Representation of a Gradient for Scalar Electromagnetic Field in MATLAB using Differential Operators

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Abstract—In this work was deduced the representation of an algorithm, developed in MATLAB, to calculate an electromagnetic scalar field utilizing differential operators to generate graphics about the electromagnetic spectrum, the distance X,Y of the field curvature, as well as the gradient example of the electromagnetic field. The main objective of this article is to compare the new GUI features of the algorithm developed in MATLAB to the previous algorithm as cited in the previous work of this paper, showing also the spectrum average of this new algorithm can reach, according to the differential operators values inserted into the code.

I.   INTRODUCTION

Electromagnetic fields were first discovered in the 19th century, when physicists noticed that electric arcs (sparks) could be reproduced at a distance, with no connecting wires in between. This led scientists to believe that it was possible to communicate over long distances without wires [1].

An electromagnetic field, sometimes referred to as an EM field, is generated when charged particles, such as electrons, are accelerated, being all electrically charged particles are surrounded by electric fields. Charged particles in motion produce magnetic fields. When the velocity of a charged particle changes, an EM field is produced.

With a title of example, regarding the gradient of scalar fields, let us consider a metal bar whose temperature varies from point to point in some complicated manner. So, the temperature will be a function of x, y, z in the Cartesian coordinate system. Hence temperature here is a scalar field represented by the function T(x,y,z). Since temperature depends on the distance it could increase in some directions and decrease in some directions. It could increase or decrease rapidly along with some directions in comparison to other directions [2].

Hence, the gradient of a scalar field is a vector field and whose magnitude is the rate of change and which points in the direction of the greatest rate of increase of the scalar field. If the vector is resolved, its components represent the rate of change of the scalar field with respect to each directional component [3].

This paper is structured presenting a related work showing the equations of scalar fields utilized in the MATLAB simulations, the methodology utilized to obtain the new based GUI algorithm, the results and discussion appointed the main contributions of this work and finally the conclusions.

II.   RELATED WORK

For a two-dimensional scalar field ϕ (x,y), we have the Equation (1) [4]:

$$\text{grad}\,\phi(x,y) = \nabla\,\phi(x,y) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right) \qquad (1)$$

And for a three-dimensional scalar field ϕ (x, y, z) we have the Equation (2):

$$\text{grad}\,\phi(x,y,z) = \nabla\,\phi(x,y,z) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right) \qquad (2)$$

The gradient of a scalar field is the derivative of f in each direction. Note that the gradient of a scalar field is a vector field. An alternative notation is to use the *del* or *nabla* operator, ∇ f = grad f.

For a three dimensional scalar, its gradient is given by the Equation (3):

$$\text{grad}(V) = \underline{a}_n \frac{dV}{dn} = \nabla V \qquad (3)$$

The gradient is a vector that represents both the magnitude and the direction of the maximum space rate of increase of a scalar.

$$dV = (\nabla V) \cdot d\underline{l}, \text{ where } d\underline{l} = \underline{a}_i \cdot dl \qquad (4)$$

In a Cartesian plane, we represent the gradient function according to the Equation (5):

$$\nabla \equiv \underline{a}_x \frac{\partial}{\partial x} + \underline{a}_y \frac{\partial}{\partial y} + \underline{a}_z \frac{\partial}{\partial z} \qquad (5)$$

In a Cylindrical plane, we represent the gradient function according to the Equation (6):

$$\nabla \equiv \underline{a}_r \frac{\partial}{\partial r} + \underline{a}_\phi \frac{\partial}{r \cdot \partial \phi} + \underline{a}_z \frac{\partial}{\partial z} \qquad (6)$$

In a Spherical plane, we represent the gradient function according to the Equation (7):

$$\nabla \equiv \underline{a}_R \frac{\partial}{\partial R} + \underline{a}_\theta \frac{\partial}{R \partial \theta} + \underline{a}_\phi \frac{\partial}{R \cdot \sin\theta \cdot \partial \phi} \qquad (7)$$

III. METHODOLOGY

The following image shows the screen of the algorithm developed in MATLAB capable to detect a gradient of a scalar electromagnetic field using differential operators ∇ (x,y,z) within four minimal differential functions defined as the patterns for this study in question.
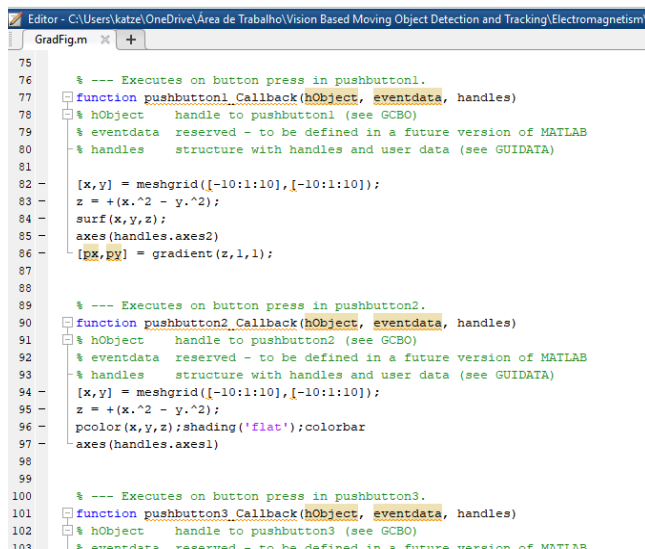

Figure 1. Algorithm for scalar electromagnetic field representation developed in MATLAB.

Also can be written in Table 1 the differential operators z = (xn + ym) for detection the gradient of the scalar electromagnetic field that will be represented by the graphics of the electromagnetic spectrum, the distance X,Y of the field curvature, as well as the gradient example inserts in the GUI developed in MATLAB.

Table 1. Differential operators considered for simulation

Equation number	Differential operator
1	$z = -(x^2 + y^2)$
2	$z = +(x^2 + y^2)$
3	$z = -(x^2 - y^2)$
4	$z = +(x^2 - y^2)$

We can see the new algorithm proposed for representation of the gradient of a scalar the electromagnetic field in MATLAB has an optimized interface with "Calculate X and Y distance", "Calculate vectors" and "Calculate spectrum" buttons inserted to the code as shows Figure 2:
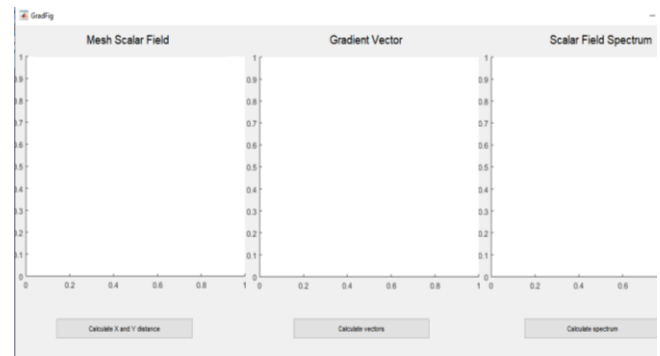

Figure 2. Improved GUI interface of he algorithm

The previous interfaces, as well as the previous algorithm for the electromagnetic field detection, is represented by the Figure 3, Figure 4 and Figure 5, as a simple "figure function" embedded at the previous code developed in MATLAB.
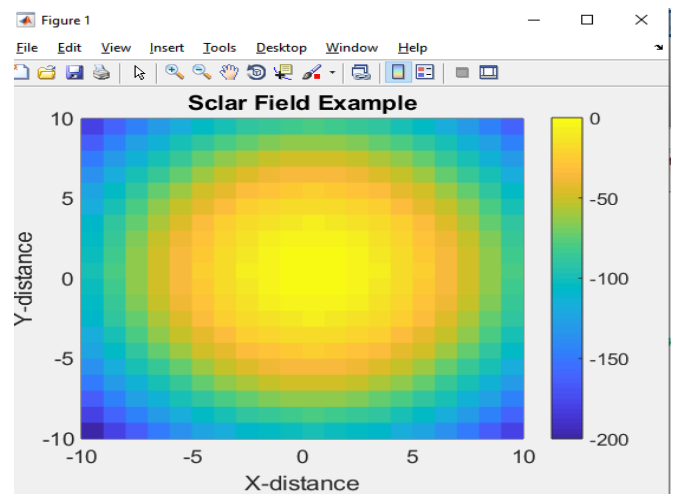

Figure 3. Scalar field example of the previous algorithm ussing "Figure interface" in MATLAB
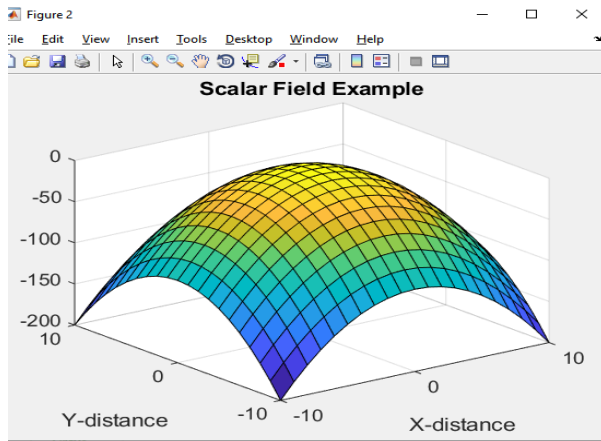
Figure 4. Curvature of the Scalar field example of the previous algorithm using "Figure interface" in MATLAB
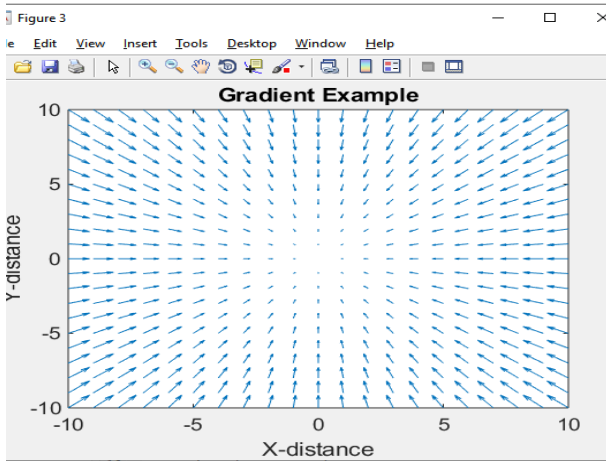

Figure 5. Gradient example of the previous algorithm using "Figure interface" in MATLAB

The previous algorithm also can be written below:

```
[x,y] = meshgrid([-10:1:10],[-
10:1:10]);
z = -(x.^2 + y.^2);
figure
pcolor(x,y,z);shading('flat');colorbar
set(gca,'Fontsize',14)
title('Sclar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
%note, you can also use surf if you
want a 3D looking plot
figure
surf(x,y,z);
set(gca,'Fontsize',14)
title('Scalar Field Example')
xlabel('X-distance')
ylabel('Y-distance')
[px,py] = gradient(z,1,1);
figure
```

```
quiver(x,y,px,py)
set(gca,'Fontsize',14)
title('Gradient Example')
xlabel('X-distance')
ylabel('Y-distance')
```

The new GUI interface developed in MATLAB for the representation of the gradient for a scalar electromagnetic field, using the differential operators showed in Table 1, are shown below for each case in the simulations:
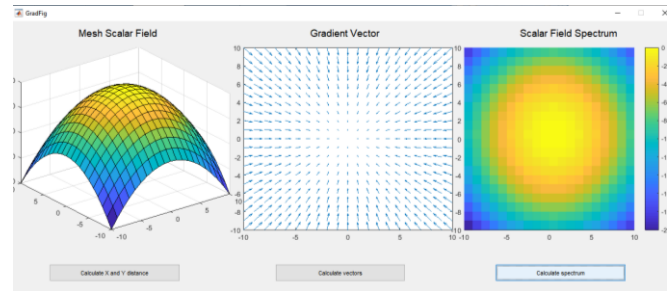
- $z = -(x^2 + y^2)$

Figure 6. Electromagnetic field represented by the differential operator $z = -(x^2 + y^2)$
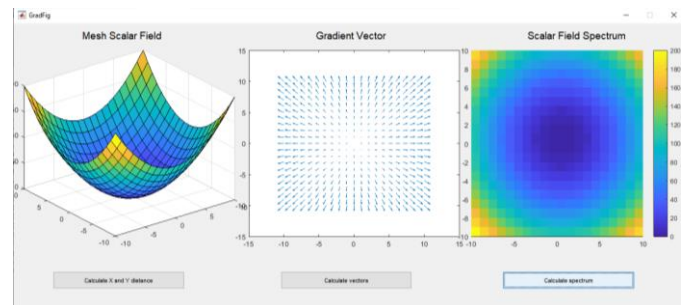
- $z = +(x^2 + y^2)$

Figure 7. Electromagnetic field represented by the differential operator $z = +(x^2 + y^2)$
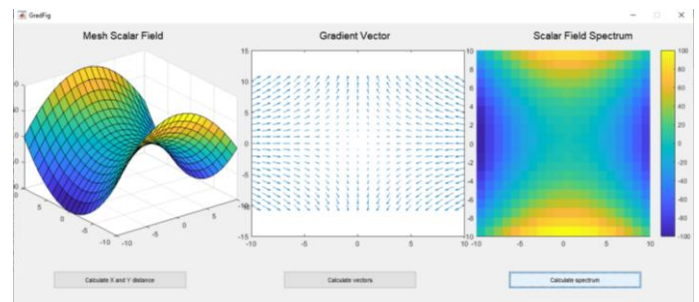
- $z = -(x^2 - y^2)$

Figure 8. Electromagnetic field represented by the differential operator $z = -(x^2 - y^2)$
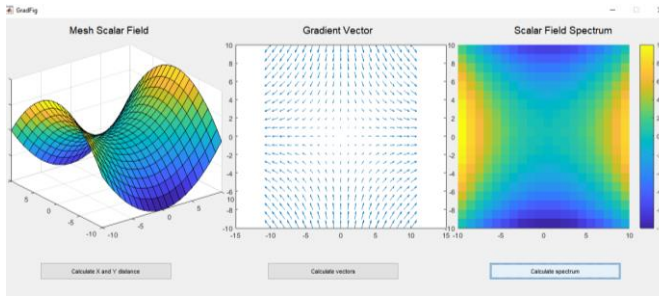
- $z = +(x^2 - y^2)$

Figure 9.   Electromagnetic field represented by the differential operator $z = + (x^2 - y^2)$

IV. RESULTS AND DISCUSSION

The results appoint a maximum electromagnetic field value about 200 kg.s^{-2}.A^{-1} and a minimum value about - 200   kg.s^{-2}.A^{-1} when we insert the differential equations $z = + (x^2 + y^2)$ and $z = - (x^2 + y^2)$, respectively, to the new algorithm developed in MATLAB. Those values represent the spectrum of the electromagnetic field in the SI unit. This means a directly proportional greatness in relation to the distance (m) and value (kg.s^{-2}.A^{-1}) of the electromagnetic field spectrum. Hence, for the differential operators $z = + (x^2 + y^2)$ and $z = - (x^2 + y^2)$, as larger as the distance from the point "zero", less intense is the electromagnetic field. The Graphic 1 and Graphic 2, indicated in the Figure 10 and Figure 11, respectively, shows the linear functions of the differential operators $z = + (x^2 + y^2)$ and $z = - (x^2 + y^2)$.
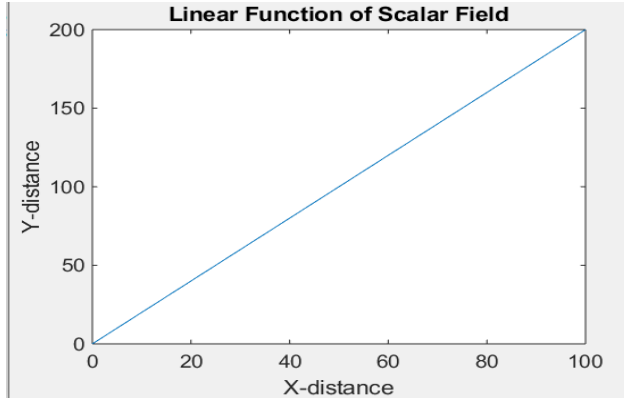

Figure 10. Electromagnetic linear function represented by the differential operator $z = + (x^2 + y^2)$
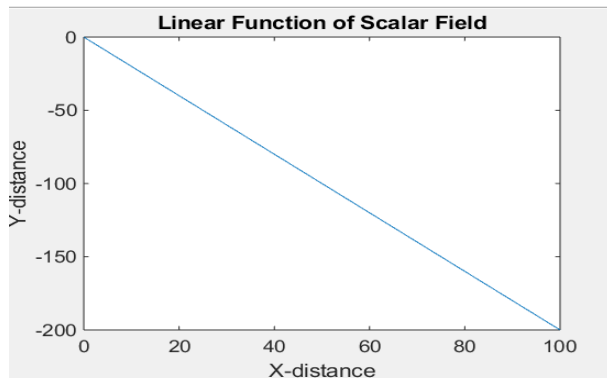

Figure 11. Electromagnetic linear function represented by the differential operator $z = - (x^2 + y^2)$

Graphic 3 and Graphic 4 also can be plotted to the differential operators $z = + (x2 - y2)$ and $z = - (x2 - y2)$, indicating proportional greatness in relation to the distance (m) and value (T) of the electromagnetic field spectrum.
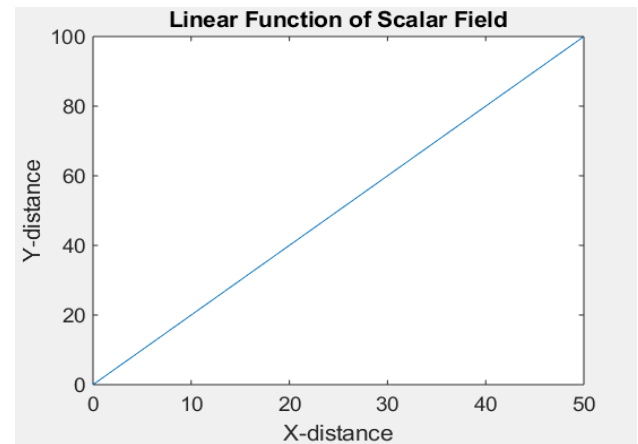

Figure 12. Positive electromagnetic linear function represented by the differential operator $z = + (x^2 - y^2)$ and $z = - (x^2 - y^2)$
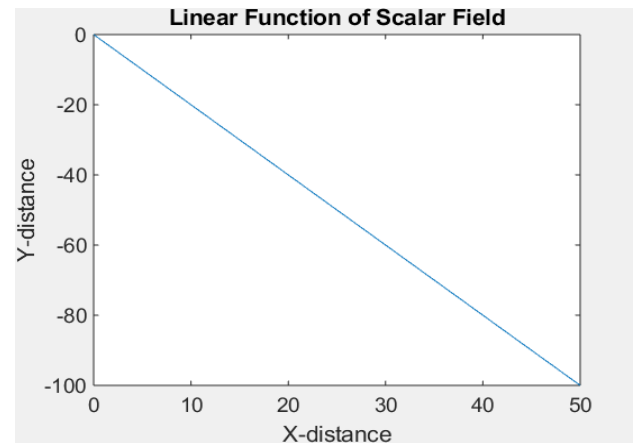

Figure 13. Negative electromagnetic linear function represented by the differential operator $z = + (x^2 - y^2)$ and $z = - (x2 - y2)$

Discussion

The idea of this article is to provide to the reader a general vision about the basics phenomena involved in the electromagnetic spectrum utilizing differential operators capable to represent gradient vectors and the X,Y curvature of the electromagnetic field in MATLAB. It is important the reader has a basic knowledge about the electromagnetism metrics, because those are the elements respond to the gradient vector generation. The presentation of the elements related to the electromagnetic field representation in this paper allows that your modeling can be incorporated into the other techniques of field representation, such as 3D images. The literature regarding the circle detection systems is very vast.

V. CONCLUSION AND FUTURE SCOPE

In this work was deduced a model commonly utilized in the field of numerical calculus to represent an

electromagnetic field (X,Y distance; spectrum and gradient vectors) of four main differential equations insert to the code developed in MATLAB. Once defined the new based GUI algorithm and the spectrum ratio detection in the code, were defined optimized implementation strategies for the linear functions for the scalar fields equations with a scale selection dependents of the spectrum average a representation showed in the new GUI Figures plotted in the sections above. After a definition of an optimization metric through the new based algorithm for electromagnetic field representation using differential operators, was verified the correlation between the distance from the electromagnetic spectrum (m) and the intensity of the electromagnetic field ($kg.s^{-2}.A^{-1}$). With these results, was possible obtain also four linear graphics in the function of the differential operators utilized in this work to calculate the electromagnetic field spectrum.

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