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Economic Effect on Variables Sampling Plans Under Second Order Auto-Correlation

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Abstract: In these paper is concerned with the effect of second order autocorrelation for three different situations on economic model Case and Bennett(1977) for variables acceptance sampling plan. This paper quantitatively combines economic and autocorrelation and clearly illustrates the effects of autocorrelation in three different situations where variables sampling plan are used.

Keywords: Sampling plan, Economic Effect, Auto Correlation

Introduction: Acceptance sampling plan is a field of statistical quality control. Tagaras (1994) pointed out that in recent years more emphasis was placed on process control and off-line quality control methods, but acceptance sampling plan still remained important functions in many practical quality control systems. Sower et al. (1993) proposed an integrated model of statistical process control(SPC) and acceptance sampling plan. Acceptance sampling plan might be useful in an SPC environment under the following three situations:

1. When a supplier has not yet achieved statistical control of the process.
2. When a process may shift from an in-control state to an out-of-control state.
3. When a customer specifies that a standard acceptance sampling plan be used.

Although there certainly exist situations in which screening inspection (i.e., 100% inspection) are feasible, there are also many cases where sampling is inevitable either because inspection is destructive, or because lot sizes are large and inspection is expensive, time-consuming, and high error rate. The classical Dodge-Romig (1959) rectifying attributes sampling plans provide the lot tolerance percent defective (LTPD) on each lot or the average outgoing quality limit (AOQL) protection for the lots. The design of Dodge-Romig (1959) AOQL single sampling plans (SSP) by attributes is based on the following four assumptions: (1) the manufacturing process is normally in binomial control with a process average fraction defective; (2) inspection is rectifying and rejected lots are totally inspected; (3) to make sure that the average quality of his product is satisfactory, the producer chooses an AOQL value, and considers only sampling plans satisfying this specification; and (4)among plans having the specified AOQL, the producer chooses the one minimizing average total inspection (ATI) for product of process average fraction defective. Montgomery (1991) pointed out that variables sampling plans usually involve smaller sample size than attributes sampling plans for the same levels of protection. Because of economic reasons, Klufa (1994) ,(1997) presented the designs of Dodge-Romig LTPD and AOQL SSP by variables. The traditional concept of conformance to specifications is that items meet the specification limits. Taguchi (1986) refined the quality of product and presented the quadratic quality loss function for reducing total losses to the society. In general, there is an optimal target value for every measurable quality characteristic. Any deviation from this target value incurs an economic loss, even if the value of the quality characteristic lies within the specification limits. The losses of quadratic function are expressed in monetary terms and are easy to understand and to apply in the evaluation of product or process improvement. The quadratic quality loss function has been succeeded in the application of SPC, sampling plans, and specification limits design. Recently, Wu and Tang (1998), Li (1997), Maghsoodloo and Li (2000) , Phillips and Cho (2000) , Li and Chou (2001), Li and Wu(2001) , Duffuaa and Siddiqi (2002), and Rahim and Tuffaha (2004) addressed the different problems of unbalanced tolerance design and optimum manufacturing target with the quadratic quality loss functions. Tagaras(1994) adopted the quadratic quality loss function for designing the parameters of acceptance sampling plan by variables. Kapur and Wang (1988), Kapur (1994), Kapur and Cho(1994), (1996) [20,21] and Chen and Chou (2003) , addressed the problems of quality loss function applied in the economic design of specification limits. Kapur and Wang pointed out that one of the short-term approaches to reduce variance of the units shipped to the customer is to put specification limits on the process and truncate the distribution by inspection. Chen (1999) and Chen and Chou (2001) considered the problems of the integrated designs of Dodge-Romig AOQL SSP by attributes and specification limits and Dodge-Romig LTPD SSP by variables and specification limits, respectively. It turns out that not only the sampling plan parameters, but also the specification limits have to be looked upon as design parameters in order to minimize the expected total cost.

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Eagle (1954) discussed the relationship between the probability of accepting non-conforming units and measurements error. He claimed that in the presence of imprecision one should set control limits inside specifications in order to protect the consumer's risk or outside specifications to protect the producer's risk. Grubbs and Coon (1954) derived formulae to adjust imprecision for a single specification when one wishes to maintain the consumer's risk and producer's risk (or the linear combination of both) at a certain level. Diviney and David (1963) examined the same type of problem with which Grubbs and Coon dealt; however, they did not attempt to compensate for bias. In addition, Walsh et al. (1959) considered the same case wherein the variance of the population was unknown. They derived an adjustment method for one specification . To the best knowledge of the authors, the case concerning both bias and imprecision has not yet been considered. Two separate research efforts in recent years have taken a new look at variables acceptance sampling by attacking traditional approaches underlying sampling plan selection. One such traditional approach has been to design the sampling plan based upon α and β in a such way that good lots are accepted a high fraction $(1-\alpha)$ of the time while poor lots accepted only a low proportion (β) of the time. The alternative procedure presented recently selects sampling plans parameters based upon costs or economic factors, and in such way eliminates the to define $\alpha, \beta,$ good and poor . the other tradition attacked has been the assumption that all measurements are error free , when ,in fact, it is well known that this is not the case . Recent results indicate that the effect of measurement error on the sampling plan operating characteristics. curve can be significant

The choice of variables sampling plans using cost effective criteria is a subjects covered by Schmidt et al. (1974).It has also been discussed by Chapman(1972) and Ailor et al. (1975).In order to base sampling plan selection upon economics, it is first necessary to develop a mathematical model of the quality control inspection process for the item or dimension of interest . such a model typically may include the following four typs of costs:

(1) Inspection Cost –the direct and indirect cost associated with inspecting unit of product.
(2) Acceptance cost- the cost of accepting some defective items in a lot which has been accepted based upon the sample inspection.
(3) Rejection cost (screening)- the cost of repairing or replacing defective items found during the screening of a rejected lot.

Rejection cost(scrapping)- the net value lost due to the scrapping of each item in a rejected lot plus the cost of the scrapping operation, per se. Schmidt et al. (1974) considers two models ,one in which lots screened upon rejection, and one in which lots are rejected. The authors then proceed to find an optimum variables sampling plan based upon minimum total expected cost.

In these chapter is concerned with the effect of second order autocorrelation given in chapter-II for three different situations on economic model Case and Bennett(1977) for variables acceptance sampling plan. This chapter quantitatively combines economic and autocorrelation and clearly illustrates the effects of autocorrelation in three different situations where variables sampling plan are used.

Economic model forvariables sampling plan: When product dimensions such as height, weight, strength, etc., are of concern, there may be an upper specification, a lower specification, or both. For example, if there are lower and upper specifications, L and U, a product item is said to be acceptable if its dimension, x, is such that L≤X≤U ; otherwise, it is defective.

A variables sampling plan, used to determine the acceptability of an individual lot, is characterized by its sample size, n, and acceptance constant, k. A random sample of size n is taken and the appropriate dimension of each item is measured. The measurements are averaged and constitute the statistics $\bar{x}$, which is then compared against decision criteria $DC_{\bar{x},L}$ and $DC_{\bar{x},U}$ .for upper and lower specification limits, this comparison proceeds as follows:

If $DC_{\bar{x},L} = L + k\sigma \leq \bar{x} \leq DC_{\bar{x},U} = U - k\sigma$, Accept the lot

Otherwise, reject the lot.

The lot standard deviation ,σ, is often assumed to be known and constant .The decision variables ,n and k , or equivalently $n, DC_{\bar{x},U}$ and $DC_{\bar{x},L}$,may then be determined using either the α , β method or cost based modeling as in Schmidt et al. (1974)

n Sample size.

k Acceptance constant,

L Lower specification limit.

U Upper specification limit

x True measurable dimension

$\bar{x}$ True sample average

μ True value of lot mean

σ true standard deviation of lot.

$f(x/\mu,\sigma)$ Lot distribution of true measurements.
$h(\bar{x}/\mu,\sigma/\sqrt{n})$ distribution of true s ample average.
$g(\mu)$ prior distribution of true mean dimension from lot.
$DC_{\bar{x},L}$ Lower decision criterion which the observed sample average must equal or exceed if the lot is to be accepted
$DC_{\bar{x},U}$ Lower decision criterion which the observed sample average must equal or below if the lot is to be accepted.

x_0 Observed dimension.

$f(x_0)$ Observed lot distribution

$\bar{x}_0$ Sample mean observed.

C_1 Cost per unit of sampling inspection.

C_a Cost per unit of a defective item contained in an accepted Lot.

C_r Cost of rejecting an entire lot.

The mathematical model to be used for illustrative purpose is very simple conceptually. This because it will later be used to illustrative the effect of autocorrelation, and presentation of more complete and difficult model would serve no purpose. From a lot of size N item a random sample of size n is drawn. Each item measured and the sample average compared against lower and upper decision criteria .If the lot accepted, all N items are accepted. If the lot is rejected, all N items in the lot will be rejected and disposed easily in three term. They include inspection, acceptance, and rejection costs.

Inspection cost is equal to the cost per unit of sampling inspection times the sample

Sizen Thus, Inspection cost = $C_1 n$ (1)

Note that the cost of inspection per item is often not a constant as inspectors are paid by the week. However ,for purposes of this model ,the cost per unit will be assumed constant , as if the number of inspectors is perfectly matched to the workload , or these inspectors can perform other duties when not busy on inspection .

Acceptance cost given that the lot is accepted and has mean μ is just the cost per unit of a defective item contained in an accepted lot times the number of such defectives. This cost may be difficult to ascertain ,particularly when loss of "goodwill " is a factor; however ,none the less it is valid cost .since the total number of defectives is unknown, an expected value is taken using the number of units in lot times the proportion of dimension which fall outside specification . Thus,

Acceptance cost given acceptance and given

$$\mu = C_1 N[1-\int_L^U f(x/\mu,\sigma)dx] \qquad (2)$$

To remove the condition on acceptance, the terms of eqn. (2) are taken times the Probability of acceptance, $P(DC_{\bar{x},L} \le \bar{x} \le DC_{\bar{x},U}\}$ .The condition on μ is removed by taking the expectation with respect to μ . The unconditional acceptance cost may then be expressed as

Acceptance cost

$$= C_a N \int_\mu [1-\int_L^U f(x/\mu,\sigma)dx][\int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}/\mu,\sigma/\sqrt{n})d\bar{x}]g(\mu)d\mu \qquad (3)$$

Finally, the rejection cost given that the lot is rejected and has mean μ is just the cost of rejecting an entire lot , C_r .The conditions on rejection and μ are removed in much the same way as for the acceptance cost term. That is,

Rejection cost

$$= C_r \int_\mu [1-\int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}/\mu,\sigma/\sqrt{n})d\bar{x}]g(\mu)d\mu. \qquad (4)$$

The total expected cost per lot for as sampling plan having parameters $n, DC_{\bar{x},L}$ and $DC_{\bar{x},U}$ is

$$TC = C_1 n + C_a N \int_\mu [1-\int_L^U f(x/\mu,\sigma)dx][\int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}/\mu,\sigma/\sqrt{n})d\bar{x}]g(\mu)d\mu$$

$$+ C_r \int_\mu [1-\int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}/\mu,\sigma/\sqrt{n})d\bar{x}]g(\mu)d\mu. \qquad (5)$$

Economic model forvariables sampling plan under second order autocorrelation: Consider a manufacturing process where a quality characteristic is measured at equidistance time points 1, 2, 3, … n. This situation may occur in a discrete manufacturing process which produces discrete time 1, 2, 3 ... n, with one quality characteristic of interest. It may also occur in a continuous manufacturing process where the quality characteristic of interest is measured at discrete equidistant time points. We denote the behavior of the quality characteristic as $x_1, x_2, \ldots x_n$. It will assumed that on EPC control action can be represented by some controllable variable or factor x_t, such that

$x_t = \mu + \xi_t$, (6)

Where μ is a constant, and ξ_t is a stationary time series with zero mean and standard deviation σ. A Durbin and Watson (1950) "d" statistic can be used to detect the presence or absence of serial correlation. The problem, however, is that to do once the suspicion of dependence via the serial correlation test is confirmed. If serial correlation exists we use identification techniques to define the nature of ξ_t. When identification is complete, the likelihood function can provide maximum likelihood estimate of the parameters of the identified model.

Suppose that a correlation test revealed the presence of data dependence and identification technique suggested autoregressive model of order two AR (2) say, then we can express ξ_t of equation (6) as

$$\xi_t = \alpha_1 \xi_{t-1} + \alpha_2 \xi_{t-2} + \in_k, \; t = 1,2, \ldots n \qquad (7)$$

Where

(i) $\in_t \sim N\left(0, \sigma_\in^2\right)$

(ii) $\operatorname{cov}\left(\in_t, \in_\gamma\right) = \begin{cases} \sigma_\in^2 & t = \gamma \\ 0 & t \neq \gamma \end{cases}$

The Class of stationary models that assume the process to remain in equilibrium about a constant mean level μ. The variance of AR (2) process is given by:

$$\sigma^2 = \left(\frac{1-\alpha_2}{1+\alpha_2}\right) \frac{\sigma_\in^2}{\left[(1-\alpha_2)^2 - \alpha_1^2\right]}. \qquad (8)$$

Following Kendall and Stuart (1976) it can be shown that for stationary, the roots of the characteristic equation of the process in equation (7)

$$\phi(B) = 1 - \alpha_1 B - \alpha_2 B^2 \qquad (9)$$

must lies outside the unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions :

$$\alpha_2 + \alpha_1 < 1$$
$$\alpha_2 - \alpha_1 < 1$$
$$-1 < \alpha_2 < 1 \qquad (10)$$

Now If G_1^{-1} and G_2^{-1} are the roots of the characteristic equation of the process given by equation (9) then

$$G_1 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \ldots.(11)$$

$$G_2 = \frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \quad (12)$$

For stationary we require that $|G_i| < 1, \; i = 1, 2.$ Thus, three situations can theoretically arise:

(i) Roots G_1 and G_2 are real and distinct $\left(i.e., \alpha_1^2 - 4\alpha_2 > 0\right)$

(ii) Roots G_1 and G_2 are real and equal $\left(i.e., \alpha_1^2 - 4\alpha_2 = 0\right)$

(iii) Roots G_1 and G_2 are complex conjugate $\left(i.e., \alpha_1^2 - 4\alpha_2 < 0\right)$.

When the serial correlation is present in the data, we have for the distribution of the sample mean $\bar{x}$, its mean and variance is given by,

$$E(\bar{x}) = \mu$$

$$\text{Var}\left(\bar{x}\right)=\frac{\sigma^2}{n}\ \lambda_{ap}\ \left(\alpha_1,\alpha_2,\ n\right), \tag{13}$$

Where $\lambda_{ap}\ \left(\alpha_1,\alpha_2,n\right)$ depends on the nature of the roots G_1 and G_2 , and for different situations is given as follows :

(i) If G_1 and G_2 are real and distinct,

$$\lambda_{ap}\ \left(\alpha_1,\alpha_2,n\right)=\left[\frac{G_1\left(1-G_2^2\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\lambda\left(G_1,n\right)-\frac{G_2\left(1-G_1^{\ 2}\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\lambda\left(G_2,n\right)\right]$$

$$=\lambda_{rd}\ \left(\alpha_1,\alpha_2,n\right), \tag{14}$$

$$\text{Where, } \lambda\left(G,n\right)=\left[\frac{1+G}{1-G}-\frac{2G}{n}\ \frac{\left(1-G^n\right)}{\left(1-G\right)^2}\right]$$

(ii) If G_1 and G_2 are real and equal

$$\lambda_{ap}\ \left(\alpha_1,\alpha_2,n\right)=\left(\frac{1+G}{1-G}\right)-\frac{2G\left(1-G^n\right)}{n\left(1-G\right)^2}\left[1+\frac{\left(1+G\right)^2\left(1-G^n\right)-n\left(1-G^2\right)\left(1+G^n\right)}{\left(1+G^2\right)\left(1-G^n\right)}\right]$$

$$=\lambda_{re}\ \left(\alpha_1,\alpha_2,n\right) \tag{15}$$

(iii) If G_1 and G_2 and complex conjugate

$$\lambda_{ap}\ \left(\alpha_1,\alpha_2,n\right)=\left[\gamma\left(d,u\right)+\frac{2d}{n}\left(W\ \left(d,u,n\right)+\ z\ \left(d,u,n\right)\right)\right] \tag{16}$$

Where

$$=\lambda_{cc}\ \left(\alpha_1,\alpha_2,n\right)$$

$$\gamma\left(d,u\right)=\frac{1-d^4+2d\left(1-d^2\right)\cos u}{\left(1+d^2\right)\left(1+d^2-2d\ \cos u\right)},$$

$$W\ \left(d,u,n\right)=\frac{2d\left(1+d^2\right)\sin u-\left(1+d^4\right)\sin 2u-d^{n+4}\ \sin\left(\left(n-2\right)u\right)}{\left(1+d^2\right)\left(1+d^2-2d\ \cos u\right)^2\ \sin u},$$

$$Z\ \left(d,u,n\right)=\frac{2d^{\ n+3}\sin\ \left(n-1\right)u-2d^{n+1}\sin\left(n+1\right)u+d^n\ \sin\ \left(\left(n+2\right)u\right)}{\left(1+d^2\right)\left(1+d^2-2d\ \cos u\right)^2\ \sin u},$$

$$d^2=-\alpha_2,$$

$$\text{And}\qquad u=\cos^{-1}\left(\frac{\alpha_1}{2d}\right).$$

The x_t denote the change in the level of the compensating variable model at the time t, i.e., the adjustment made at the time point t. The ε_t is Gaussian white noise with variance $\sigma_\epsilon^{\ 2}$. throughout, we suppose that the noise variance is known. In practice, this is justified if reliable estimates of $\sigma_\epsilon^{\ 2}$ are available from the evaluation of a large number of previous values of the process, e.g., during the setup phase. The real - valued parameters α_1 and α_2 (the autoregressive parameters) determines the influence of the preceding time point (t - 1) and (t - 2) on the present time point t. We assume an in-control value $\alpha_1 = \alpha_2 = O$ for the auto regression parameters. It is possible that the auto regression parameters may shift to an out-of-control value $(\alpha_1,\alpha_2) \neq 0$.

Further, the distribution of the sample average will have mean μ and standard deviation $\frac{\sigma^2}{n}\ \lambda(\alpha_1,\alpha_2,n)$

Now the using equation (13)Reconsidering eqn.(2) in light of AR(2) process the only terms which will change those concerned with sampling inspection..Namely $h(\bar{x}/\mu,\frac{\sigma}{\sqrt{n}})$ will change to $h[\bar{x}^*/\mu,\frac{\sigma}{\sqrt{n}}\lambda(\alpha_1,\alpha_2,n)]$ .The true proportion defective and the distribution of true lot means will not be affected. Thus the current model under second order autocorrelation processes is written as

$$TC = C_1 n + C_a N \int_{\mu} [1 - \int_L^U f(\frac{x}{\mu}, \sigma) dx][\int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}^* / \mu, \frac{\sigma}{\sqrt{n}} \lambda(\alpha_1, \alpha_2, n))] g(\mu) d\mu$$
$$+ C_r \int_{\mu} [1 - \int_{DC_{\bar{x},L}}^{DC_{\bar{x},U}} h(\bar{x}^* / \mu, \frac{\sigma}{\sqrt{n}} \lambda(\alpha_1, \alpha_2, n)) d\bar{x}] g(\mu) d\mu. \tag{17}$$

We calculate the total cost for the help of following terms

$$T_1 = 1 - \int_L^U f(x, \mu, \sigma) dx$$

$$T_2 = \int_L^U h(\bar{x}^*, \mu, \sigma\lambda(\alpha_1, \alpha_2, n) / \sqrt{n}) d\bar{x}$$

$$T_3 = 1 - T_2,$$

We take the value $T_4 = g(\mu)\Delta\mu$ from Kenneth and Bennett (1977)

Numerical example and conclusion: It is difficult to appreciate the effect of AR(2) without referring to some typical specifications, plans ,and costs. Consider a manufacturing situation where variables acceptance sampling is used to obtain a decision to accept or reject lots. The lot size is $N = 5000$, and if accepted, all items in the lot are passed. If rejected, all items in the lot are scrapped. Each item is to meet specifications, of $150 \pm 0.3(149.7, 150.3)$.The true process or lot distribution is normal with standard deviation $\sigma = 0.1$. Note that ,if the manufacturing process is properly centered, virtually all of the items produced will be within specifications.

A sampling plan $n = 5, 10, 15, 20, 26$ $DC_{\bar{x},L} = 149.9003$ *and* $DC_{\bar{x},U}$ =15 $0.0997 (k = 2.00330)$ is to be used. The three costs are $C_i = 1.5$, $C_a = 10$ and C_r= 10000.We calculate total cost for different sample size n, when no auto-correlation present, when roots are real and distinct, real and equal, and roots are complex conjugate by the help equation (17) and arrange in the tables 1.1 to 1.21, for visual comparison we plot the total cost versus different situations for different sample size n.

Table:1.1 Values of terms in cost of model of equation (17) when no autocorrelation present (n=5)

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.053459	0.946541	0.0048	46.71599784
149.8751	50000	10000	0.039984	0.286551	0.713449	0.0418	322.1676874
149.9326	50000	10000	0.010129	0.764836	0.235164	0.2315	634.0728908
150	50000	10000	0.0027	0.97421	0.02579	0.4438	172.8193521
150.0674	50000	10000	0.010129	0.764836	0.235164	0.2315	634.0728908
150.1249	50000	10000	0.039984	0.286551	0.713449	0.0418	322.1676874
150.1718	50000	10000	0.099923	0.053459	0.946541	0.0048	46.71599784
							2178.732504
						C_in =	7.5
					Total cost		2186.232504

Table:1.2 Values of terms in cost of model of equation (17) when no autocorrelation present (n= 10)

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.011304	0.988696	0.0048	47.72849909
149.8751	50000	10000	0.039984	0.212756	0.787244	0.0418	346.8470393
149.9326	50000	10000	0.010129	0.846471	0.153529	0.2315	454.6589476
150	50000	10000	0.0027	0.998383	0.001617	0.4438	66.98862101
150.0674	50000	10000	0.010129	0.846471	0.153529	0.2315	454.6589476
150.1249	50000	10000	0.039984	0.212756	0.787244	0.0418	346.8470393
150.1718	50000	10000	0.099923	0.011304	0.988696	0.0048	47.72849909
							1765.457593
						C_in =	15
					Total cost		1780.457593

Table:1.3 Values of terms in cost of model of equation (17) when no autocorrelation present (n=15)

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.002616	0.997384	0.0048	47.93717307
149.8751	50000	10000	0.039984	0.164534	0.835466	0.0418	362.9741342
149.9326	50000	10000	0.010129	0.894528	0.105472	0.2315	349.0416439
150	50000	10000	0.0027	0.999887	0.000113	0.4438	60.40212893
150.0674	50000	10000	0.010129	0.894528	0.105472	0.2315	349.0416439
150.1249	50000	10000	0.039984	0.164534	0.835466	0.0418	362.9741342
150.1718	50000	10000	0.099923	0.002616	0.997384	0.0048	47.93717307
							1580.308031
						C_in =	22.5
					Total cost		1602.808031

Table:1.4 Values of terms in cost of model of equation (17) when no autocorrelation present (n= 20)

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.000631	0.999369	0.0048	47.98484036
149.8751	50000	10000	0.039984	0.129876	0.870124	0.0418	374.5650935
149.9326	50000	10000	0.010129	0.925701	0.074299	0.2315	280.5311821
150	50000	10000	0.0027	0.999992	8.25E-06	0.4438	59.94457232
150.0674	50000	10000	0.010129	0.925701	0.074299	0.2315	280.5311821
150.1249	50000	10000	0.039984	0.129876	0.870124	0.0418	374.5650935
150.1718	50000	10000	0.099923	0.000631	0.999369	0.0048	47.98484036
							1466.106804
						C_in =	30
					Total cost		1496.106804

Table:1.5 Values of terms in cost of model of equation (17) when no autocorrelation present (n= 26)

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.000118	0.999882	0.0048	47.99715914
149.8751	50000	10000	0.039984	0.099404	0.900596	0.0418	384.7557909
149.9326	50000	10000	0.010129	0.950219	0.049781	0.2315	226.6454679
150	50000	10000	0.0027	1	3.7E-07	0.4438	59.91009501
150.0674	50000	10000	0.010129	0.950219	0.049781	0.2315	226.6454679
150.1249	50000	10000	0.039984	0.099404	0.900596	0.0418	384.7557909
150.1718	50000	10000	0.099923	0.000118	0.999882	0.0048	47.99715914
							1378.706931
						C_in =	39
					Total cost		1417.706931

Table:1.6 Values of terms in cost of model of equation (17) When roots are real and equal λ(α1=-0.8,α2=0.16,n=5)=2.775

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.266284	0.733716	0.0048	41.60424
149.8751	50000	10000	0.039984	0.384381	0.615619	0.0418	289.4498
149.9326	50000	10000	0.010129	0.513598	0.486402	0.2315	1186.235
150	50000	10000	0.0027	0.578241	0.421759	0.4438	1906.409
150.0674	50000	10000	0.010129	0.513598	0.486402	0.2315	1186.235
150.1249	50000	10000	0.039984	0.384381	0.615619	0.0418	289.4498
150.1718	50000	10000	0.099923	0.266284	0.733716	0.0048	41.60424
							4940.987
						C_in =	7.5
					Total cost		4948.487

Table:1.7 Values of terms in cost of model of equation (17) When roots are real and equal λ(α1=-0.8,α2=0.16,n=10)=3.345

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.242608	0.757392	0.0048	42.17291
149.8751	50000	10000	0.039984	0.388986	0.611014	0.0418	287.9099
149.9326	50000	10000	0.010129	0.562867	0.437133	0.2315	1077.953
150	50000	10000	0.0027	0.654083	0.345917	0.4438	1574.365
150.0674	50000	10000	0.010129	0.562867	0.437133	0.2315	1077.953
150.1249	50000	10000	0.039984	0.388986	0.611014	0.0418	287.9099
150.1718	50000	10000	0.099923	0.242608	0.757392	0.0048	42.17291
							4390.437
						C_in =	15
					Total cost		4405.437

Table: 1.8 Values of terms in cost of model of equation (17) When roots are real and equal$\lambda(\alpha1=-0.8,\alpha2=0.16,n=15)=3.544$

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.213866	0.786134	0.0048	42.86325
149.8751	50000	10000	0.039984	0.384453	0.615547	0.0418	289.4258
149.9326	50000	10000	0.010129	0.604033	0.395967	0.2315	987.4796
150	50000	10000	0.0027	0.724088	0.275912	0.4438	1267.874
150.0674	50000	10000	0.010129	0.604033	0.395967	0.2315	987.4796
150.1249	50000	10000	0.039984	0.384453	0.615547	0.0418	289.4258
150.1718	50000	10000	0.099923	0.213866	0.786134	0.0048	42.86325
							3907.412
						C_in =	22.5
					Total cost		3929.912

Table:1.9 Values of terms in cost of model of equation (17) When roots are real and equal $\lambda(\alpha1=-0.8,\alpha2=0.16,n=20)=3.643$

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.187622	0.812378	0.0048	43.49357
149.8751	50000	10000	0.039984	0.375611	0.624389	0.0418	292.3829
149.9326	50000	10000	0.010129	0.634019	0.365981	0.2315	921.5783
150	50000	10000	0.0027	0.779015	0.220985	0.4438	1027.399
150.0674	50000	10000	0.010129	0.634019	0.365981	0.2315	921.5783
150.1249	50000	10000	0.039984	0.375611	0.624389	0.0418	292.3829
150.1718	50000	10000	0.099923	0.187622	0.812378	0.0048	43.49357
							3542.309
						C_in =	30
					Total cost		3572.309

Table:1.10 Values of terms in cost of model of equation (17) When roots are real and equal $\lambda(\alpha1=-0.8,\alpha2=0.16,n=26)=3.7126$

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.16093	0.83907	0.0048	44.13468
149.8751	50000	10000	0.039984	0.363613	0.636387	0.0418	296.3955
149.9326	50000	10000	0.010129	0.660475	0.339525	0.2315	863.4338
150	50000	10000	0.0027	0.829099	0.170901	0.4438	808.1266
150.0674	50000	10000	0.010129	0.660475	0.339525	0.2315	863.4338
150.1249	50000	10000	0.039984	0.363613	0.636387	0.0418	296.3955
150.1718	50000	10000	0.099923	0.16093	0.83907	0.0048	44.13468
							3216.055
						C_in =	39
					Total cost		3255.055

Table:1.11 Values of terms in cost of model of equation (17) when roots are real and distinct λ(α1=0.3,α2=0.6,n=5)=4.03

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.278581	0.721419	0.0048	41.30886
149.8751	50000	10000	0.039984	0.338055	0.661945	0.0418	304.9428
149.9326	50000	10000	0.010129	0.394196	0.605804	0.2315	1448.651
150	50000	10000	0.0027	0.419867	0.580133	0.4438	2599.782
150.0674	50000	10000	0.010129	0.394196	0.605804	0.2315	1448.651
150.1249	50000	10000	0.039984	0.338055	0.661945	0.0418	304.9428
150.1718	50000	10000	0.099923	0.278581	0.721419	0.0048	41.30886
							6189.588
						C_in =	7.5
					Total cost		6197.088

Table:1.12 Values of terms in cost of model of equation (17) when roots are real and distinct λ(α1=0.3,α2=0.6,n=10)=7.26

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.258255	0.741745	0.0048	41.79707
149.8751	50000	10000	0.039984	0.292335	0.707665	0.0418	320.233
149.9326	50000	10000	0.010129	0.322589	0.677411	0.2315	1606.026
150	50000	10000	0.0027	0.335907	0.664093	0.4438	2967.37
150.0674	50000	10000	0.010129	0.322589	0.677411	0.2315	1606.026
150.1249	50000	10000	0.039984	0.292335	0.707665	0.0418	320.233
150.1718	50000	10000	0.099923	0.258255	0.741745	0.0048	41.79707
							6903.483
						C_in =	15
					Total cost		6918.483

Table:1.13 Values of terms in cost of model of equation (17) when roots are real and distinct λ(α1=0.3,α2=0.6,n=15)=9.91

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.244727	0.755273	0.0048	42.12201
149.8751	50000	10000	0.039984	0.27074	0.72926	0.0418	327.4554
149.9326	50000	10000	0.010129	0.293365	0.706635	0.2315	1670.253
150	50000	10000	0.0027	0.3032	0.6968	0.4438	3110.564
150.0674	50000	10000	0.010129	0.293365	0.706635	0.2315	1670.253
150.1249	50000	10000	0.039984	0.27074	0.72926	0.0418	327.4554
150.1718	50000	10000	0.099923	0.244727	0.755273	0.0048	42.12201
							7190.225
						C_in =	22.5
					Total cost		7212.725

Table:1.14 Values of terms in cost of model of equation (17) when roots are real and distinct λ(α1=0.3,α2=0.6,n=20)=12.10

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.237115	0.762885	0.0048	42.30483
149.8751	50000	10000	0.039984	0.25966	0.74034	0.0418	331.1607
149.9326	50000	10000	0.010129	0.279093	0.720907	0.2315	1701.621
150	50000	10000	0.0027	0.287492	0.712508	0.4438	3179.336
150.0674	50000	10000	0.010129	0.279093	0.720907	0.2315	1701.621
150.1249	50000	10000	0.039984	0.25966	0.74034	0.0418	331.1607
150.1718	50000	10000	0.099923	0.237115	0.762885	0.0048	42.30483
							7329.508
						C_in =	30
					Total cost		7359.508

Table:1.15 Values of terms in cost of model of equation (17)when roots are real and distinct λ(α1=0.3,α2=0.6,n=26)=14.25

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.232556	0.767444	0.0048	42.41433
149.8751	50000	10000	0.039984	0.253283	0.746717	0.0418	333.2934
149.9326	50000	10000	0.010129	0.271062	0.728938	0.2315	1719.27
150	50000	10000	0.0027	0.278723	0.721277	0.4438	3217.726
150.0674	50000	10000	0.010129	0.271062	0.728938	0.2315	1719.27
150.1249	50000	10000	0.039984	0.253283	0.746717	0.0418	333.2934
150.1718	50000	10000	0.099923	0.232556	0.767444	0.0048	42.41433
							7407.682
						C_in =	39
					Total cost		7446.682

Table:1.16 Values of terms in cost of model of equation (17) when roots are complex and conjugate λ(α1=0.8,α2=-0.6,n=5)=1.40

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.124741	0.875259	0.0048	45.0039
149.8751	50000	10000	0.039984	0.343494	0.656506	0.0418	303.124
149.9326	50000	10000	0.010129	0.69323	0.30677	0.2315	791.4454
150	50000	10000	0.0027	0.888705	0.111295	0.4438	547.1677
150.0674	50000	10000	0.010129	0.69323	0.30677	0.2315	791.4454
150.1249	50000	10000	0.039984	0.343494	0.656506	0.0418	303.124
150.1718	50000	10000	0.099923	0.124741	0.875259	0.0048	45.0039
							2826.314
						C_in =	7.5
					Total cost		2833.814

Table:1.17 Values of terms in cost of model of equation (17) when roots are complex and conjugate λ(α1=0.8,α2=-0.6,n=10)=1.18

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.026667	0.973333	0.0048	47.35949
149.8751	50000	10000	0.039984	0.249732	0.750268	0.0418	334.4812
149.9326	50000	10000	0.010129	0.806643	0.193357	0.2315	542.1912
150	50000	10000	0.0027	0.992457	0.007543	0.4438	92.93368
150.0674	50000	10000	0.010129	0.806643	0.193357	0.2315	542.1912
150.1249	50000	10000	0.039984	0.249732	0.750268	0.0418	334.4812
150.1718	50000	10000	0.099923	0.026667	0.973333	0.0048	47.35949
							1940.997
						C_in =	15
					Total cost		1955.997

Table:1.18 Values of terms in cost of model of equation (17) when roots are complex and conjugate λ(α1=0.8,α2=-0.6,n=15)=1.06

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.004215	0.995785	0.0048	47.89877
149.8751	50000	10000	0.039984	0.178591	0.821409	0.0418	358.273
149.9326	50000	10000	0.010129	0.881032	0.118968	0.2315	378.7015
150	50000	10000	0.0027	0.99973	0.00027	0.4438	61.08921
150.0674	50000	10000	0.010129	0.881032	0.118968	0.2315	378.7015
150.1249	50000	10000	0.039984	0.178591	0.821409	0.0418	358.273
150.1718	50000	10000	0.099923	0.004215	0.995785	0.0048	47.89877
							1630.836
						C_in =	22.5
					Total cost		1653.336

Table:1.19 Values of terms in cost of model of equation (17) when roots are complex and conjugate λ(α1=0.8,α2=-0.6,n=20)=1.03

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.000873	0.999127	0.0048	47.97904
149.8751	50000	10000	0.039984	0.136944	0.863056	0.0418	372.2013
149.9326	50000	10000	0.010129	0.919606	0.080394	0.2315	293.9258
150	50000	10000	0.0027	0.999985	1.5E-05	0.4438	59.9741
150.0674	50000	10000	0.010129	0.919606	0.080394	0.2315	293.9258
150.1249	50000	10000	0.039984	0.136944	0.863056	0.0418	372.2013
150.1718	50000	10000	0.099923	0.000873	0.999127	0.0048	47.97904
							1488.186
						C_in =	30
					Total cost		1518.186

Table:1.20 Values of terms in cost of model of equation (17) when roots are complex and conjugate λ(α1=0.8,α2=-0.6,n=26)=1.01

μ	CaN	Cr	T1	T2	T3	T4	**Partial TC for row**
149.8282	50000	10000	0.099923	0.000136	0.999864	0.0048	47.99673
149.8751	50000	10000	0.039984	0.101646	0.898354	0.0418	384.0062
149.9326	50000	10000	0.010129	0.948521	0.051479	0.2315	230.3785
150	50000	10000	0.0027	1	4.82E-07	0.4438	59.91058
150.0674	50000	10000	0.010129	0.948521	0.051479	0.2315	230.3785
150.1249	50000	10000	0.039984	0.101646	0.898354	0.0418	384.0062
150.1718	50000	10000	0.099923	0.000136	0.999864	0.0048	47.99673
							1384.674
						C_in =	39
					Total cost		1423.674

Table:1.21 Values of terms in cost of model of equation (17) for different situations of autocorrelation

Sample size n	**no autocorrelation**	**real and equal**	**real and distinct**	**complex conjugate**
5	2186.2325	4948.487	6197.088	2833.814
10	1780.4575	4405.437	6918.483	1955.997
15	1602.808	3929.912	7212.725	1653.336
20	1496.1068	3572.309	7359.508	1518.186
26	1417.7	3255.055	7446.682	1423.674

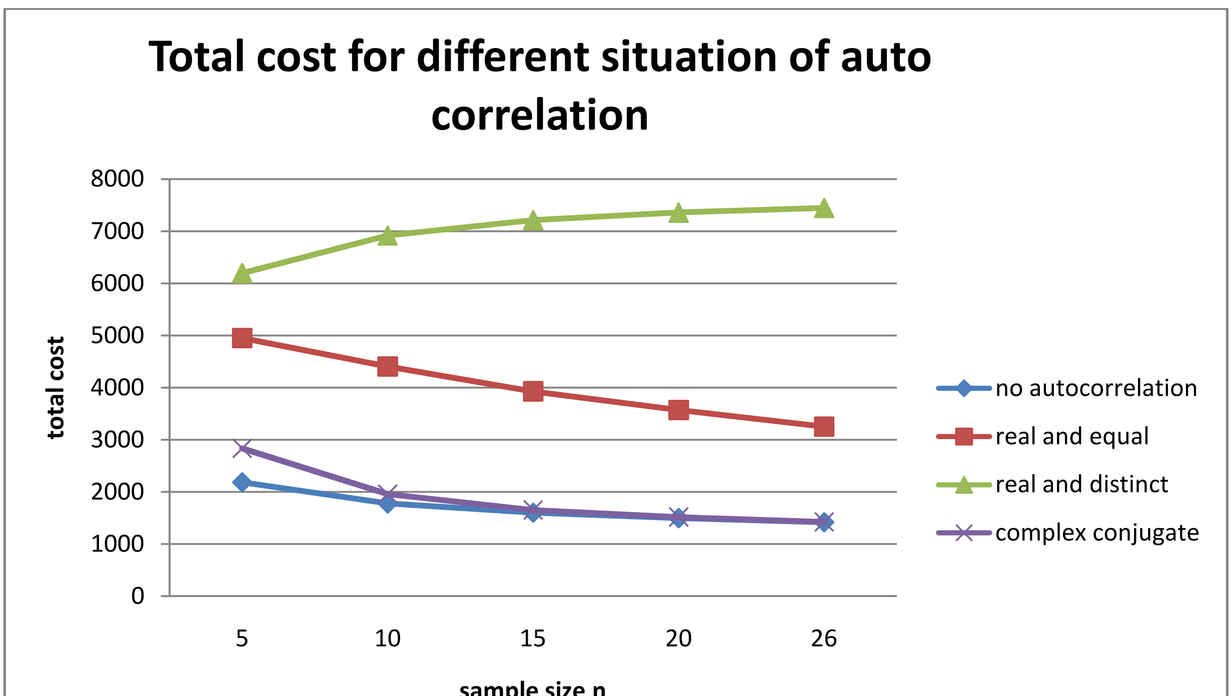

fig. 1.1

The importance of autocorrelation is clearly seen in table 1.1 to 1.20 and fig. 1.1. Fig 1.1 shows the when sample size increases total cost also increases when roots are real and distinct, decreases when roots are real and equal, complex conjugate and no autocorrelation is presented. For visual comparison between all four different situation total cost was found large in case of roots are real and distinct as compare to other three different situations (real and equal, complex conjugate and no auto correlation), total cost was found large as compare to other two different situation (complex conjugate and no auto correlation), and there was minor difference found between total cost when roots are complex conjugate and no autocorrelation present.

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