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Variable Sampling Plan for Correlated Data

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Abstract: In this paper an attempt has been made to study the sampling plan for mean with known CV. The OC of the variable sampling plans for mean with known CV are calculated and compared with the OC of known standard deviation case.

Keywords: *Sampling, Variable, MatLab*

I. Introduction

The theory of inspection by variables is applicable when the quality characteristic of sampled items is measurable on a continuous scale and the functional form of the probability distribution is assumed to be known. A variable sampling is advantageous in the sense that is generates more information from each item inspected, requires small sample and provides same protection when compared to attribute sampling (Boowker and Goode and Montgomery).

Some of early works on variable sampling inspection are seen in Lieberman and Resnikoff, Schilling, Owen and Hamaker. Purkar et al. (2011) designed an OC curve for acceptance sampling plan to minimize a consumer's risk. Ferrell and Chhoker (2010) developed economically optimal acceptance sampling plans with inspection error. Purkal and Khandwawala (2012) constructed OC curve for acceptance sampling plan by using MATLAB software. Vispute and Singh (2014) examined economic effect of variable sampling plan for auto correlated data. Narayanan and Rajarathinam (2013) provided the procedure for selection of single sampling plans by variable for pareto distribution.

Statistical process control for auto correlated processes has received a great deal of attention, due in part to the increasing prevalence of autocorrelation in process inspection data. Apley and Tsung (2002) proposed the autoregressive chart for auto correlated processes. Montgomery and Mastragelo (1991) discuss some methods for auto correlated data and Zou et al. (2008) used the auto correlated data for variable sampling plan.

II. Model Description

The procedure of acceptance sampling consists in the first drawing a random sample of size n from the lot submitted for inspection and measuring the item quality x of the sampled items. Let $x_1, x_2, x_3 \ldots x_n$ denote the values of the characteristic x. An estimator $\bar{X}'$ has been constructed by Searls (1964), where

$$\bar{X}' = w\sum_{j=1}^{n} X_j \quad (2.1)$$

We now calculate the OC function of single sampling plan described as:

Accept the lot if $\bar{X}' + k\sigma \leq U$ and reject the lot otherwise, where $\bar{X}'$ is the weighted sample mean and U is the upper specification limit. The values of n and k are determined for a given set of values of the producer's risk α, consumers risk β, acceptable quality level (AQL) p_1 and the lot tolerance proportion defective (LTPD) p_2 . From the OC function of the sampling plan making use of the relationship between p_1, α, p_2 and β. The OC function of the plan is

$$L_p = P[\bar{X}' + k\sigma \leq U] \quad (2.2)$$

After some mathematical simplification the values of n and k are determined as

$$n = \left[\frac{K_\alpha + K_\beta}{K_{p1} - K_{p2}}\right]^2 \quad (2.3)$$

and

$$k = \left[\frac{K_\alpha K_{p2} + K_\beta K_{p1}}{K_\alpha + K_\beta}\right] \quad (2.4)$$

The expression for probability of acceptance $L_p = P\left[\bar{X}' + k\sigma \leq U = \mu + k_p\sigma\right]$ is derived by recalling then ormality of the statistic $(\bar{X}' + k\sigma)$. Following Schilling (1982) the OC function with known CV after some simplification works out to be

$$L_p = \Phi\left[\sqrt{n+v^2}\,(k_p - k)\right] \quad (2.5)$$

where

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}}\, exp\left(\frac{-x^2}{2}\right)dx \quad (2.6)$$

The usual OC of single sampling plan for known standard deviation is

$$L_p = \Phi\left[\sqrt{n}\,(k_p - k)\right] \quad (2.7)$$

III. OC Function for Correlated Data under Known Coefficient of Variation

Suppose that observations $x_1, x_2, x_3 \ldots x_n$ have a multivariate normal distribution with $E(x_i) = \mu$ and $Var(x_i) = \sigma^2$ and ρ as the common correlation coefficient between any x_i and x_j, i≠j. Then

$$E(\bar{x}) = \mu$$

and

$$Var(\bar{x}) = \frac{\sigma^2}{n+v^2}[1 + (n-1)\rho]$$

$$= \frac{\sigma^2}{n+v^2} T^2$$

where $T^2 = [1 + (n-1)\rho]$

The OC function of single sampling plan can now be calculated. The acceptance criterion for correlated data mean plan is, for upper specification limit U, accept the lot if

$$\bar{x} + \frac{K\sigma T}{\sqrt{n+v^2}} \le U$$

reject the lot otherwise.

The values of n and k are determined for a given set of values of the producer risk α and consumer risk β, p_1 and p_2 by formulae

$$n = \left[\frac{K_\alpha + K_\beta}{K_{p1} - K_{p2}}\right]^2 \quad (3.1)$$

and

$$k = \left[\frac{K_\alpha K_{p2} + K_\beta K_{p1}}{K_\alpha + K_\beta}\right] \quad (3.2)$$

The expression for probability of acceptance (OC) function of the plan in normal case is

$$L(p) = p[\bar{x} + \sqrt{MSE(\bar{x})} \le U = \mu + k_p \sigma \frac{T}{\sqrt{n+v^2}}]$$

Following Schilling (1982) the OC function for correlated data works out to be as

$$L_p = \Phi\left[\frac{\sqrt{n+v^2}}{T}\,(k_p - k)\right]$$

(3.3)

where

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}}\, exp\left(\frac{-x^2}{2}\right)dx$$

IV. Numerical Illustration and Result

For illustration, consider an example of producers and consumers oriented single sampling plan p_1 = 0.01, α= 0.05, p_2 = 0.08 and β = 0.10. The values of n and k have been determined from the equation (3.1) and (3.2) and are 10 and 1.808 respectively. The values of OC function for the above plan have been calculated for correlated data as well as for known standard deviation by using equation (2.7) and (3.3).

From all the tables it is evident that the value of OC increases as the CV increases. As ρ increases, a significant effect is seen in producers risk as well as consumers risk, which is not acceptable. Hence one should maintain the correlation between the observations as low as possible, so as to protect producer as well as consumer.

The results of example considered are quite adequate to be generalized, certainly lead to some conclusions of general character. For a known CV case, when the normal theory variable inspection plan is applied, an overall improvement is likely to result over known standard deviation case. However, the author recommends that such a sampling plan be used by quality control practitioner.

TABLE-I

P	***ρ*=0**	***ρ*=0.2**	***ρ*=0.5**	***ρ*=0.6**	***ρ*=0.8**
0.005	0.992344	0.926355	0.84943	0.831104	0.801453
0.01	0.94908	0.835888	0.757284	0.741082	0.716107
0.02	0.780524	0.67815	0.629307	0.620173	0.606528
0.03	0.5898	0.553962	0.53856	0.535754	0.531596
0.04	0.426848	0.456124	0.468663	0.470946	0.474327
0.05	0.301854	0.378202	0.412416	0.418715	0.428078
0.06	0.210717	0.315456	0.365875	0.375325	0.389453
0.08	0.100742	0.222626	0.292995	0.306811	0.327775
0.1	0.047664	0.159435	0.238477	0.254848	0.280125
0.12	0.022485	0.115425	0.196302	0.21403	0.241917
0.14	0.010603	0.084244	0.162914	0.181187	0.210498
0.16	0.005	0.061861	0.136031	0.154297	0.18419
0.18	0.002357	0.045628	0.114106	0.131988	0.161857
0.2	0.00111	0.033762	0.096046	0.113289	0.142695

TABLE-II

P	***ρ*=0**	***ρ*=0.2**	***ρ*=0.5**	***ρ*=0.6**	***ρ*=0.8**
0.005	0.994509	0.928998	0.850433	0.831792	0.801715
0.01	0.956905	0.839071	0.75819	0.741679	0.716322
0.02	0.79153	0.680343	0.629826	0.620506	0.606643
0.03	0.594103	0.554672	0.53872	0.535856	0.531631
0.04	0.423321	0.455545	0.468533	0.470863	0.474299
0.05	0.293079	0.37664	0.412058	0.418486	0.427999
0.06	0.199566	0.313198	0.365339	0.374981	0.389333
0.08	0.090175	0.219624	0.292216	0.306305	0.327597
0.1	0.040115	0.156251	0.237561	0.254245	0.279908
0.12	0.017742	0.112366	0.19532	0.213371	0.241674
0.14	0.00783	0.08146	0.16191	0.180503	0.210241
0.16	0.003451	0.059412	0.135036	0.153608	0.183924
0.18	0.001519	0.043524	0.113142	0.131308	0.161588
0.2	0.000667	0.031985	0.095125	0.112627	0.142427

TABLE-III

P	***ρ*=0**	***ρ*=0.2**	***ρ*=0.5**	***ρ*=0.6**	***ρ*=0.8**
0.005	0.999585	0.940438	0.854744	0.834735	0.802825
0.01	0.987935	0.85347	0.762113	0.744241	0.717234
0.02	0.856977	0.690552	0.632078	0.621935	0.607133
0.03	0.622837	0.558004	0.539415	0.536294	0.53178
0.04	0.399677	0.45283	0.467967	0.470507	0.474178
0.05	0.237151	0.36934	0.4105	0.417502	0.427664
0.06	0.133899	0.302691	0.363009	0.373501	0.388825
0.08	0.039146	0.205845	0.28884	0.304131	0.326837
0.1	0.01075	0.141869	0.233601	0.251655	0.278984
0.12	0.002858	0.09878	0.191079	0.21055	0.240645
0.14	0.000746	0.069312	0.157589	0.177576	0.209148
0.16	0.000192	0.048918	0.130768	0.150663	0.182798
0.18	4.9E-05	0.034672	0.109014	0.128407	0.160451
0.2	1.24E-05	0.024646	0.091194	0.109812	0.141296

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