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Robustness to Non-Normality and Ar (2) Process of Control Charts

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Abstract- In this paper we investigate the effect of non-normality and auto-correlation on the OC function of mean chart with known coefficient of variation. We synthesize the second order auto-correlation process by its three different roots. In particular, the shift in the auto-correlation structure from independent data to a random walk, this is a special case of the structural shift occurring in the process. For various values of roots the values of OC functions are tabulated with known coefficient of variation.

Key words: Mean chart, non-normality, OC function, auto-correlation, coefficient of variation.

1. Introduction

Control charts are widely used to monitor manufacturing processes with the objective of detecting any change in a process that may affect the quality of the output. The statistic plotted on a control chart is frequently based on samples (subgroups) of n > 1 observations that are taken at regular sampling intervals. For example, a sample of n = 5 observations might be taken hourly. There are many practical situations, however, where control charts are applied using individual observations (n = 1), such as when repeated process measurements differ very little (as in many chemical and process industries) or when the rate of production is slow (Montgomery, 1997) and (Ryan , 2000) for other situations where using individual observations is appropriate. In most applications in which a continuous process characteristic, say x is being measured, it is assumed that x is approximately normally distributed, and that the observations from the process at different sampling times are independent random variables. If x is approximately normally distributed and a special cause affects the distribution of x, then the special cause may change the mean μ of x, the standard deviation σ of x, or both μ and σ. When individual observations are taken to monitor μ and σ, the standard practice to simultaneously use a Shewhart X chart for monitoring μ and a. Shewhart moving range (MR) chart for monitoring σ. These two Shewhart charts are simple to implement and effective in detecting large changes in μ or σ, but have two drawbacks which can seriously hinder their practical usefulness.

(i)The X and MR charts are ineffective in detecting small or moderate shifts in μ or σ. There is also recent evidence that there is little or no benefit to using an MR chart when an X chart for μ is also being used,(Albin et al., 1997) and (Reynolds and Stoumbos, 2000).

(ii)The statistical performance of the X and MR charts is very sensitive to deviation from the normality assumption, (Montgomery, 1997) and (Borror et al., 1999).

The usual independence assumption may be unrealistic in industries, such as the pharmaceutical and chemical industries, where many processes inherently produce auto correlated data. The presence of autocorrelation in the process data can result in considerable changes in the statistical performance of the X and MR charts, when these charts are developed under the assumption of independence, (Maragah and Woodall , 1992) , (Lu and Reynolds , 1999), and references there in efficient detection of small and moderate shifts in μ or σ requires that the control statistics in some way incorporate information from current and past sample statistics. Runs rules, which are based on patterns of points in a Shewhart chart, improve the ability of Shewhart charts to detect small and moderate shifts in μ or σ (Champ and Woodall, 1987) and (Lowary et al., 1995), but use of these rules is not the best method for detecting small and moderate shifts in μ or σ. A much better method of accumulating information across sampling points employs a control statistic that is an exponential weighted moving average (EWMA) of current and past sample statistics. (Reynolds and Stoumbos , 2001) considered various combination of Shewhart and other control charts, in order to determine which chart combinations are most effective for detecting shifts in μ or σ when single independent and normally distributed observations are taken. They showed that the common practice of using the Shewhart X and MR charts to monitor μ and σ is ineffective at detecting small and moderate shifts in these parameters, and that much better alternative for monitoring μ and σ are based on combinations of charts that involve at least one EWMA chart. There are many other contributions by (Amin and Ethridge, 1998), (Box et al., 1994), (Champ and Woodall , 1987), (Domangue and Patch, 1991), (Lucas and Saccucci , 1990), (Macgregor and Harris, 1993), (Reynolds, 1996b), (Regdon et al., 1994), (Stoumbos and Reynolds, 1997), (Sullivan and Woodall, 1996), and (Zhang , 1998) on control charts to detect shift in mean chart. If the underlying distribution is not normal, three different

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approaches are suggested to deal this problem. One approach is to transform the original data so that the transformed data more closely modeled by the normal distribution, and then proceed with the standard control charts using the transformed data. However, the difficulties of this approach is how to identify an appropriate transformation and justify the transformation appear more suitable than with the following alternatives. Another approach is to design control charts based on heuristic methods. Some heuristic control charts are developed for monitoring the non-normal process, for example, the weighted variance (WV) control chart proposed by (Chang and Bai,1995), the weighted standard deviation (WSD) control chart proposed by (Chang and Bai, 2001) and Skewness correction (SC) control chart proposed by (Chan and Cui, 2003). The design of heuristic control charts depend on some approximation procedures so that these control charts may not work well for some specified processes, for example, the skew normal process. A third approach is to increase the sample size until the distribution of the sample average is well modeled by the normal distribution, regardless of the amount of deviation from normality of individual units. Larger sample sizes, however, may not be operationally feasible and they are obviously more costly. Measurements come from production process often follow skewed distribution. This situation makes the standard $\overline{X}$ control chart results in a high false alam rate (Azzalini ,1985, 1986, 1999), (Bittanti et al., 1998), (Chang ,1994), (Chou et al.,2005), (Cowden, 1957), (Dodge and Rousson, 1999), (Genton et al., 2001), (Gunter,1989), (Gupta and Brown, 2001), and (Pyzdek, 1995)) for more details.

The objective of this paper to investigate the effect of non-normality and autocorrelation on the statistical performance of control charts for monitoring the mean μ with known *cv* of a continuous process characteristic x. For the numerical results that pertain to investigating the effects of non-normality, both heavy-tailed symmetric and skewed non-normal distributions are used as first four terms an Edgeworth series. While for the numerical result in this chapter that pertain to investigating the effect of autocorrelation, the observations are represented as a second order autoregressive (AR(2)) process. This time - series model is relatively simple, but it has been used in a wide variety of applications. The level of autocorrelation considered in the numerical results were chosen with the general objective of considering autocorrelation that is sufficiently high to have a significant effect on the performance of the charts, yet not so high that the patterns of points in a control chart look very unusual.

2. Materials And Methods

2.1 Second order Autoregressive mode

Consider a manufacturing process where a quality characteristic is measured at equidistance time points 1, 2, 3, ... n. This situation may occur in a discrete manufacturing process which produces discrete time 1, 2, 3, ... n, with one quality characteristic of interest. It may also occur in a continuous manufacturing process where the quality characteristic of interest is measured at discrete equidistant time points. We denote the behavior of the quality characteristic as x_1, x_2, ... x_n. It will assumed that on EPC control action can be represented by some controllable variable or factor x_t, such that

$$x_t = \mu + \xi_t, \qquad (1)$$

Where μ is a constant, and ξ_t is a stationary time series with zero mean and standard deviation σ. A (Durbin and Watson , 1950) "d" statistic can be used to detect the presence or absence of serial correlation. The problem, however, is that to do once the suspicion of dependence via the serial correlation test is confirmed. If serial correlation exist we use identification techniques to define the nature of ξ_t. When identification is complete, the likelihood function can provide maximum likelihood estimate of the parameters of the identified model.

Suppose that a correlation test revealed the presence of data dependence and identification technique suggested autoregressive model of order two AR(2) say, then we can express ξ_t of equation (1) as

$$\xi_t = \alpha_1\, \xi_{t-1} + \alpha_2\, \xi_{t-2} + \in_k,\ t = 1, 2, \ldots n \qquad (2)$$

where

(i) $\in_t \sim N\left(0,\ \sigma_\in{}^2\right)$

(ii) $\operatorname{cov}\left(\in_t, \in_\gamma\right) = \begin{cases} \sigma_\in{}^2 & t = \gamma \\ 0 & t \neq \gamma \end{cases}$

(3)

The Class of stationary models that assume the process to remain in equilibrium about a constant mean level μ. The variance of AR (2) process is given by:

$$\sigma^2 = \left(\frac{1-\alpha_2}{1+\alpha_2}\right) \frac{\sigma_\in{}^2}{\left[(1-\alpha_2)^2 - \alpha_1{}^2\right]}. \qquad (4)$$

Following Kendall and Stuart (1976) it can be shown that for stationarity, the roots of the characteristic equation of the process in equation (2)

$$\phi(B) = 1 - \alpha_1\, B - \alpha_2 B^2 \qquad (5)$$

must lies outside the unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions :

$$\begin{aligned} \alpha_2 + \alpha_1 &< 1 \\ \alpha_2 - \alpha_1 &< 1 \\ -1 < \alpha_2 &< 1 \end{aligned} \qquad (6)$$

Now If G_1^{-1} and G_2^{-1} are the roots of the characteristic equation of the process given by equation (5) then

$$G_1 = \frac{\alpha_1 + \sqrt{\alpha_1{}^2 + 4\alpha_2}}{2} \qquad (7)$$

$$G_2 = \frac{\alpha_1 - \sqrt{\alpha_1{}^2 + 4\alpha_2}}{2} \qquad (8)$$

For stationarity we require that $|G_i| < 1,\ i = 1, 2.$ Thus, three situations can theoretically arise :

(i) Roots G_1 and G_2 are real and distinct $\left(i.e., \alpha_1^2 + 4\alpha_2 > 0\right)$

(ii) Roots G_1 and G_2 are real and equal $\left(i.e., \alpha_1^2 + 4\alpha_2 = 0\right)$

(iii) Roots G_1 and G_2 are complex conjugate $\left(i.e., \alpha_1^2 + 4\alpha_2 < 0\right)$.

When the serial correlation is present in the data, we have for the distribution of the sample mean $\bar{x}$, its mean and variance is given by,

$$E\left(\bar{x}\right) = \mu$$

$$\text{Var}\left(\bar{x}\right) = \frac{\sigma^2}{n}\ \lambda_{ap}\left(\alpha_1, \alpha_2,\ n\right), \qquad (9)$$

where $\lambda_{ap}\left(\alpha_1, \alpha_2, n\right)$ depends on the nature of the roots G_1 and G_2 , and for different situations is given as follows :

(i) If G_1 and G_2 are real and distinct,

$$\lambda_{ap}\left(\alpha_1, \alpha_2, n\right) = \left[\frac{G_1\left(1-G_2^2\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\ \lambda\left(G_1, n\right) - \frac{G_2\left(1-G_1^{\,2}\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\ \lambda\left(G_2, n\right)\right]$$

$$= \lambda_{rd}\left(\alpha_1, \alpha_2, n\right), \qquad (10)$$

$$\text{Where, } \lambda\left(G, n\right) = \left[\frac{1+G}{1-G} - \frac{2G}{n}\ \frac{\left(1-G^n\right)}{\left(1-G\right)^2}\right]$$

(ii) If G_1 and G_2 are real and equal

$$\lambda_{ap}\left(\alpha_1, \alpha_2, n\right) = \left(\frac{1+G}{1-G}\right) - \frac{2G\left(1-G^n\right)}{n\left(1-G\right)^2}\left[1 + \frac{\left(1+G\right)^2\left(1-G^n\right) - n\left(1-G^2\right)\left(1+G^n\right)}{\left(1+G^2\right)\left(1-G^n\right)}\right]$$

$$= \lambda_{re}\left(\alpha_1, \alpha_2, n\right) \qquad (11)$$

(iii) If G_1 and G_2 and complex conjugate

$$\lambda_{ap}\left(\alpha_1, \alpha_2, n\right) = \left[\gamma\left(d, u\right) + \frac{2d}{n}\left(W\left(d,u,n\right) + z\left(d,u,n\right)\right)\right] \qquad (12)$$

$$= \lambda_{cc}\left(\alpha_1, \alpha_2, n\right)$$

$$\text{Where } \gamma\left(d, u\right) = \frac{1-d^4+2d\left(1-d^2\right)\cos u}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)},$$

$$W\left(d,u,n\right) = \frac{2d\left(1+d^2\right)\sin u - \left(1+d^4\right)\sin 2u - d^{n+4}\sin\left(\left(n-2\right)u\right)}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)^2\sin u},$$

$$Z\left(d,u,n\right) = \frac{2d^{\,n+3}\sin\left(n-1\right)u - 2d^{\,n+1}\sin\left(n+1\right)u + d^n\sin\left(\left(n+2\right)u\right)}{\left(1+d^2\right)\left(1+d^2-2d\cos u\right)^2\sin u},$$

$$d^2 = -\alpha_2,$$

$$\text{and} \qquad u = \cos^{-1}\left(\frac{\alpha_1}{2d}\right).$$

The x_t denote the change in the level of the compensating variable model at the time t, i.e., the adjustment made at the time point t. The ε_t is Gaussian white noise with variance $\sigma_\epsilon^{\,2}$. Throughout, we suppose that the noise variance is known. In practice, this is justified if reliable estimates of $\sigma_\epsilon^{\,2}$ are available from the evaluation of a large number of previous values of the process, e.g., during the setup phase. The real - valued parameters α_1 and α_2 (the autoregressive parameters) determines the influence of the preceding time point (t - 1) and (t - 2) on the present time point t. We assume an in-control value $\alpha_1 = \alpha_2 = 0$ for the autoregression parameters. It is possible that the autoregression parameters may shift to an out-of-control value $(\alpha_1,\alpha_2) \neq 0$.

2.2 The effect of Non-Normality and AR (2) process on control charts for monitoring the mean : We consider the effect of autocorrelation equation (9). The AR(2) process will be used to model the data taken from auto correlated process of interest with known *cv* Further, we assume that the non-normal population is represented by the first four terms of an Edgeworth series. To study the robustness of the control chart to non normality under AR(2) process, we examined the effect of non-normality and dependency on the OC and error of the first kind with known *cv* mean chart. We assumes that the observations $x_t \ (t = 1,2 \ldots n)$ are address the problem of non-normality and dependency with known *cv* in the control statistics $\bar{x}^*$. Following (Srivastava and Banarasi, 1982)

$$MSE \ (\bar{x}^*) = \frac{\sigma^2}{n} \left(1 - \frac{v * \lambda_{ap}(\alpha_1, \alpha_2, n)}{n}\right)$$

where $v = \frac{\sigma}{\mu}$. The OC of the control chart is derived from the sampling distribution of mean with the probability density function of the non-normal variables as the first four terms of an Edgeworth series. The control chart for mean is set up by drawing the control line at the process average μ and the control limits at $\mu \pm k \frac{\sigma}{\sqrt{n}}$, where *cv* is known. The OC function gives the probability that the control chart indicates the process average as μ when it is actually not μ but $\mu' = \mu + \gamma \sigma \frac{\sqrt{1 - \frac{v * \lambda_{ap} \ (\alpha_1, \alpha_2, n)}{n}}}{\sqrt{n}}$ (say) and it is device by integrating the distribution of mean with μ' as the process average between the limits of the control chart. In case of known *cv* the non-normal population is represented by the first four terms on Edgeworth series by (Rao and Bhatt, 1989) as,

$$f(x) = \frac{1}{\sigma}\left[\phi\left(\frac{x-\mu}{\sigma}\right) - \frac{\lambda_3}{6\sqrt{n}}\phi^{(3)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_4}{24n}\phi^{(4)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_3^2}{72n}\phi^{(6)}\left(\frac{x-4}{\sigma}\right)\right] \quad (13)$$

Following the $MSE(\bar{x}^*)$ the distribution of sample mean $\bar{x}^*$ is given by Gayen (1949), as

$$g\ (\bar{x}^*) = \frac{\sqrt{n'}}{\sigma}\left[\phi\left(\frac{\bar{x}^* - \mu}{\sigma/\sqrt{n'}}\right) - \frac{\lambda_3}{6\sqrt{n'}}\phi^{(3)}\left(\frac{\bar{x}^* - \mu}{\sigma/\sqrt{n'}}\right) + \frac{\lambda_4}{24n^1}\phi^{(4)}\left(\frac{\bar{x}^* - \mu}{\sigma/\sqrt{n'}}\right)\right]$$

$$+ \frac{\lambda_3^2}{72n'}\phi^{(6)}\left(\frac{\bar{x}^* - \mu}{\sigma/\sqrt{n'}}\right) \quad (14)$$

$$\text{where, } M^2 = \left(1 - \frac{v^* \lambda_{ap}\ (\alpha_1, \alpha_2, n)}{n}\right),\ n' = \frac{M^2}{n},\ \phi(x) = \frac{1}{\sqrt{2x}} \exp\left(\frac{-x^2}{2}\right)$$

$$\text{and} \qquad \phi^{(r)}(x) = \left(\frac{d}{dx}\right)^r \phi\ (x).$$

The OC function is obtained, after replacing μ in equation (14) by n' and integrating between the limits of control charts we get

$$L' = L_N - L'_u + L'_b \quad (15)$$

$$\text{Where, } L_N = \phi\left(\frac{k}{m} + \mu\right) + \phi\left(\frac{k}{m} - \mu\right) - 1, \phi(x) = \int_{-\infty}^{x} \phi\ (t)\ dt.$$

L_N is the OC of the control chart when the underlying population is normal with know *cv* and

$$L'_u = \frac{n'}{72}\left[\frac{12\ \lambda_3}{\sqrt{n'}}\phi^{(3)}\left(\frac{k}{m} - \mu\right) - 3\lambda_4\ \phi^{(3)}\left(\frac{k}{m} - \mu\right) - \lambda_3^2 \phi^{(5)}\left(\frac{k}{m} - \mu\right)\right] \quad (16)$$

$$L'_b = \frac{n'}{72}\left[\frac{12\ \lambda_3}{\sqrt{n'}}\phi^{(3)}\left(\frac{k}{m} + \mu\right) + 3\lambda_4\ \phi^{(3)}\left(\frac{k}{m} + \mu\right) + \lambda_3^2 \phi^{(5)}\left(\frac{k}{m} + \mu\right)\right] \quad (17)$$

$$\alpha' = 1 - \int_{\mu - k\,\sigma/\sqrt{n}}^{\mu + k\frac{\sigma}{\sqrt{n}}} g\left(\bar{x}^*\right) d\bar{x}^*$$

$$= \alpha_N - c \tag{18}$$

where,

$$\alpha_N = 2\Phi\left(-\frac{k}{m}\right) \tag{19}$$

and $$C = \frac{n'}{36}\left[3\lambda_4\, \phi^{(3)}\left(\frac{k}{m}\right) + \lambda_3^2\, \phi^{(5)}\left(\frac{k}{m}\right)\right] \tag{20}$$

is the non-normality correction for the error of the first kind.

The error of the first kind gives the probability of searching for assignable causes when is fact there are no such causes exist.

If α'_u is the probability of exceeding, the upper control limit and α'_b, that lying below the lower control limit, when the process is in control, then positively skewed population $\alpha'_u \geq \alpha'_b$ and negatively skewed population $\alpha'_u \leq \alpha'_b$ are given as the expressions for α'_u *and* α'_b are given as

$$\alpha'_u = \frac{\alpha}{2} + \frac{n'}{72}\left[12\,\lambda_3\sqrt{n'}\,\phi^{(2)}\left(\frac{k}{m}\right) - 3\lambda_4\,\phi^{(3)}\left(\frac{k}{m}\right) - \lambda_3^2\,\phi^{(5)}\left(\frac{k}{m}\right)\right] \tag{21}$$

$$\alpha'_b = \frac{\alpha}{2} - \frac{n'}{72}\left[12\,\lambda_3\sqrt{n'}\,\phi^{(2)}\left(\frac{k}{m}\right) + 3\lambda_4\,\phi^{(3)}\left(\frac{k}{m}\right) + \lambda_3^2\,\phi^{(5)}\left(\frac{k}{m}\right)\right] \tag{22}$$

it is evident from the above equation that α'_u becomes α'_b and α'_b becomes α'_u, when the sign of λ_3 changed. That is $\alpha' = \alpha'_u + \alpha'_b$ is independent of the sign of skewness.

(3) Results And Discussion

To investigate the robustness of the control chart schemes for the normality assumption, both heavy-tailed symmetric and skewed distribution i.e. (λ_3, λ_4) will be used. For non-normal with three different situation of autoregressive parameter along with independent observation, the values of type-I error and OC function have been computed and given in Table (1) and Table (4) respectively.

Table-1: Values of OC Function under AR(2) Process with Known cv for (n=7, k=1.0232)

v	Independent Observations (α_1=0.0, α_2=0.0)									
	λ_3=0,λ_4=0		λ_3=-0.6,λ_4=0		λ_3=0.6,λ_4=0		λ_3=0,λ_4=-1.0		λ_3=0,λ_4=2.0	
	p	L(p)	p	L(p)	p	L(p)	p	L(p)	p	L(p)
0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9500	0.0284	0.9572	0.0636	0.9439	0.0521	0.9497	0.0458	0.9506
	0.1000	0.7529	0.0902	0.7449	0.1127	0.7576	0.1127	0.7496	0.0746	0.7594
	0.1500	0.5140	0.1548	0.4988	0.1583	0.5289	0.1694	0.5137	0.1112	0.5145
	0.2000	0.3155	0.2181	0.3067	0.2018	0.3274	0.2225	0.3183	0.1550	0.3098
	0.2500	0.1781	0.2787	0.1779	0.2441	0.1808	0.2727	0.1812	0.2046	0.1720
	0.3000	0.0935	0.3364	0.0982	0.2860	0.0889	0.3207	0.0951	0.2586	0.0902
	0.3500	0.0457	0.3912	0.0518	0.3281	0.0384	0.3670	0.0459	0.3161	0.0454
	0.4000	0.0208	0.4432	0.0261	0.3709	0.0142	0.4120	0.0201	0.3761	0.0222
	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9549	0.0284	0.9620	0.0636	0.9489	0.0521	0.9550	0.0458	0.9551

0.4	0.1000	0.7593	0.0902	0.7520	0.1127	0.7635	0.1127	0.7623	0.0746	0.7654
	0.1500	0.5144	0.1548	0.4996	0.1583	0.5288	0.1694	0.5146	0.1112	0.5149
	0.2000	0.3104	0.2181	0.3021	0.2018	0.3217	0.2225	0.3077	0.1550	0.3050
	0.2500	0.1710	0.2787	0.1712	0.2441	0.1730	0.2727	0.1682	0.2046	0.1653
	0.3000	0.0871	0.3364	0.0920	0.2860	0.0821	0.3207	0.0857	0.2586	0.0843
	0.3500	0.0411	0.3912	0.0470	0.3281	0.0339	0.3670	0.0411	0.3161	0.0411
	0.4000	0.0180	0.4432	0.0229	0.3709	0.0119	0.4120	0.0187	0.3761	0.0194
0.8	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9597	0.0284	0.9667	0.0636	0.9540	0.0521	0.9598	0.0458	0.9596
	0.1000	0.7662	0.0902	0.7596	0.1127	0.7699	0.1127	0.7633	0.0746	0.7720
	0.1500	0.5148	0.1548	0.5005	0.1583	0.5289	0.1694	0.5146	0.1112	0.5153
	0.2000	0.3049	0.2181	0.2971	0.2018	0.3155	0.2225	0.3074	0.1550	0.2997
	0.2500	0.1635	0.2787	0.1641	0.2441	0.1648	0.2727	0.1661	0.2046	0.1583
	0.3000	0.0804	0.3364	0.0854	0.2860	0.0752	0.3207	0.0816	0.2586	0.0781
	0.3500	0.0365	0.3912	0.0421	0.3281	0.0296	0.3670	0.0363	0.3161	0.0368
	0.4000	0.0152	0.4432	0.0197	0.3709	0.0097	0.4120	0.0145	0.3761	0.0167
1.2	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9646	0.0284	0.9713	0.0636	0.9591	0.0521	0.9648	0.0458	0.9642
	0.1000	0.7737	0.0902	0.7678	0.1127	0.7769	0.1127	0.7709	0.0746	0.7791
	0.1500	0.5153	0.1548	0.5015	0.1583	0.5289	0.1694	0.5151	0.1112	0.5158
	0.2000	0.2988	0.2181	0.2915	0.2018	0.3088	0.2225	0.3013	0.1550	0.2939
	0.2500	0.1554	0.2787	0.1564	0.2441	0.1560	0.2727	0.1578	0.2046	0.1507
	0.3000	0.0736	0.3364	0.0787	0.2860	0.0681	0.3207	0.0745	0.2586	0.0718
	0.3500	0.0319	0.3912	0.0373	0.3281	0.0253	0.3670	0.0316	0.3161	0.0324
	0.4000	0.0126	0.4432	0.0167	0.3709	0.0077	0.4120	0.0119	0.3761	0.0141

Table-2: Values of OC Function under AR(2) Process with Known cv for (n=7, k=1.0232)

v	Roots are Real and Distinct (α_1=0.3, α_2=0.6)									
	λ_3=0,λ_4=0		λ_3=-0.6,λ_4=0		λ_3=0.6,λ_4=0		λ_3=0,λ_4=-1.0		λ_3=0,λ_4=2.0	
	p	L(p)	p	L(p)	p	L(p)	p	L(p)	p	L(p)
0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9500	0.0284	0.9572	0.0636	0.9439	0.0521	0.9497	0.0458	0.9506
	0.1000	0.7529	0.0902	0.7449	0.1127	0.7576	0.1127	0.7496	0.0746	0.7594
	0.1500	0.5140	0.1548	0.4988	0.1583	0.5289	0.1694	0.5137	0.1112	0.5145
	0.2000	0.3155	0.2181	0.3067	0.2018	0.3274	0.2225	0.3183	0.1550	0.3098
	0.2500	0.1781	0.2787	0.1779	0.2441	0.1808	0.2727	0.1812	0.2046	0.1720
	0.3000	0.0935	0.3364	0.0982	0.2860	0.0889	0.3207	0.0951	0.2586	0.0902
	0.3500	0.0457	0.3912	0.0518	0.3281	0.0384	0.3670	0.0459	0.3161	0.0454
	0.4000	0.0208	0.4432	0.0261	0.3709	0.0142	0.4120	0.0201	0.3761	0.0222
	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9760	0.0284	0.9817	0.0636	0.9714	0.0521	0.9765	0.0458	0.9752
	0.1000	0.7945	0.0902	0.7906	0.1127	0.7964	0.1127	0.7922	0.0746	0.7989
	0.1500	0.5168	0.1548	0.5042	0.1583	0.5292	0.1694	0.5166	0.1112	0.5172

0.4	0.2000	0.2817	0.2181	0.2758	0.2018	0.2899	0.2225	0.2839	0.1550	0.2774
	0.2500	0.1336	0.2787	0.1357	0.2441	0.1325	0.2727	0.1353	0.2046	0.1301
	0.3000	0.0562	0.3364	0.0613	0.2860	0.0505	0.3207	0.0566	0.2586	0.0555
	0.3500	0.0212	0.3912	0.0257	0.3281	0.0157	0.3670	0.0207	0.3161	0.0222
	0.4000	0.0072	0.4432	0.0101	0.3709	0.0038	0.4120	0.0066	0.3761	0.0084
0.8	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9961	0.0390	0.9900	0.0636	0.9945	0.0521	0.9964	0.0458	0.9900
	0.1000	0.8653	0.0902	0.8680	0.1127	0.8639	0.1127	0.8644	0.0746	0.8673
	0.1500	0.5226	0.1548	0.5132	0.1583	0.5318	0.1694	0.5224	0.1112	0.5229
	0.2000	0.2188	0.2181	0.2166	0.2018	0.2221	0.2225	0.2200	0.1550	0.2163
	0.2500	0.0680	0.2787	0.0716	0.2441	0.0641	0.2727	0.0683	0.2046	0.0673
	0.3000	0.0165	0.3364	0.0196	0.2860	0.0128	0.3207	0.0162	0.2586	0.0171
	0.3500	0.0032	0.3912	0.0046	0.3281	0.0017	0.3670	0.0029	0.3161	0.0037
	0.4000	0.0005	0.4432	0.0009	0.3709	0.0001	0.4120	0.0004	0.3761	0.0007
1.2	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	1.0000	0.0290	1.0000	0.0636	1.0000	0.0521	1.0000	0.0458	1.0000
	0.1000	0.9939	0.0902	0.9670	0.1127	0.9930	0.1127	0.9940	0.0746	0.9780
	0.1500	0.5511	0.1548	0.5470	0.1583	0.5551	0.1694	0.5510	0.1112	0.5512
	0.2000	0.0391	0.2181	0.0408	0.2018	0.0372	0.2225	0.0390	0.1550	0.0391
	0.2500	0.0004	0.2787	0.0005	0.2441	0.0002	0.2727	0.0003	0.2046	0.0004
	0.3000	0.0000	0.3364	0.0000	0.2860	0.0000	0.3207	0.0000	0.2586	0.0000
	0.3500	0.0000	0.3912	0.0000	0.3281	0.0000	0.3670	0.0000	0.3161	0.0000
	0.4000	0.0000	0.4432	0.0000	0.3709	0.0000	0.4120	0.0000	0.3761	0.0000

Table-3: Values of OC Function under AR(2) Process with Known cv for (n=7, k=1.0232)

v	Roots are Real and Equal (α_1=0.8, α_2=-0.16)									
	λ_3=0,λ_4=0		λ_3=-0.6,λ_4=0		λ_3=0.6,λ_4=0		λ_3=0,λ_4=-1.0		λ_3=0,λ_4=2.0	
	p	L(p)	p	L(p)	p	L(p)	p	L(p)	p	L(p)
0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9500	0.0284	0.9572	0.0636	0.9439	0.0521	0.9497	0.0458	0.9506
	0.1000	0.7529	0.0902	0.7449	0.1127	0.7576	0.1127	0.7496	0.0746	0.7594
	0.1500	0.5140	0.1548	0.4988	0.1583	0.5289	0.1694	0.5137	0.1112	0.5145
	0.2000	0.3155	0.2181	0.3067	0.2018	0.3274	0.2225	0.3183	0.1550	0.3098
	0.2500	0.1781	0.2787	0.1779	0.2441	0.1808	0.2727	0.1812	0.2046	0.1720
	0.3000	0.0935	0.3364	0.0982	0.2860	0.0889	0.3207	0.0951	0.2586	0.0902
	0.3500	0.0457	0.3912	0.0518	0.3281	0.0384	0.3670	0.0459	0.3161	0.0454
	0.4000	0.0208	0.4432	0.0261	0.3709	0.0142	0.4120	0.0201	0.3761	0.0222
0.4	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9651	0.0284	0.9717	0.0636	0.9596	0.0521	0.9653	0.0458	0.9647
	0.1000	0.7744	0.0902	0.7686	0.1127	0.7775	0.1127	0.7717	0.0746	0.7798
	0.1500	0.5154	0.1548	0.5016	0.1583	0.5289	0.1694	0.5152	0.1112	0.5158
	0.2000	0.2982	0.2181	0.2910	0.2018	0.3081	0.2225	0.3007	0.1550	0.2933
	0.2500	0.1546	0.2787	0.1557	0.2441	0.1552	0.2727	0.1570	0.2046	0.1500

	0.3000	0.0729	0.3364	0.0780	0.2860	0.0674	0.3207	0.0738	0.2586	0.0712
	0.3500	0.0314	0.3912	0.0368	0.3281	0.0249	0.3670	0.0311	0.3161	0.0320
	0.4000	0.0124	0.4432	0.0164	0.3709	0.0075	0.4120	0.0117	0.3761	0.0138
0.8	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9796	0.0284	0.9848	0.0636	0.9753	0.0521	0.9801	0.0458	0.9787
	0.1000	0.8024	0.0902	0.7991	0.1127	0.8038	0.1127	0.8003	0.0746	0.8065
	0.1500	0.5174	0.1548	0.5052	0.1583	0.5293	0.1694	0.5172	0.1112	0.5178
	0.2000	0.2751	0.2181	0.2696	0.2018	0.2827	0.2225	0.2771	0.1550	0.2710
	0.2500	0.1256	0.2787	0.1280	0.2441	0.1240	0.2727	0.1271	0.2046	0.1225
	0.3000	0.0504	0.3364	0.0553	0.2860	0.0447	0.3207	0.0506	0.2586	0.0500
	0.3500	0.0179	0.3912	0.0221	0.3281	0.0129	0.3670	0.0174	0.3161	0.0189
	0.4000	0.0057	0.4432	0.0082	0.3709	0.0028	0.4120	0.0051	0.3761	0.0067
1.2	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9918	0.0420	0.9820	0.0636	0.9892	0.0521	0.9922	0.0458	0.9910
	0.1000	0.8407	0.0902	0.8401	0.1127	0.8402	0.1127	0.8393	0.0746	0.8434
	0.1500	0.5204	0.1548	0.5100	0.1583	0.5306	0.1694	0.5202	0.1112	0.5207
	0.2000	0.2417	0.2181	0.2383	0.2018	0.2466	0.2225	0.2432	0.1550	0.2386
	0.2500	0.0892	0.2787	0.0926	0.2441	0.0858	0.2727	0.0899	0.2046	0.0877
	0.3000	0.0271	0.3364	0.0311	0.2860	0.0224	0.3207	0.0269	0.2586	0.0276
	0.3500	0.0069	0.3912	0.0093	0.3281	0.0042	0.3670	0.0065	0.3161	0.0077
	0.4000	0.0015	0.4432	0.0025	0.3709	0.0005	0.4120	0.0012	0.3761	0.0019

Table-4: Values of OC Function under AR(2) Process with Known cv for (n=7, k=1.0232)

v	Roots are Complex Conjugate (α_1=0.8, α_2=-0.6)									
	λ_3=0,λ_4=0		λ_3=-0.6,λ_4=0		λ_3=0.6,λ_4=0		λ_3=0,λ_4=-1.0		λ_3=0,λ_4=2.0	
	p	L(p)	p	L(p)	p	L(p)	p	L(p)	p	L(p)
0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9500	0.0284	0.9572	0.0636	0.9439	0.0521	0.9497	0.0458	0.9506
	0.1000	0.7529	0.0902	0.7449	0.1127	0.7576	0.1127	0.7496	0.0746	0.7594
	0.1500	0.5140	0.1548	0.4988	0.1583	0.5289	0.1694	0.5137	0.1112	0.5145
	0.2000	0.3155	0.2181	0.3067	0.2018	0.3274	0.2225	0.3183	0.1550	0.3098
	0.2500	0.1781	0.2787	0.1779	0.2441	0.1808	0.2727	0.1812	0.2046	0.1720
	0.3000	0.0935	0.3364	0.0982	0.2860	0.0889	0.3207	0.0951	0.2586	0.0902
	0.3500	0.0457	0.3912	0.0518	0.3281	0.0384	0.3670	0.0459	0.3161	0.0454
	0.4000	0.0208	0.4432	0.0261	0.3709	0.0142	0.4120	0.0201	0.3761	0.0222
0.4	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9556	0.0284	0.9627	0.0636	0.9497	0.0521	0.9555	0.0458	0.9558
	0.1000	0.7603	0.0902	0.7532	0.1127	0.7645	0.1127	0.7573	0.0746	0.7664
	0.1500	0.5145	0.1548	0.4998	0.1583	0.5288	0.1694	0.5142	0.1112	0.5149
	0.2000	0.3095	0.2181	0.3013	0.2018	0.3207	0.2225	0.3122	0.1550	0.3042
	0.2500	0.1698	0.2787	0.1701	0.2441	0.1717	0.2727	0.1726	0.2046	0.1642
	0.3000	0.0860	0.3364	0.0909	0.2860	0.0810	0.3207	0.0874	0.2586	0.0833
	0.3500	0.0404	0.3912	0.0462	0.3281	0.0332	0.3670	0.0403	0.3161	0.0404

	0.4000	0.0175	0.4432	0.0224	0.3709	0.0115	0.4120	0.0168	0.3761	0.0190
0.8	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9613	0.0284	0.9681	0.0636	0.9556	0.0521	0.9614	0.0458	0.9611
	0.1000	0.7685	0.0902	0.7622	0.1127	0.7721	0.1127	0.7657	0.0746	0.7742
	0.1500	0.5150	0.1548	0.5008	0.1583	0.5289	0.1694	0.5148	0.1112	0.5155
	0.2000	0.3030	0.2181	0.2954	0.2018	0.3134	0.2225	0.3055	0.1550	0.2979
	0.2500	0.1609	0.2787	0.1617	0.2441	0.1620	0.2727	0.1635	0.2046	0.1559
	0.3000	0.0782	0.3364	0.0833	0.2860	0.0729	0.3207	0.0793	0.2586	0.0761
	0.3500	0.0350	0.3912	0.0406	0.3281	0.0282	0.3670	0.0348	0.3161	0.0354
	0.4000	0.0144	0.4432	0.0187	0.3709	0.0090	0.4120	0.0136	0.3761	0.0158
1.2	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
	0.0500	0.9669	0.0284	0.9734	0.0636	0.9616	0.0521	0.9672	0.0458	0.9665
	0.1000	0.7775	0.0902	0.7720	0.1127	0.7804	0.1127	0.7748	0.0746	0.7827
	0.1500	0.5156	0.1548	0.5020	0.1583	0.5289	0.1694	0.5154	0.1112	0.5160
	0.2000	0.2957	0.2181	0.2887	0.2018	0.3053	0.2225	0.2981	0.1550	0.2909
	0.2500	0.1513	0.2787	0.1526	0.2441	0.1516	0.2727	0.1536	0.2046	0.1469
	0.3000	0.0702	0.3364	0.0753	0.2860	0.0646	0.3207	0.0710	0.2586	0.0686
	0.3500	0.0297	0.3912	0.0349	0.3281	0.0233	0.3670	0.0293	0.3161	0.0304
	0.4000	0.0114	0.4432	0.0153	0.3709	0.0068	0.4120	0.0107	0.3761	0.0128

The table(1) clearly indicates that the effect of autocorrelation and non-normality on type - I error is quite substantial and an increasing function of n. it is seen from table that for heavy tailed and skewed distributions, type - I error (false alarms) will occur much more often than anticipated. An excessive number of false alarms can lead to unneeded process adjustment, loss of confidence in the control chart scheme, and eventually low productivity and increased cost. From Table (2) it is observed that the OC function is very close to the independent observation while the OC function increases when the roots are (real and distinct, and real and equal) for increasing value of *v*. It is very difficult to imagine any applications where the process *cv* would be monitored alone without also monitoring the process mean. Thus, deviating from assumptions of normality and independence seemingly causes more harm to the performance of the $\bar{x}-chart$. The performance of $\bar{x}-chart$ is quite robust to low and moderate autocorrelation of course, these findings for the performance of the $\bar{x}-chart$ should be taken cautiously become of the increase in the number of false alarms resulting from the autocorrelation. In general, we conclude that autocorrelation and non-normality can have significant effect on the statistical performances of control charts for monitoring μ with known *cv*, even when the process parameters have been estimated with negligible error by a large enough sample.

REFERENCE

[1]. Albin, S. L., Kang, L., Shea, G. (**1997**). An X and EWMA chart for individual observations, Journal of quality Technology. **29**:**41–48**.

[2]. Amin,R.W. and Ethridge,R.A.(**1998**). A note on Individual and Moving Range Control Vhart, Journal of Quality Technology,**30**,**70-74**

[3]. Azzalini, A. and Capitanio, A. (**1999**). Statistical applications of the multivariate skew normal distribution, J. R. Statist. Soc. B. **61, 579–602**.

[4]. Azzalini, A. (**1985**). A class of distributions which includes the normal ones, Scand. J. Statist. **171–178.**

[5]. Azzalini, A. (**1986)**. Futher results on a class of distributions which includes the normal ones, Statistica **46**, **199–208**.

[6]. Bittanti, S., Lovera, M. and Moiraghi, L. (**1998**). Application of non-normal process capability indices to semiconductor quality control, IEEE Transactions On Semiconductor Manufacturing 11, **296–303**.

[7]. Borror, C. M., Montgomery, D. C., Runger, G. C. (**1999**). Robustness of the EWMA control chart to non-normality, Journal of quality Technology. **31:309–316**.

[8]. Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (**1994)**. Time Series Analysis: Forecasting and Control, 3rd ed. (Prentice-Hall: Englewood Cliffs, NJ).

[9]. Champ, C.W.,Woodall, W.H. (**1987**). Exact results for shewhart's control charts with supplementary runs rules, Journal of quality Technology **19:388–399**

[10]. Chan, L. K. and Cui, H. J. (**2003).** Skewness correction X and R charts for skewed distributions, Naval Research Logistics **50, 1–19**.

[11]. Chang, Y. S. and Bai, D. S. (**1995**). X and R control charts for skewed populations, Journal of Quality Technology **27**, **120–131**.

[12]. Chang, Y. S. and Bai, D. S. (**2001**). Control charts for positively-skewed populations with weighted standard deviations, Quality and Reliability Engineering International **17**, **397–406**.

[13]. Chang, W. (**1994)**. Practical implementation of the process capability indices, Quality Engineering 7, 239–259.

[14]. Chou, C.-Y. , Chen, C.-H and Liu, H.-R. (**2005**). Acceptance control charts for non-normal data, Journal of Applied Statistics **32, 25–36**.

[15]. Cowden, D. J. (**1957**). Statistical Method in Quality Control (Prentice-Hall, Englewood Cliffs, NJ,

[16]. Dodge, Y. and Rousson, V. (**1999**). The complications of the fourth central moment, The American Statistician **53, 267–269**.

[17]. Domangue, R., Patch, S.C. (**1991**). Some omnibus exponentially weighted moving average statistical process monitoring schemes,Technometrics **33:299–313**

[18]. Durbin, J. and Watson, G. S. (**1950**).Testing for serial correlation in least squares regression Biometrika, **37**.

[19]. Gayen, A. K. (**1949**). Mathematical Investigation in to effect of Non-normality on standard test, Ph.D. Thesis, Combrig University, USA.

[20]. Genton, M. G., He, L. and Liu, X. (**2001)**. Moments of skew-normal random vectors and their quadratic forms, Statistics and Probability Letters **51 319–325**.

[21]. Gunter, W. H. (**1989**). The use and abuse of Cpk, Quality Progress **22 108 109**.

[22]. Gupta, R. C. and Brown, N. (**2001**). Reliability studies of the skew-normal distribution and its application to a strength-stress model, Commun. Statist.-Theory Math. **30 2427–2445**.

[23]. Lowry, C.A., Champ, C.W., Woodall, W.H. (**1995**). The Performance of Control Charts for Monitoring Process Variation, Communications in Statistics Simulation and Computation, **24**(**2**)**, 409–437**.

[24]. Lu, C.W. and Reynolds, M.R. Jr. (**1999)**.Control charts for monitoring the mean and variance of autocorrelated processes, Journal of Quality Technology, **31, 259–274**.

[25]. Lucas, J.M., Saccucci, M.S. (**1990**). Exponential weighted moving average control schemes: properties and enhancements,Technometrics 32:1–12

[26]. MacGregor, J. F. and Harris, T. J. (**1993**). The Exponentially Weighted Moving Variance, Journal of Quality Technology 25: 106–118.

[27]. Maragah, H.O. and Woodall, W.H. (**1992**).The effects of autocorrelation on the retrospective X-chart, J. Statist. Compu. Simul**. 40, 29–42**.

[28]. Montgomery, D. C. (**1997**). Statistical Quality Control, 3rd ed. New York: John Wiley.

[29]. Pyzdek, T. (1995). Why normal distribution aren't – all that normal, Quality Engineering **7 769–777.**

[30]. Rao, K. A. and Bhatta, A. R. S. (**1989**). A note on test for coefficient of variation, University of Agricultural Science, Dharwad.

[31]. Reynolds, M. R. Jr. (**1996**). Variable Sampling Interval Control Charts with Sampling at Fixed Times, IIE Transactions **28: 497–510**.

[32]. Reynolds, M.R. Jr. and Stoumbos, Z.G. (**2001**). Monitoring the process mean and variance using individual observations and variable sampling intervals, Journal of Quality Technology, **33**, pp. **181–205**.

[33]. Rigdon, S. E., Cruthis, E. N., Champ, C. W. (**1994**). Design strategies for individuals and moving range control charts, Journal of quality Technology. **26:274–287**.

[34]. Ryan, T.P. (**2000**). Statistical Methods for Quality Improvements. John Wiley & Sons: New York,.

[35]. Srivastava, S. R. and Banarasi (**1980**). A note on the estimation of mean of asymmetrical population, Jar. Ind. Soc. Agri. Stat. **26(2), 33-36**.

[36]. Stoumbos, Z. G. and Reynolds, M. R. Jr. (**1997**). Control Charts Applying a Sequential Test at Fixed Sampling Intervals, Journal of Quality Technology **29: 21–40**.

[37]. Sullivan, J.H., Woodall, W.H. (**1996**). A control chart for preliminary analysis of individual observations, Journal of quality Technology, **28:265–278**

[38]. Zhang, N.F. (**1998)**. A statistical control chart for stationary process data, Technometrics, **40: 24 – 38**.