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# Decision Support Model with Default Risk under Conditional Delay

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***Abstract*****:** To establish long term relationship with the customers and survive in the business scenario the supplier frequently offers a credit period to its customers to make the payment. This attracts the new customer and hence boosts market demand. On the contrary the presence of credit period leads to default risk for the supplier. In this paper an inventory model is developed which deals with credit period dependent quadratic demand and default risk associated with sales revenue. Deterioration rate under natural environment is also incorporated with the inventory model. Seller's optimal decision to set the permissible credit period for the customer and purchase quantity is discussed here. For profit maximization, environmental concerns of deterioration are also incorporated in to purchasing decision. A solution procedure is given for finding the optimal solution of total profit. Numerical example is given to show the effectiveness of the model. Finally sensitivity analysis is carried out to explore the managerial implications.

***Keywords***: Inventory, Environment, Trade Credit, Default Risk, Deterioration

AMS classification No: MSC: 90B05

## I. INTRODUCTION

In any business organization, the control of inventory plays a vital role in smooth functioning of the enterprise as lack of it leads to shortage and excess of it leads to spoilage, damage and obsolete. Food grains, fruits, milk, vegetables, fertilizers, pesticides, drugs, radioactive chemicals are some examples of deteriorating items whose effectiveness reduces when stored for a long time resulting damage, spoilage, obsolete and ultimately degradation of the environment.

In addition to the environmental concerns of deterioration an EOQ model is developed when the buyer is offered a credit period to make the payment without paying any extra interest charges. To survive in the business scenario, the seller frequently offers a time period to the buyer to make the payment.Within this allowable period, the buyer can earn interest on the items he sold. If the credit period is too small then the buyer may not capable to make his payment which increases default risk. On the contrary larger credit period helps the buyer to make the payment and thus decreases default risk.

Researchers in different time periods have worked on inventory problem and developed new EOQ models. Goyal first developed an EOQ model for the buyer when the seller presents a fixed credit period [1]. Thereafter; many more articles on credit period came into existence. Jaggi et al. discussed the inventory model with credit period dependent demand under permissible delay in payments [5]. Shah and Pandey derived a model with constant demand with trade credit and effect of deterioration [6]. Shah et al. added effect of deterioration in Jaggi et al.'s model [7]. Shah et al. reviewed the inventory articles with trade credits [8]. Tripathy and Pradhan studied inventory model of constant demand with a permissible delayperiod in payment [9]. Lou and Wang worked on supply chain model between vendor and buyer with a permissible credit period [11]. Tripathy and Pradhan investigate an inventory model with ramp type demand under trade credit system [12]. Shah et al. has considered credit period with default risk to maximize profit [14].

Revised paper of the same was presented at $19^{th}$Odisha Bigyan 'O' Paribesh Congress (OBPC), organized by KIIT

 

University and Orissa Environmental Society, dated November 25-26, 2017.

Sujatha and Parvathi derived an EOQ model with weibull demand for variable decaying items when the buyer is offered a trade credit period [15]. Tripathy and Sukla have taken linear demand with permissible delay in payment in the inventory model [17].

In this paper, we consider the deteriorating items with credit period dependent quadratic demand .The purpose of the study is to gain maximum prof it per unit time with respect to credit period and cycle time for ordering of deteriorating items. Environmental concern with deterioration of perishable inventory is incorporated in the study.

The organization of the paper is started with the introduction part. Review of literature is mentioned here with a tabular form. Then notations and assumptions are given in the next section to understand the model. In third section, mathematical model with derivation followed by the solution procedure is given. In fourth section numerical example with graph is illustrated. Sensitivity analysis is carried out in fifth section to show the effect of changes of some system parameters. In the final section the conclusion is demonstrated with emphasizing on future scopes.

**Table 1: Table of comparison for review of literature**

| Author(s) & year of publication | Type of demand | Trade Credit | Default risk | Effect of deterioration |
|---|---|---|---|---|
| Jaggi et al.(2008) | Credit linked | Present | Absent | Absent |
| Shah &Pandey(2008) | Constant | Present | Absent | Present |
| Tripathy and Pradhan (2011) | Weibull | Present | Absent | Present |
| Tripathy and pradhan (2014) | Ramp | Present | Absent | Present |
| Shah et al.(2014) | Quadratic | Present | Default risk increases with time | Present |
| Sujatha & Parvathi(2015) | Weibull | Present | Absent | Present |
| Sarkar et al.(2015) | Credit period dependent | Present | Absent | Present |
| Tripathy &Mohanty(2017) | Exponential | Absent | Absent | Present |
| Tripathy and Sukla(2017) | Linear | Present | Absent | Absent |
| Present paper | Quadratic | Present | Default risk decreases with time | Present |

## II. NOTATIONS AND ASSUMPTIONS

We will take the following notations and assumptions in the proposed inventory model.

**Notations**

A: Ordering cost per order

p: Purchase cost per unit

s: Selling price per unit , $s > p$

h: Holding cost per unit per annum

M: Credit period offered by the seller to his buyer (a decision variable)

R (M, T): Time and credit period dependent annual demand rate

I (t): Inventory level at any instant of time t. $(0 \leq t \leq T)$

$\theta$ : Deterioration rate under natural environment

T: Cycle time (a decision variable)

Q: Seller's purchase quantity

$\emptyset$ (M, T): Seller's profit per unit time

**Assumptions**

1. The inventory model deals with single item.

 

2. The items in the inventory deteriorate at a constant rate. There is no repair or replacement of deteriorating items during the cycle time.

3. The replenishment rate is infinite.

4. Shortages are not allowed. Lead time is zero or negligible.

5. In global market, sellers keep selling price constant to bind his retailers.

6. Demand rate is considered as a function of time and credit period as

$$R(M,t) = a(1+bt-ct^2)M^{\beta} \quad \ldots(1)$$

Where a > 0 is scale demand, $0 \le b < 1$ denotes linear rate of change of demand with respect to time , $0 \le c < 1$ denotes quadratic rate of change of demand and $\beta > 0$ is constant.

7. For seller, default risk decreases if longer credit period is given to the buyer. Hence the rate of default risk for given credit period M is assumed to be

$$F(M) = 1 - M^{\gamma}, \text{ where } \gamma > 0 \text{ is a constant} \quad \ldots(2)$$

## III. MATHEMATICAL MODEL

The mathematical model deals with the perishable products deteriorate with a constant rate θ .The buyer is offered a credit period with a quadratic demand.

The inventory level at any instant of time t is governed by the differential equation

$$\frac{dI(t)}{dt} = -aM^{\beta}(1+bt-ct^2) - \theta I(t) \quad \ldots(3)$$

With initial condition I (T) = 0, the solution of differential equation (3) is given by

$$I(t) = (-1/\theta^3)\left[aM^{\beta}\right]\left[\theta^2(1+bt-ct^2) - \theta(b-2ct) - 2c + \left(\theta^2(cT^2-Tb-1) - 2\theta\left(-\frac{b}{2}+Tc\right) + 2c\right)e^{\theta T}.e^{-\theta t}\right] \quad \ldots(4)$$

At the beginning, the supplier has Q units in the inventory system. i .e

$$Q = I(0) = (-1/\theta^3)\left[aM^{\beta}\right]\left[\theta^2 - \theta b - 2c + e^{\theta T}\left\{\theta^2(cT^2 - Tb - 1) - 2\theta\left(-\frac{b}{2}+Tc\right) + 2c\right\}\right] \quad \ldots(5)$$

The different costs per cycle for the supplier are as follows.

Net revenue after default risk:

SR = Selling price*Total demand

$$= s\int_0^T R(M,t)dt M^{\gamma}$$

$$= (-1/6)\, sM^{\gamma+\beta} aT(2cT^2 - 3Tb - 6) \quad \ldots(6)$$

Purchase cost (PC) = p Q

Ordering cost (OC) = A

Holding cost (HC) = h $\int_0^T I(t)dt$

$$= -a\,(M^{\beta}h/\theta^3)\left[\theta^2T + \left(\frac{b}{2}\right)\theta^2T^2 - \left(\frac{c}{3}\right)\theta^2T^3 + \theta - b - \frac{2c}{\theta} + (c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2c}{\theta})e^{\theta T}\right] \quad \ldots(7)$$

The supplier's total profit per unit time is given by

$$\emptyset(M,T) = (I/T)[SR - PC - OC - HC] \quad \ldots(8)$$

$$= -\frac{1}{6}sM^{\gamma+\beta}a\,(2cT^2\text{-}3bT\text{-}6) - \frac{A}{T} + \frac{aM^{\beta}}{\theta^3T}[p\,(\theta^2 - \theta b - 2c) + h(\theta^2T + \frac{b}{2}\theta^2T^2 - \frac{C}{3}\theta^2T^3 + \theta - b - \frac{2c}{\theta})$$

$$+ e^{\theta T}\{p\theta^2(cT^2 - bT - 1) + p\theta b - 2p\theta cT + 2pc + h(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2C}{\theta})\}]$$

To find out the optimal value of total profit with respect to credit period and cycle time, the necessary conditions are

$$\frac{\partial\emptyset(M,T)}{\partial M} = 0 \quad \text{and} \quad \frac{\partial\emptyset(M,T)}{\partial T} = 0 \quad \ldots(9)$$

Equation (9) is equivalent to

$$(-\frac{1}{6}s\,a)\,(2cT^2\text{-}3\,b\,T\text{-}\,6)\,(\gamma+\beta)\,M^{\gamma+\beta-1} + \frac{a\beta M^{\beta-1}}{\theta^3T}[p\,(\theta^2 - \theta b - 2c) + h(\theta^2T + \frac{b}{2}\theta^2T^2 - \frac{C}{3}\theta^2T^3 + \theta - b - \frac{2c}{\theta}) + e^{\theta T}\{p\theta^2(cT^2 - bT - 1) + p\theta b - 2p\theta cT + 2pc + h(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2C}{\theta})\}] = 0$$

and

$$(-\frac{1}{6}sM^{\gamma+\beta})a(4cT-3b) + \frac{A}{T^2} + \frac{aM^\beta}{\theta^3 T}\Big[h(\theta^2 + b\theta^2 T - c\theta^2 T^2) + e^{\theta T}\{p\theta^2(2cT - b) - 2p\theta c + h(2c\theta T - b\theta - 2c)\} + \theta\, e^{\theta T}\Big\{p\theta^2(c\,T^2 - bT - 1) + p\theta b - 2p\theta cT + 2pc + h\Big(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2c}{\theta}\Big)\Big\}\Big]$$

$$- \frac{aM^\beta}{\theta^3 T^2}[p\,(\theta^2 - \theta b - 2c) + h(\theta^2 T + \frac{b}{2}\theta^2 T^2 - \frac{C}{3}\theta^2 T^3 + \theta - b - \frac{2c}{\theta}) + e^{\theta T}\{p\theta^2(cT^2 - bT - 1) + p\theta b - 2p\theta cT + 2pc + h(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2C}{\theta})\}] = 0$$

… (10)

Further to test the concavity of the total cost function Ø (M, T), the following conditions must be satisfied.

$$\frac{\partial^2 \emptyset(M,T)}{\partial M^2} > 0\,, \frac{\partial^2 \emptyset(M,T)}{\partial T^2} > 0\,,$$

and $(\frac{\partial^2 \emptyset(M,T)}{\partial M^2})\,(\frac{\partial^2 \emptyset(M,T)}{\partial T^2}) - (\frac{\partial^2 \emptyset(M,T)}{\partial M \partial T}) > 0$

To find out the solution for M and T in equation (9), we have to take the help of mathematical software because of the nonlinearity form .We have used here MATHEMATICA-5.1 to get the values of M and T.

The solution procedure is described as follows.

Step 1: Assign values to the inventory parameters

Step 2: Solve the simultaneous equations (10) with MATHEMATICA-5.1

Step 3: Test the conditions for sufficiency (concavity test)

Step 4: Work out profit $\emptyset(M,T)$ per unit time and purchase quantity Q from equation (8) and equation (5) respectively.

## IV. NUMERICAL EXAMPLE

Example: Let a = 1100 units, β = 5, b = 0.3, c = 0.1, γ = 4, p = $10 per unit, s = $25 per unit, h=$0.1 per unit per annum, A = $400 per order and θ = 0.1

With the above inventory parameters we found the optimal credit period M=1.47517 year and cycle time T=1.67554 for the supplier. This results in profit of $954883.00 with purchase of 16271.3 units.

The sufficiency condition i.e. the second order partial derivatives w .r .t. M and T are tested that reflects the concavity nature of total profit with respect to cycle time T and credit period M.

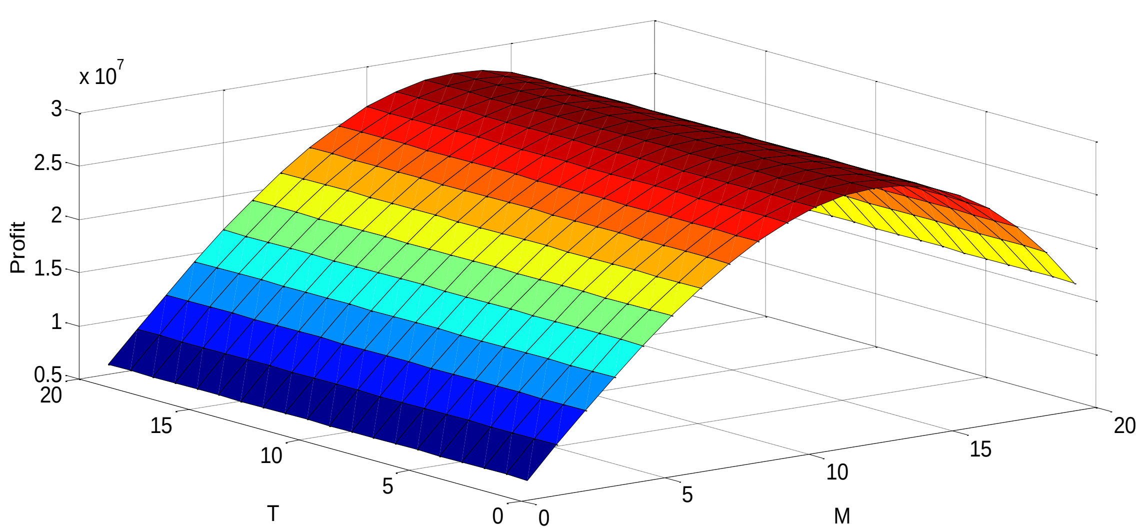


Fig.1 Total Profit w .r .t . Cycle Time (T) and Credit Period (M)

## V. SENSITIVITY ANALYSIS

**Table 2**

| **Parameters** | ***% change*** | ***T*** | ***M*** | ***∅ (M,T)*** | ***Q*** |
|---|---|---|---|---|---|
| $a$ | -5 | 1.67562 | 1.47517 | 954885 | 16272.2 |
| | 5 | 1.67546 | 1.47517 | 954881 | 16270.4 |
| | 10 | 1.67539 | 1.47517 | 954879 | 16269.6 |
| | -10 | 1.67572 | 1.47516 | 954827 | 16272.7 |
| $\boldsymbol{\beta}$ | -5 | 0.5306 | 1.58657 | 1751740 | 21113.2 |
| | 5 | 1.68363 | 1.48285 | 1002910 | 16791.5 |
| | 10 | 0.49652 | 1.60919 | 1990350 | 22085.3 |
| | -10 | 1.65724 | 1.45801 | 854754 | 15155.1 |
| $b$ | -5 | 1.60234 | 1.47831 | 972076 | 15626.1 |
| | 5 | 1.75017 | 1.47184 | 936535 | 16909.8 |
| | 10 | 1.82608 | 1.46833 | 917140 | 17538.3 |
| | -10 | 1.53071 | 1.48124 | 987944 | 14976.9 |
| $c$ | -5 | 1.76613 | 1.45605 | 846266 | 16188.9 |
| | 5 | 1.59385 | 1.49359 | 1070420 | 16350.8 |
| | 10 | 1.5198 | 1.51138 | 1193050 | 16428.8 |
| | -10 | 1.86717 | 1.43618 | 744557 | 16103.4 |
| $\gamma$ | -5 | 1.68403 | 1.51434 | 1221590 | 18656.8 |
| | 5 | 1.66721 | 1.44077 | 764118 | 14378.4 |
| | 10 | 1.65903 | 1.41035 | 624008 | 12850.7 |
| | -10 | 1.6927 | 1.5593 | 1606120 | 21722.9 |
| $p$ | -5 | 1.67245 | 1.45802 | 855170 | 15314.8 |
| | 5 | 1.67836 | 1.49174 | 1060720 | 17239 |
| | 10 | 1.68095 | 1.50777 | 1172710 | 18217.4 |
| | -10 | -0.514696 | 1.55653 | - | - |
| $s$ | -5 | 1.67544 | 1.49421 | 1077240 | 17347.6 |
| | 5 | 1.67564 | 1.45728 | 851157 | 15309.3 |
| | 10 | -0.51781 | 1.55706 | - | - |
| | -10 | 1.67534 | 1.51455 | 1222880 | 18559.7 |
| $h$ | -5 | 1.67868 | 1.47357 | 945258 | 16217.9 |
| | 5 | 1.67242 | 1.47676 | 964532 | 16324.3 |
| | 10 | 1.66933 | 1.47835 | 974265 | 16377.7 |
| | -10 | 1.68184 | 1.47197 | 935718 | 16164.8 |
| $A$ | -5 | 1.67546 | 1.47517 | 954881 | 16270.4 |
| | 5 | 1.67562 | 1.47517 | 954885 | 16272.2 |
| | 10 | 1.6757 | 1.47516 | 954827 | 16272.5 |
| | -10 | 1.67538 | 1.47517 | 954879 | 16269.5 |
| $\theta$ | -5 | 1.65961 | 1.51256 | 1207370 | 18240.2 |
| | 5 | 1.69415 | 1.44067 | 764161 | 14639.2 |
| | 10 | 1.71523 | 1.40872 | 618069 | 13272.6 |
| | -10 | -0.437814 | 1.6691 | - | - |

We make sensitivity analysis of the model by changing the parameter values of a ,β ,b ,c ,γ ,p ,s ,h ,A and θ at a percentage of -5%,5%,10%,-10% and observe its effects on the cycle time (T) ,credit period (M), total profit ∅ (M, T), and purchase quantity Q. The variations are tabulated above.

The keen observations of the table can be summarized below

.

1. Variation in the values of a and b has a little impact on T, M, π (M, T), and Q.
2. Variation in β value has a great impact on T, M, ∅ (M, T), and Q.
3. Increment in the value of c, results in decrement in T, but increment in M, ∅ (M, T), and Q.
4. Increment of the value of γ, results in decrement in T, M, ∅ (M, T), and Q.

5. There should be a definite gap between cost of purchase per unit (p) and selling price per unit else it will give irrelevant result.
6. Holding cost per unit (h) has a very little effect on decision parameters.
7. Increment of the value of θ results in decrement in total profit and purchase quantity.

## VI. CONCLUSION

In this research paper an inventory model with deteriorating items is considered with quadratic demand. The model is analyzed with a permissible credit period. Default risk is considered to be decreased with increase in time. An optimal credit period is found that maximizes the total profit per unit time of the system. This EOQ model enables in preventing deterioration of the inventory by increasing credit period and order size that ensures environmental concern. Hence for higher demand rate it is advised to the seller to give more credit period to get greater order.

This study will help significantly to the seller for setting optimal credit period. In future this research can be extended to study inventory models with stochastic demand. This paper can be extended if we allow shortages. Also different preservation technologies can be incorporated to reduce deterioration and enhance environment protection.

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Note: Utkal University is an A+ grade University as per NAAC and tier 1 University by the UGC .It is the oldest university of the state and the mother University of the State.

 