (20e)$$

Upon integration of (20a) and (20e), yields

$$\frac{A}{B} = k_9 \exp\left\{k_8 \int \frac{1}{ABC\phi} dt\right\}, \qquad (21a)$$

Similarly

$$\frac{B}{A} = k_{11} \exp\left\{k_{10} \int \frac{1}{ABC\phi} dt\right\}, \qquad (21b)$$

We can express the values of A and B in the following form

$$A = (ABC)^{1/3} k_{12} \exp\left\{k_{11} \int \frac{1}{ABC\phi} dt\right\}, \qquad (22a)$$

$$B = (ABC)^{1/3} k_{14} \exp\left\{k_{13} \int \frac{1}{ABC\phi} dt\right\}, \qquad (22b)$$

Equation (17) implies C is scalar multiple of A

$$C = (ABC)^{1/3} k_{16} \exp\left\{k_{15} \int \frac{1}{ABC\phi} dt\right\}, \qquad (22c)$$

Using equations (15) and (22), we get,

$$\frac{\ddot{D}}{\dot{D}} + \frac{\dot{A}}{A} = 0, \qquad (23)$$

Integrating above equation, we get

$$D = k_{17} \int \frac{1}{A} dt + k_{18}, \qquad (24)$$

Using (24) the equation (13) reduces to

$$v_1 = k_{19} \int \frac{1}{A} dt + k_{20}, \qquad (25)$$

$$v_2 = k_{21} \int \frac{1}{A} dt + k_{22}, \qquad (26)$$

$$v_3 = k_{23} \int \frac{1}{A} dt + k_{24}, \qquad (27)$$

v_4 is undetermined.

The metric in (1), with the help of (22) can be redefined in the form

$$ds^2 = (ABC)^{2/3}\left[K^{'}\exp K^{''}\int \frac{1}{(ABC)\phi}dt\right]^2 \left(dx^2 + e^{-2mx}dy^2 + dz^2\right) - dt^2, \qquad (28)$$

where $K^{'} = k_{12}k_{14,}k_{16}$ and $K^{''} = k_{11}k_{13,}k_{15}$ are constants.

III. CONCLUSION

In this present paper, we have presented Bianchi Type-III space time with electromagnetic field tensor and relativistic charged perfect fluid in the context of Brans-Dicke theory of gravity. We have derived and solved the gravitational field equations corresponding to B-D theory. It is observed that the convergent, non-singular, isotropic solutions can be obtained along with the components of vector potential. It is also interesting to note that the investigated models are free from singularity.

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