Review Article

A Brief Review in the Fifth State of Matter (Bose-Einstein Condensation States)

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Abstract— In this review, the fifth state of the matter is discussed. There are five states of the matter; solid, liquid, gas, plasma, and Bose-Einstein condensation state (BECs). The fifth state of the matter is called Bose-Einstein condensation state. It is a low temperature state of Bose gases. First observations of the exotic state of matter in an ultracold vapor of rubidium atoms were made in the 1920s by Satyendra Nath Bose and Albert Einstein. On June 5, 1995, Eric Cornell and Carl Wieman conducted an experimental observation of the first gaseous condensate at Boulder NIST-JILA lab, University of Colorado, when rubidium atoms cooled to 170 nanokelvins. Wolfgang Ketterle used sodium atoms to observe BECs experimentally at MIT in the United States at the end of the same year. These three distinguished scientists share the 2001 Nobel Prize in Physics. Then, both theoretical and experimental scientists have used BECs as the beginning point to produce different exotic states of matter like super-solids, superfluidity, excitonic, quantum ball lightning, and fluids exhibiting negative mass.

Keywords— Plasma; Bose-Einstein condensation state; Bose gases; rubidium atoms; sodium atoms; superfluidity

1. Introduction

The matter is something that has mass and occupies space. The matter has five different phases such as solids, liquids, gases, plasma, and Bose-Einstein condensates (BECs). Atoms are binding to each other with a strong intermolecular force of attraction in solids. Therefore, solids have definite shape and volume. The solids are characterized by high density, low kinetic energy, incompressible, rigid, and low rate of diffusion. The solids are two types crystalline and amorphous. In crystalline solids, the atoms are arranged in periodic order whereas, in amorphous solids, atoms are disordered in structure. Again, crystalline solids are divided into four categories based on particles and nature of bonding: metallic, molecular, network, and ionic. The atomic structure and composition in solid will determine the physical properties like density, solubility, electrical and thermal conductivity [1-2].

The liquid is the second phase of the matter, which has a fixed volume but no definite shape and size because of the weak molecular force of attraction. The liquid is an intermediate phase between solid and gaseous states. The particles are moving randomly in liquid, which makes the liquid to be variable in shape. When liquid pours into a vessel, it will flow and fill the lowest part of the vessel making shape without changing the volume. The liquids are characterized by incompressibility, weak interatomic force,

fixed volume, and less density compared to solid. The structure and composition of liquid will decide the different properties like cohesion & adhesion, viscosity, evaporation, and volatility, etc. The 3rd phase of the matter is called a gaseous state which has no fixed volume and shape due to the absence of the intermolecular force of attraction. The gaseous phase has a lower density compared to a solid and liquid state. The gas molecules are randomly moving inside the container and colliding among themselves along with a wall of the container. These molecules exert force on the wall of the container due to collision and produce pressure. The gaseous phase is characterized by no definite volume & shape, random molecular motion, no molecular force of attraction and pressure, diffusivity, highly compressible, etc. The plasma is the fourth state of the matter and was first discovered by Irving Langmuir in 1920 [3].

The plasma is a superheated state of the matter. Due to heating, the electrons are detached from the atoms forming the ionized gas. The plasma is formed with ionized gas. We know when energy is given to liquid, it will change to gas. Likewise, when we heat the gas molecules, plasma (a combination of positively ionized gas and negatively electrons gas) is formed. The whole universe (approximately 99%) is covered with plasma.

Interestingly, in the night sky, stars glow due to the presence of plasma on its surface. Even the lightning that splits the sky is made up of plasma. Frankly say the existence of the living



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being in the universe is directly or indirectly related to the plasma. The researchers took a keen interest in the topic of plasma because most of the universe is made of plasma and control the different physical properties. Since all the matter is composed of plasma, the scientists used the existing properties of the plasma as a charged gas to control by application of a suitable magnetic field. The temperature of the plasma is hotter than the core of the sun. The plasmas are very exciting in applications for making rocket propulsion, biological hazards. computer cleaning chips. and environmental cleaning reagents, etc. Now, the fifth state of the matter so-called Bose-Einstein condensate state will be discussed. The properties of gases can be studied by three different perspectives i.e., a kinetic theory which includes molecular interactions, thermodynamics which includes behavior under the action of heat treatment, and statistical mechanics which include the law of averages. The behavior of gas molecules can be explained in three different ways like an ideal gas, classical gas, and quantum gas. Any gas that exists in nature is called real gas. A real gas is said to be ideal gas when it satisfies the conditions; negligible molecular interaction and collision with the wall of the container, no internal structure of the molecules (negligible volume), and obeying gas laws including Boyle's law and Charles's law. One step ahead to an ideal gas, the classical gas includes the collisions as the interaction with gravitational and electromagnetic fields. The classical gas molecules include the translational, rotational, and vibrational degrees of freedom. Again, one step ahead to classical gas, the quantum gas includes kinetics of collisions, particle scattering, electron spin, and particle statistics (Fermi-Dirac and Bose-Einstein). In the quantum mechanical approach, the complex discrete internal structure of gas molecules is considered with neglecting relativistic effect. The particles are divided into two categories; fermions and bosons. The quantum gas which has a half-integral spin and obeys Fermi-Dirac statistics is called fermion. Examples of fermions are electrons, neutrinos, quarks (up and down), baryons (protons, neutrons, etc.) [4-5]. Again, the role of the microstate is much more important compared to macrostate in the quantum mechanical approach [6]. In the context of the classical mechanics, the maxwell displacement establishes the equation of continuity and got credit for finding the velocity of the em waves [7].

Similarly, the quantum gas which has zero or integral spin and follows Bose-Einstein statistics is called a boson. The examples of bosons particles are photons, gluons, W bosons, Z bosons, Higgs bosons, gravitons, plasmons, and phonons, etc [8]. The first sighting of the exotic state of matterpredicted by Satyendra Nath Bose and Albert Einstein in the 1920s-in an ultracold vapor of rubidium atoms was documented in a science publication published on July 14, 1995 [9]. When the bosons are brought down almost to the coldest temperature possible (near absolute zero, 0 Kelvin, or minus 273.15 Celsius), bosons stop behaving as independent particles and collapse into a single quantum state called as BECs. Thereafter many groups of scientists tried to observe BECs experimentally. When rubidium atoms cooled to 170 nanokelvins, the first gaseous condensate was observed experimentally by Eric Cornell and Carl Wieman at Boulder NIST-JILA lab, University of Colorado on 5th June-1995 [10]. At the end of the same year, Wolfgang Ketterle observed experimentally BECs using sodium atoms in MIT, USA [11]. In the year 2001, these three eminent scientists share the Nobel prize in Physics for their invaluable contribution towards the experimental verification of the fifth state of the matter [12]. At the Bose-Einstein condensate state, quantum phenomena (micro-scale) can be observed at the macro scale. Then, both theoretical and experimental scientists have used BECs as the beginning point to produce different exotic states of matter like super-solids, excitonic, quantum ball lightning, and fluids exhibiting negative mass.

2. Formalism

A Bose-Einstein Condensate (BEC) is the fifth state of matter which is typically formed when a gas of bosons at low densities is cooled to temperatures very close to absolute zero (-273.15 degree centigrade). Under such conditions, a large fraction of bosons occupy the lowest quantum state, particularly wave function interference become apparent macroscopically. A BEC is formed by cooling a gas of extremely low density, about one-hundred-thousand (1/100,000) the density of normal air to ultra-low temperature. The 1st experimental gaseous BEC state was produced by Eric Cornell and Carl Wieman at the University of Colorado at Boulder NIST-JILA lab, in a gas of rubidium atoms cooled to 170 nano-Kelvin. Shortly thereafter, Wolfgang Ketterle at MIT realized a BEC in a gas of sodium atoms. For their achievements Cornell, Wieman and Ketterle received Novel Prize 2001 in Physics. Summarize the fundamental concepts and characteristics of BECs highlight its unique properties, such as superfluidity and coherence, and emphasize its significance in the field of physics [13]. Here, we want to do some mathematics to understand formalism of the BECs at below certain absolute temperature.

- N = no of the particles present in the system
- V = volume of the system

 ϵ_i = degeneracy or the no of quantum states available to the particles

The partition function of the boson gases can be written as $n_i = \frac{g_i}{\alpha + \beta \epsilon_{i-1}}$

Where
$$\alpha = -\frac{\mu}{k_B T}$$
, μ = chemical potential, $\beta = \frac{1}{k_B T}$ and T = absolute temperature

Again, the partition function can be simplified by one step as $n_{i} = -\frac{g_{i}}{\sqrt{g_{i}}}$

$$e^{\frac{(\epsilon_i - \mu)}{k_B T}} - 1$$

Now total number of particles present in the system can be written as

$$N = \sum_{i} n_{i} = \sum_{i} \frac{g_{i}}{e^{\alpha + \beta \epsilon_{i-1}}}$$

To simplify the above summation, let we replace summation by integration with density of state function $g(\epsilon)d\epsilon$, so that the above expression can be reduce as

The number of the quantum state having energy in between ϵ and $\epsilon + d\epsilon$ can be written as

$$g(\epsilon)d\epsilon = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} \epsilon^{\frac{1}{2}} d\epsilon \quad \dots \dots \dots (2)$$

Now, substitute eq. (2) in eq. (1), we get

$$N = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{\epsilon^{\frac{1}{2}}}{e^{\alpha+\beta\epsilon}-1} d\epsilon$$
$$N = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{\epsilon^{\frac{1}{2}}}{e^{\alpha}e^{\beta\epsilon}-1} d\epsilon$$

Let us assume, $z = e^{-\alpha}$, so that expression of N can be written as

$$N = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{\epsilon^{\frac{1}{2}}}{\frac{1}{2}e^{\beta\epsilon}-1} d\epsilon \quad \dots \dots (3)$$

To evaluate the integration, we take

Let $x = \beta \epsilon = \frac{\epsilon}{k_B T}$ $\epsilon = x k_B T$ $d\epsilon = k_B T dx$

Substitute the value of ϵ and $d\epsilon$ in the equation (3), we get

$$N = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{x^{\frac{1}{2}}(k_BT)^{\frac{1}{2}}k_BT}{\frac{1}{z}e^{x}-1} dx$$

$$\Rightarrow N = (2s+1)2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_BT)^{\frac{3}{2}} \int_0^\infty \frac{x^{\frac{1}{2}}}{\frac{1}{z}e^{x}-1} dx$$

$$N = (2s+1)V \left(\frac{2\pi mk_BT}{h^2}\right)^{3/2} \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{\frac{1}{2}}}{\frac{1}{z}e^{x}-1} dx$$

$$N = (2s+1)V \left(\frac{2\pi mk_BT}{h^2}\right)^{3/2} g_{3/2}(z)$$
(4)
Where,

 $g_{3/2}(z) = Bose \ function = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{\frac{1}{2}}}{\frac{1}{z}e^{x}-1} dx$ Now, we want to evaluate the Bose function,

$$g_{3/2}(z) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{\frac{1}{2}}}{\frac{1}{z}e^{x-1}} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{zx^{\frac{1}{2}}}{e^{x-z}} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{zx^{\frac{1}{2}}}{e^{x(1-ze^{-x})}} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty zx^{\frac{1}{2}} e^{-x} dx (1-ze^{-x})^{-1}$$

[since $(1-y)^{-1} = 1+y+y^2+y^3+\cdots$]

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty zx^{\frac{1}{2}} e^{-x} dx (1+ze^{-x}+z^2e^{-2x}$$

At the mining value (maximum),
$$Z = 1, 07 \ \mu = 0$$

 $g_{3/2}(z = 1) = \sum_{n=1}^{\infty} \frac{z^n}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
 $= 1 + \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} + \frac{1}{4^{3/2}} + \cdots$
 $= \zeta \binom{3}{2}$
 $g_{3/2}(z = 1) = 2.61$ (6)

Where ζ is called Riemann zeta function Now, consider eq.(4) at z =1, we get

$$N = (2s+1)V\left(\frac{2\pi mk_BT}{h^2}\right)^{3/2} \times g_{3/2}(z=1)$$

$$N = (2s+1)V\left(\frac{2\pi mk_BT}{h^2}\right)^{3/2} \times 2.61 \quad \text{[using eq.(6)]}$$

$$\Rightarrow \left(\frac{2\pi m k_B T_b}{h^2}\right)^{3/2} = \frac{N}{V(2s+1)(2.61)}$$

[At $T = T_b$ = Bose temperature]
$$\Rightarrow T_b = \frac{h^2}{2\pi m k_B} \left(\frac{1}{2.61}\right)^{2/3} \left(\frac{N}{V}\right)^{2/3}$$
.....(7)

Here, the particle number N should not be the function of temperature particularly in the condition $T < T_b$, $T \downarrow N \uparrow$. So, this indicates that we made mistake during the time when the summation is placed by integration i.e., $\Sigma \rightarrow \int$ so that we miss the ground state energy calculation.

According to quantum mechanics, in each quantum state should occupy a non-degenerate single particle state.

Therefore, in the range $T < T_b$, $N = N_0 + N_e$ Where $N_0 =$ no of particle present in ground state $N_e =$ no of particle in the excited state

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$$= N \left[1 - \left(\frac{T}{T_b}\right)^{3/2} \right]$$

Therefore, $N_0 = N \left[1 - \left(\frac{T}{T_b}\right)^{3/2} \right]$ (9)

In conclusion, it is observed that decreasing the temp., the number of the gas molecules increase in the ground state i.e., $T \downarrow$, $N_0 \uparrow$. These particles lose their individuality by overlapping the wave functions and formed a condense state, so called Bose-Einstein condensate state.

3. Results and discussion

After reaching the number of bosons depending upon the temperature, it is important to draw the results and immediately followed by proper discussion. In this section, it is discussed that how the particle and wave overlap to each other and form a single quantum state losing all the identities so called BEC state and become a strong reason for superfluidity and coherence

Now, equation (4) can be simplified further as

$$N = (2s + 1)V\left(\frac{2\pi mk_BT}{h^2}\right)^{3/2} \times g_{3/2}(z)$$

$$\frac{N}{V} = (2s + 1)\frac{1}{\left(\frac{h}{\sqrt{2\pi mk_BT}}\right)^3} \times g_{3/2}(z)$$

$$\Rightarrow n = \frac{(2s+1)}{\left(\frac{h}{\sqrt{2\pi mk_BT}}\right)^3} g_{3/2}(z) \qquad \text{(where } \lambda = \text{de-Brogile wavelength)}$$

$$\Rightarrow n \propto \frac{1}{\lambda^3} \qquad (10)$$
We know that $n = \frac{N}{V} = \frac{N}{\frac{4}{3}\pi d^3} \qquad \text{(where, d = inter-atomic distance between boson gases)}$

$$\Rightarrow n \propto \frac{1}{d^3} \qquad (11)$$
From eq.(10) and eq. (11), we get
$$\frac{1}{\lambda^3} = \frac{1}{d^3}$$

$$\Rightarrow \lambda^3 = d^3$$

$$\Rightarrow \boxed{\lambda = d}$$

Condition for BEC:

If the distance between the two bosons (d) is equal to de-Broglie wavelength (λ), then BEC state is formed. Therefore, it is important to remember the condition of the BECs and relevant applications in different field of science and

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technology. Now, the equation of state of the 5th state of the matter i.e., BEC state is discussed below-

matter i.e., BEC state is discussed below-
We know that,
$$\frac{p}{k_B T} = \frac{2s+1}{\lambda^3} g_{5/2}(z) - \frac{2s+1}{\nu} ln(1-z)$$

At thermodynamic limit, $V \to \infty$, $\frac{2s+1}{\nu} ln(1-z) \to 0$
Therefore, $\frac{p}{k_B T} = \frac{2s+1}{\lambda^3} g_{5/2}(z)$ (3)
Here we take $g_{5/2}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^{5/2}}$ (12)
At the limiting value (maximum), $z = 1$, or $\mu = 0$
 $g_{5/2}(z = 1) = \sum_{n=1}^{\infty} \frac{z^n}{n^{5/2}}$
 $= 1 + \frac{1}{2^{5/2}} + \frac{1}{3^{5/2}} + \frac{1}{4^{5/2}} + \cdots$
 $= \zeta {\binom{5}{2}}$
 $g_{5/2}(z = 1) = 1.314$ (13)

Where ζ is called Riemann zeta function

Now, we use eq.(3) to write equation of state as

Now, the internal energy of the BECs state can be calculated as follows-

Internal energy of BEC

$$U = \frac{s}{2} pV$$

$$\Rightarrow U = \frac{3}{2} V \times \frac{p}{k_B T} \times k_B T$$

$$\Rightarrow U = \frac{3}{2} V \times k_B T \times \frac{2s+1}{\lambda^3} g_{5/2}(z) \quad \text{(using eq.(3))}$$

$$\Rightarrow U \ge \frac{3Vk_B T}{2} \times \frac{2s+1}{\lambda^3} g_{5/2}(z=1) \qquad \dots \dots \dots (15)$$

This is the correspondence of the intermal energy of the REC state.

This is the expression of the internal energy of the BEC state.

5. Conclusion and Future Scope

In conclusion, the exploration of Bose-Einstein Condensate (BEC) has marked a transformative journey in the realm of quantum physics, opening doors to unprecedented insights into the nature of matter and the fundamental principles governing the quantum world. This exotic state, initially envisioned by Bose and Einstein, has proven to be a captivating playground for physicists, pushing the boundaries of our understanding and challenging conventional notions. A continuous effort by the worldwide scientists brings the idea into practical application. As far as experimental methods go, creating atomic BECs in the lab has become second nature. Commercial table-top BEC systems are already available, as are small systems suitable for free-fall experiments or portable sensor applications. A publication released last 3 years detailed the next phase of these studies, which included the first BEC created in space in the International Space Station's Cold Atom Laboratory. Discuss current and

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potential applications of BEC in various scientific and technological domains, such as quantum computing, precision measurements, and fundamental physics research. Therefore, it is a great achievement for scientific community to have BECs for the development of the human civilization.

Data Availability

Data will made available upon the reasonable request from the corresponding author.

Conflict of Interest

No conflict of interest to declare

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Authors' Contributions

S.K. Parida: Data Collection, Writing - Original Draft, Reviewing & Editing, Conceptualization, Methodology and Validation of Software

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