Two Phase Sampling Exponential Type Estimators for Ratio and Product of two Population Means in the Presence of Non-response

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Abstract— In this paper, two phase sampling exponential type estimators for ratio and product of two population means in the presence of non-response have been proposed and the expressions for the mean suare error of the proposed estimators for ratio and product of two population means in case of fixed sample sizes and also in case of fixed cost, are obtained. The expressions for optimum values of sample sizes are obtained in case of fixed cost and also in case of specified variance. The proposed estimators have been found to be more efficient than the relevant estimators for the fixed values of sample sizes, under the specified conditions. The proposed estimators are also more efficient than the relevant estimators in case of the fixed cost and have less total cost in comparison to the cost incurred in case of relevant estimators for the specified variance. Empirical studies have been given in support of the problem under investigation.

I.   INTRODUCTION

Estimation of ratio and product of two population means is widely used in the field of agriculture, socio-economics and medical sciences. A lot of research works for estimating the ratio and product of two population means using one auxiliary character, have been done by Singh [1, 2, 3], Shah and Shah [4], Tripathi [5], Singh [6], Singh [7], Birader and Singh [8] and Upadhayay *et al.*[9].

For estimating the population mean by using sample values sometimes the information on all units selected in the sample is not collected due to the problem of non-response which occurs due to lack of interest, not at home, refusal and lack of the knowledge regarding to the surveys. Hansen and Hurwitz [10] first suggested a technique of sub sampling from non-respondent to deal with the problem of non-response. Further by using the known population mean of auxiliary character, various ratio, product and regression type estimators for population mean in the presence of non-response have been proposed by Rao [11,12], Khare and Srivastava [13,14], Singh et al. [15], Kumar and Bhougal [16] and Kumar and Kumar [17]. Sometimes the population mean of auxiliary character is not known. In this situation, two phase sampling ratio, product and regression type estimators have been proposed by Khare and Srivastava [18, 19], Singh and Kumar [20] and Khare et al. [21]. The research works for estimating the ratio of two population means using auxiliary characters in the presence of non-response have also been

done by Khare and Pandey [22] and Khare and Sinha [23, 24].

In this paper, we have proposed two phase sampling exponential type estimators for ratio and product of two population means in the presence of non-response. The expressions for the mean suare error of the proposed estimators for ratio and product of two population means in case of fixed sample sizes are obtained. The conditions in which proposed estimators are more efficient in comprision to the relevant estimators are obtained. The expressions for the optimum values of sample sizes are also obtained in case of fixed cost and also in case of specified variance. Empirical studies are also given to show the performance of the proposed estimators in comprision to the relevant estimators.

II.   THE ESTIMATORS

Let (y_1, y_2) denote study characters and x denote the auxiliary character having *lth* values $Y_{il}, ; i = 1, 2,$ $X_l; l = 1, 2, ..., N$ with their population means $\bar{Y}_1, \bar{Y}_2$ and $\bar{X}$ respectively. The population is supposed to be divided in N_1 responding and N_2 non-responding units such that $N_1 + N_2 = N$. By using the Hansen and Hurwitz [10] technique's, a sample of size $n(< N)$ is drawn from the population of size N by simple random sampling without

replacement (SRSWOR) method of sampling and found that n_1 units respond and n_2 units do not respond in a sample of size n for study character $y_i ; i = 1,2$. Further from n_2 non-responding units, a subsample of size $m(= n_2 / k, k > 1)$ is drawn by using SRSWOR method of sampling and the information on m units is collected by personal interview. By using the Hansen and Hurwitz [10] technique's, the estimator for the population mean $\bar{Y}_i$ of study character y_i based on $n_1 + m$ units is defined as follows:

$$\bar{y}_i^* = \frac{n_1}{n} \bar{y}_{i(1)} + \frac{n_2}{n} \bar{y}_{i(2)}', i = 1, 2, \qquad (2.1)$$

where $\bar{y}_{i(1)}$ and $\bar{y}_{i(2)}'$ denote the means of study character y_i based on n_1 and m units respectively.

The mean square error (MSE) of the estimator $\bar{y}_i^* : i = 1, 2$ is obtained as:

$$MSE(\bar{y}_i^*) = \frac{f}{n} S_{yi}^2 + \frac{W_2(k-1)}{n} S_{yi}'^2, \qquad (2.2)$$

where $f = 1 - \dfrac{n}{N}$, $W_2 = \dfrac{N_2}{N}$, $(S_{yi}^2, S_{yi}'^2)$ are population mean squares of study character $y_i : i = 1, 2$ for the entire population and for the non responding part of the population. Hence the conventional estimators for the ratio of two population means $R(= \bar{Y}_1 / \bar{Y}_2)$ and for the product of two population means $P(= \bar{Y}_1 * \bar{Y}_2)$ are defined as:

$$\hat{R} = \frac{\bar{y}_1^*}{\bar{y}_2^*} \quad \text{and} \quad \hat{P} = \bar{y}_1^* * \bar{y}_2^* \qquad (2.3)$$

When the population mean $\bar{X}$ of auxiliary character x is not known, a first phase sample of size $n'(< N)$ is drawn from population of size N by using simple random sampling without replacement (SRSWOR) method of sampling and collects the information on n' units for auxiliary character x. Further, a second phase sample of size $n(< n')$ is drawn from first phase sample of size n' by using simple random sampling without replacement (SRSWOR) method of sampling and found that n_1 units are responding and n_2 units are not responding in a sample of size n for study character y_i. Again a subsample of size $m(= n_2 / k, k > 1)$ is drawn from n_2 responding units and collects the information on m units for study character y_i by personal interview.

Now we define the estimator for population mean $\bar{X}$ of auxiliary character x based on $n_1 + m$ units corresponding to $n_1 + m$ units of study character y_i which is given as:

$$\bar{x}^* = \frac{n_1}{n} \bar{x}_1 + \frac{n_2}{n} \bar{x}_2' , \qquad (2.4)$$

where $\bar{x}_1$ and $\bar{x}_2'$ denote the means of auxiliary character x based on n_1 and m units corresponding to n_1 and m units of study character y_i.

The mean square error (MSE) of the estimator $\bar{x}^*$ is obtained as:

$$MSE(\bar{x}^*) = \frac{f}{n} S_x^2 + \frac{W_2(k-1)}{n} S_x'^2 , \qquad (2.5)$$

where $(S_x^2, S_x'^2)$ are population mean squares of auxiliary character x for the entire population and for the non responding part of the population.

The classical ratio and product estimators are useful only when the linear relationship between study character and auxiliary character is very strong. Sometimes the linear relationship between study character and auxiliary character is not very strong. In this situation, Singh and Vishwakarma [25] proposed the two phase sampling exponential ratio (t_R) and exponential product (t_P) estimators for the population mean which are given as follows:

$$t_R = \bar{y}_1 \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \qquad (2.6)$$

and

$$t_P = \bar{y}_1 \exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right), \qquad (2.7)$$

where $\bar{x} = \dfrac{1}{n} \sum_{l=1}^{n} x_l$ and $\bar{x}' = \dfrac{1}{n'} \sum_{l=1}^{n'} x_l$.

Further Tailor et al. [26] and Lakhre [27] studied Singh and Vishwakarma [25] ratio and product type exponential estimators in double sampling for stratification.

Now using two phase sampling exponential product and exponential ratio estimators, we propose conventional two phase sampling exponential type estimators for ratio and product of two population means in the presence of non-response which are given as follows:

$$t_{TR1} = \frac{\bar{y}_1^*}{\bar{y}_2^*} \exp\left(\frac{\bar{x}^* - \bar{x}'}{\bar{x}^* + \bar{x}'}\right) \qquad (2.8)$$

and

$$t_{TP1} = \bar{y}_1^* * \bar{y}_2^* \exp\left(\frac{\bar{x}' - \bar{x}^*}{\bar{x}' + \bar{x}^*}\right) \qquad (2.9)$$

Using two phase sampling exponential product and exponential ratio estimators, we propose the alternate two phase sampling exponential type estimators for ratio and product of two population means in the presence of non-response which are given as follows:

$$t_{TR2} = \frac{\bar{y}_1^*}{\bar{y}_2^*} \exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right) \tag{2.10}$$

and

$$t_{TP2} = \bar{y}_1^* * \bar{y}_2^* \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \tag{2.11}$$

III. THE EXPRESSIONS FOR THE MEAN SQUARE ERROR (MSE) OF THE PROPOSED ESTIMATORS

In order to derive the expressions for the mean square error of the proposed estimators.

Le $\bar{y}_1^* = \bar{Y}_1(1 + \varepsilon_0)$, $\bar{y}_2^* = \bar{Y}_2(1 + \varepsilon_1)$, $\bar{x}^* = \bar{X}(1 + \varepsilon_2)$, $\bar{x} = \bar{X}(1 + \varepsilon_3)$, $\bar{x}' = \bar{X}(1 + \varepsilon_4)$ Such that

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_3) = E(\varepsilon_4) = 0.$$

By using simple random sampling without replacement method of sampling, we have

$$E(\varepsilon_0^2) = \frac{1}{\bar{Y}_1^2} V(\bar{y}_1^*) = \frac{1}{\bar{Y}_1^2} \left\{ \frac{f}{n} S_{y1}^2 + \frac{W_2(k-1)}{n} S_{y1}'^2 \right\},$$

$$E(\varepsilon_1^2) = \frac{1}{\bar{Y}_2^2} V(\bar{y}_2^*) = \frac{1}{\bar{Y}_2^2} \left\{ \frac{f}{n} S_{y2}^2 + \frac{W_2(k-1)}{n} S_{y2}'^2 \right\},$$

$$E(\varepsilon_2^2) = \frac{1}{\bar{X}^2} V(\bar{x}^*) = \frac{1}{\bar{X}^2} \left\{ \frac{f}{n} S_x^2 + \frac{W_2(k-1)}{n} S_x'^2 \right\},$$

$$E(\varepsilon_3^2) = \frac{1}{\bar{X}^2} V(\bar{x}) = \frac{1}{\bar{X}^2} \frac{f}{n} S_x^2,$$

$$E(\varepsilon_4^2) = \frac{1}{\bar{X}^2} V(\bar{x}') = \frac{1}{\bar{X}^2} \frac{f'}{n'} S_x^2,$$

$$E(\varepsilon_0 \varepsilon_1) = \frac{1}{\bar{Y}_1 . \bar{Y}_2} Cov(\bar{y}_1^*, \bar{y}_2^*)$$

$$= \frac{1}{\bar{Y}_1 \bar{Y}_2} \left\{ \frac{f}{n} S_{y1y2} + \frac{W_2(k-1)}{n} S_{y1y2}' \right\},$$

$$E(\varepsilon_0 \varepsilon_2) = \frac{1}{\bar{Y}_1 . \bar{X}} Cov(\bar{y}_1^*, \bar{x}^*)$$

$$= \frac{1}{\bar{Y}_1 \bar{X}} \left\{ \frac{f}{n} S_{y1x} + \frac{W_2(k-1)}{n} S_{y1x}' \right\},$$

$$E(\varepsilon_1 \varepsilon_2) = \frac{1}{\bar{Y}_2 . \bar{X}} Cov(\bar{y}_2^*, \bar{x}^*)$$

$$= \frac{1}{\bar{Y}_2 \bar{X}} \left\{ \frac{f}{n} S_{y2x} + \frac{W_2(k-1)}{n} S_{y2x}' \right\},$$

$$E(\varepsilon_0 \varepsilon_3) = \frac{1}{\bar{Y}_1 . \bar{X}} Cov(\bar{y}_1, \bar{x}) \quad = \frac{1}{\bar{Y}_1 \bar{X}} \frac{f}{n} S_{y1x},$$

$$E(\varepsilon_1 \varepsilon_3) = \frac{1}{\bar{Y}_2 . \bar{X}} Cov(\bar{y}_2, \bar{x}) \quad = \frac{1}{\bar{Y}_2 \bar{X}} \frac{f}{n} S_{y2x},$$

$$E(\varepsilon_0 \varepsilon_4) = \frac{1}{\bar{Y}_1 . \bar{X}} Cov(\bar{y}_1, \bar{x}') \quad = \frac{1}{\bar{Y}_1 \bar{X}} \frac{f'}{n'} S_{y1x},$$

$$E(\varepsilon_1 \varepsilon_4) = \frac{1}{\bar{Y}_2 . \bar{X}} Cov(\bar{y}_2, \bar{x}') \quad = \frac{1}{\bar{Y}_2 \bar{X}} \frac{f'}{n'} S_{y2x},$$

$$E(\varepsilon_2 \varepsilon_4) = \frac{1}{\bar{X} . \bar{X}} Cov(\bar{x}^*, \bar{x}') \quad = \frac{1}{\bar{X}^2} \frac{f'}{n'} S_x^2,$$

$$E(\varepsilon_3 \varepsilon_4) = \frac{1}{\bar{X} . \bar{X}} Cov(\bar{x}, \bar{x}') = \frac{1}{\bar{X}^2} \frac{f'}{n'} S_x^2,$$

where $f' = 1 - \frac{n'}{N}$.

Using the large sample approximations, the expressions for the mean square error of the proposed estimators t_{TR1}, t_{TP1}, t_{TR2} and t_{TP2} up to the terms of order $(1/n)$ are given as follows:

$$MSE(t_{TR1}) = MSE(\hat{R}) + R^2 \left[A_1 \left\{ \frac{1}{4} C_x^2 + \left(C_{y1x} - C_{y2x}\right) \right\} \right.$$
$$\left. + A_2 \left\{ \frac{1}{4} C_x'^2 + \left(C_{y1x}' - C_{y2x}'\right) \right\} \right], \tag{3.1}$$

$$MSE(t_{TP1}) = MSE(\hat{P}) + P^2 \left[A_1 \left\{ \frac{1}{4} C_x^2 - \left(C_{y1x} + C_{y2x}\right) \right\} \right.$$
$$\left. + A_2 \left\{ \frac{1}{4} C_x'^2 - \left(C_{y1x}' + C_{y2x}'\right) \right\} \right], \tag{3.2}$$

$$MSE(t_{TR2}) = MSE(\hat{R}) + R^2 \left[A_1 \left\{ \frac{1}{4} C_x^2 + \left(C_{y1x} - C_{y2x}\right) \right\} \right] \tag{3.3}$$

and

$$MSE(t_{TP2}) = MSE(\hat{P}) + P^2 \left[A_1 \left\{ \frac{1}{4} C_x^2 - \left(C_{y1x} + C_{y2x}\right) \right\} \right], \tag{3.4}$$

where

$$MSE(\hat{R}) = R^2 \left[\frac{f}{n} \left(C_{y1}^2 + C_{y2}^2 - 2C_{y1y2}\right) \right.$$

$$+ \frac{W_2(k-1)}{n}\left(C_{y1}'^2 + C_{y2}'^2 - 2C_{y1y2}'\right)\Bigg], \quad (3.5)$$

$$MSE(\hat{P}) = P^2\Bigg[\frac{f}{n}\left(C_{y1}^2 + C_{y2}^2 + 2C_{y1y2}\right)$$

$$+ \frac{W_2(k-1)}{n}\left(C_{y1}'^2 + C_{y2}'^2 + 2C_{y1y2}'\right)\Bigg], \quad (3.6)$$

$$A_1 = \frac{1}{n} - \frac{1}{n'}, \qquad A_2 = \frac{W_2(k-1)}{n}, \quad C_{y1y2} = \frac{S_{y1y2}}{\bar{Y}_1\bar{Y}_2},$$

$$C_{y1y2}' = \frac{S_{y1y2}'}{\bar{Y}_1\bar{Y}_2}, \ S_{y1y2} = \rho_{y1y2}S_{y1}S_{y2}, \ S_{y1y2}' = \rho_{y1y2}'S_{y1}'S_{y2}',$$

$$C_{yi} = \frac{S_{yi}}{\bar{Y}_i}, \quad C_x = \frac{S_x}{\bar{X}}, \quad C_{yi}' = \frac{S_{yi}'}{\bar{Y}_i}, \quad C_x' = \frac{S_x'}{\bar{X}},$$

$(\rho_{y1y2}, \rho_{y1y2}')$ are correlation coefficients between two study characters y_1 and y_2 for responding and non-responding parts of the population and $(\rho_{yix}, \rho_{yix}')$ are the correlation coefficients between $y_i\,(i=1,2)$ and x for the entire population and for the non-responding part of the population.

IV. COMPARISONS OF THE PROPOSED ESTIMATORS WITH RELEVANT ESTIMATORS

Comparing the proposed estimators (t_{TR1}, t_{TR2}) and (t_{TP1}, t_{TP2}) with respect to $\hat{R}$ and $\hat{P}$.

$$MSE(t_{TR1}) < MSE(\hat{R}) \quad \text{If} \quad (4.1)$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} - \rho_{y2x}\frac{C_{y2}}{C_x}\right) < -\frac{1}{4}$$

and $\quad \left(\rho_{y1x}'\frac{C_{y1}'}{C_x'} - \rho_{y2x}'\frac{C_{y2}'}{C_x'}\right) < -\frac{1}{4}$,

$$MSE(t_{TR2}) < MSE(\hat{R}) \quad \text{If} \quad (4.2)$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} - \rho_{y2x}\frac{C_{y2}}{C_x}\right) < -\frac{1}{4}$$

and

$$MSE(t_{TP1}) < MSE(\hat{P}) \quad \text{If} \quad (4.3)$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} + \rho_{y2x}\frac{C_{y2}}{C_x}\right) > \frac{1}{4}$$

and $\quad \left(\rho_{y1x}'\frac{C_{y1}'}{C_x'} + \rho_{y2x}'\frac{C_{y2}'}{C_x'}\right) > \frac{1}{4}$,

$$MSE(t_{TP2}) < MSE(\hat{P}) \quad \text{If} \quad (4.4)$$

$$\left(\rho_{y1x}\frac{C_{y1}}{C_x} + \rho_{y2x}\frac{C_{y2}}{C_x}\right) > \frac{1}{4}$$

V. DETERMINATION OF n', n AND k FOR THE FIXED COST $C \le C_0$

Let the fixed total cost apart from overhead cost is $C \le C_0$ and the cost function is given by

$$C' = c_1'n' + c_1 n + c_2 n_1 + c_3\frac{n_2}{k}. \quad (5.1)$$

The expected cost C to be incurred on the survey apart from overhead expenses, is given by

$$C = E(C') = c_1'n' + n\left(c_1 + c_2 W_1 + c_3\frac{W_2}{k}\right), \quad (5.2)$$

where

c_1' − the cost per unit of identifying and observing auxiliary character x at the first phase,

c_1 − the cost per unit of mailing questionnaire visiting the units at the second phase,

c_2 − the cost per unit of collecting and processing data for the study characters y_1 and y_2 obtained from n_1 responding units,

c_3 − the cost per unit of obtaining and processing data for the study characters y_1 and y_2 (after extra effort) from the sub-sampled units and

$W_1 = N_1/N$ is the response rate in population of size N.

The $MSE(t(j)), j = 1,2,3,4$ can be written in term of the notation V_{0j}, V_{1j}, V_{2j} and V_{3j} which is given as

$$MSE(t(j)) = \left[\frac{1}{n}V_{0j} + \frac{1}{n'}V_{1j} + \frac{k}{n}V_{2j} - \frac{1}{N}V_{3j}\right], (5.3)$$

where V_{0j}, V_{1j}, V_{2j} and V_{3j} are respectively the coefficients of terms of $n^{-1}, n'^{-1}, k\,n^{-1}$ and N^{-1} in the expression of $MSE(t(j))$. Here $t(1) = t_{TR1}$, $t(2) = t_{TP1}$, $t(3) = t_{TR2}$ and $t(4) = t_{TP2}$.

Let us define a function ϕ for obtaining the optimum values of n', n, k and for minimizing the $MSE(t(j))$ in case of fixed cost $C \le C_0$ which is given as

$$\phi = MSE(t(j)) + \lambda_j \left\{ c_1'n' + n \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k} \right) - C_0 \right\}$$

$$(5.4)$$

where λ_j is Lagrange's multiplier.

By differentiating ϕ with respect to n', n, k and equating to zero, we get the optimum values of n', n and k in case of fixed cost $C \le C_0$ which are given as follows:

$$n' = \sqrt{\frac{V_{1j}}{\lambda_j c_1'}}, \qquad (5.5)$$

$$n = \sqrt{\frac{(V_{0j} + kV_{2j})}{\lambda_j \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k} \right)}} \qquad (5.6)$$

and

$$k_{opt} = \sqrt{\frac{c_3 W_2 V_{0j}}{(c_1 + c_2 W_1) V_{2j}}}, \qquad (5.7)$$

where

$$\sqrt{\lambda_j} = \frac{1}{C_0} \left[\sqrt{V_{1j} c_1'} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right]$$

$$(5.8)$$

By putting the optimum values of n', n and k from (5.5), (5.6) and (5.7) in equation (5.3) and neglecting the term of order $(1/N)$, we get the minimum mean square error of $t(j)$ in case of the fixed cost $C \le C_0$ which is given as:

$$MSE(t(j))_{min} = \left[\frac{1}{C_0} \left\{ \sqrt{V_{1j} c_1'} + \right. \right.$$

$$\left. \left. \left(V_{0j} + k_{opt} V_{2j} \right) \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k_{opt}} \right) \right\}^2 \right]$$

$$(5.9)$$

VI. DETERMINATION OF n', n AND k FOR THE SPECIFIED VARIANCE $V = V_0$

Let V_0 be the specified variance of the estimator $t(j)$ which is fixed in advance. So we have

$$V_0 = \frac{1}{n} V_{0j} + \frac{1}{n'} V_{1j} + \frac{k}{n} V_{2j} - \frac{1}{N} V_{3j}, \qquad (6.1)$$

For minimizing the average total cost C for the specified variance (i.e. $MSE(t(j)) = V_0$) of the estimator $t(j)$ and

for obtaining the optimum values of n', n and k, we define a function ψ which is given as:

$$\psi = \left\{ c_1'n' + n \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k} \right) \right\} + \mu_j \left(MSE(t(j)) - V_0 \right),$$

$$(6.2)$$

where μ_j is Lagrange's multiplier.

By differentiating ψ with respect to n', n, k and equating to zero, we get the optimum values of n', n and k in case of specified variance which are given as follows:

$$n' = \sqrt{\frac{\mu_j V_{1j}}{c_1'}}, \qquad (6.3)$$

$$n = \sqrt{\frac{\mu_j (V_{0j} + kV_{2j})}{\left(c_1 + c_2W_1 + c_3 \frac{W_2}{k} \right)}} \qquad (6.4)$$

and

$$k_{opt} = \sqrt{\frac{c_3 W_2 V_{0j}}{(c_1 + c_2 W_1) V_{2j}}}, \qquad (6.5)$$

where

$$\sqrt{\mu_j} = \frac{\left[\sqrt{c_1' V_{1j}} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right]}{\left[V_0 + \frac{1}{N} V_{3j} \right]}$$

$$(6.6)$$

By putting the optimum values of n', n and k from (6.3), (6.4) and (6.5) in equation (5.2) and neglecting the term of order $(1/N)$, we get the minimum expected total cost of the estimator $t(j)$ for the specified variance $V = V_0$ which is given as follows:

$$C(t(j))_{min} = \frac{\left[\sqrt{c_1' V_{1j}} + \sqrt{(V_{0j} + k_{opt} V_{2j}) \left(c_1 + c_2W_1 + c_3 \frac{W_2}{k_{opt}} \right)} \right]^2}{V_0}$$

$$(6.7)$$

VII. EMPIRICAL STUDIES
VII.1 Data Set I [Khare and Sinha [24]]

The present data belong to the data on growth of upper socio- economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics,

BHU during 1983-84 has been taken under study. The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y_1 (the height of the children in cm) and y_2 (weight of the children in kg) and auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\overline{Y}_1 = 115.9526$, $\qquad \overline{Y}_2 = 19.4968$, $\qquad \overline{X} = 55.8611$,

$C_{y1} = 0.0515$, $\qquad C_{y2} = 0.1561$, $\qquad C_x = 0.0586$,

$C'_{y1} = 0.0440$, $\qquad C'_{y2} = 0.1208$, $\qquad C'_x = 0.0540$,

$\rho_{y1x} = 0.620$, $\qquad \rho_{y2x} = 0.846$, $\qquad \rho_{y1y2} = 0.713$,

$\rho'_{y1x} = 0.401$, $\qquad \rho'_{y2x} = 0.729$, $\qquad \rho'_{y1y2} = 0.678$

.

The problem considered is to estimate the ratio between height and weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 1. Relative efficiency (RE) and mean square error (MSE) of the proposed estimators with respect to $\hat{R}$ ($N = 95$, $n' = 65$, $n = 30$)

Estimators	1/k					
	1/4		1/3		1/2	
	RE	MSE	RE	MSE	RE	MSE
$\hat{R}$	100.00	0.02088	100.00	0.01810	100.00	0.01533
t_{R1}	139.64	0.01495	138.27	0.01309	136.46	0.01123
t_{R2}	117.98	0.01769	121.32	0.01492	126.19	0.01215

Table 2. Relative efficiency (RE) and mean square error (MSE) of the estimators with respect to $\hat{R}$ for the fixed cost $C_0 = $ Rs. 250

Estimators	$c'_1 = Rs.\ 0.50$, $\quad c_1 = Rs.\ 5$, $\quad c_2 = Rs.\ 15$, $\quad c_3 = Rs.\ 25$				
	k_{opt}	n'_{opt} (approx.)	n_{opt} (approx.)	RE (in %)	MSE
$\hat{R}$	1.48	---	12	100.00	0.0483
t_{R1}	1.35	50	11	138.56	0.0349
t_{R2}	1.16	47	10	118.75	0.0407

Table 3. Expected cost (in Rs.) of the estimators for the specified variance $V_0 = 0.0561$

Estimators	$c'_1 = Rs.\ 0.50$, $\quad c_1 = Rs.\ 5$, $\quad c_2 = Rs.\ 15$, $\quad c_3 = Rs.\ 25$			
	k_{opt}	n'_{opt} (approx.)	n_{opt} (approx.)	Expected cost (in Rs.)
$\hat{R}$	1.48	---	11	215.43
t_{R1}	1.35	31	7	155.48
t_{R2}	1.16	34	8	181.41

From table 1, It has been observed that for the fixed values of N , n' , n and different values of k , the proposed estimator's t_{R1} and t_{R2} are more efficient in comparison to the usual estimator $\hat{R}$. The estimator t_{R1} is more efficient in comparison to the estimator t_{R2} for different values of k . The values of *MSE* of all estimators decrease as the values of k decrease.

From table 2, it has been observed that for the fixed cost, the proposed estimators (t_{R1} , t_{R2}) have less MSE in comparison to the usual estimator $\hat{R}$ and t_{R1} have less MSE in comparison to the estimator t_{R2} .

From table 3, it has been observed that for the specified variance V_0, the proposed estimators (t_{R1} , t_{R2}) have less total cost in comparision to the relevant estimator $\hat{R}$.

VII.2 Data Set II [Sinha [28]]

109 Village/Town/Ward wise population of urban area under Police-station-Baria, Tahasil Champua, Orissa has been taken under consideration from District Census Handbook, 1981, Orissa, published by Govt. of India. The last 25% villages (i.e. 27 villages) have been considered as non-response group of the population. Here we have take the study characters and auxiliary character as follows:

y_1 - Number of occupied residential house in the village.

y_2 - Average number of person in a house in the village.

x - Number of Cultivator's in the village.

The values of the parameters of the population under study are given as follows:

$$\bar{Y}_1 = 88.3670 , \quad \bar{Y}_2 = 5.5832 , \quad \bar{X} = 100.5505,$$
$$C_{y1} = 0.6713 , \quad C_{y2} = 0.1079 , \quad C_x = 0.7314,$$
$$C'_{y1} = 0.5123 , \quad C'_{y2} = 0.0900 , \quad C'_x = 0.5678 ,$$
$$\rho_{y1x} = 0.795 , \quad \rho_{y2x} = -0.084 , \quad \rho_{y1y2} = -0.194,$$
$$\rho'_{y1x} = 0.658 , \quad \rho'_{y2x} = 0.092 , \quad \rho'_{y1y2} = 0.023$$

Table 4. Relative efficiency (RE) and mean square error (MSE) of the proposed estimators with respect to $\hat{P}$ (N =109, n' =70, n =40)

Estimators	1/ k					
	1/4		1/3		1/2	
	RE	MSE	RE	MSE	RE	MSE
$\hat{P}$	100.00	2905.62	100.00	2494.60	100.00	2083.58
t_{P1}	167.82	1731.37	166.92	1494.46	165.69	1257.54
t_{P2}	128.93	2253.69	135.38	1842.67	145.54	1431.65

Table 5. Relative efficiency (RE) and mean square error (MSE) of the estimators with respect to $\hat{P}$ for the fixed cost C_0 =Rs. 250

Estimators	$c'_1 = Rs.$ 0.50, $c_1 = Rs.$ 3, $c_2 = Rs.$ 10, $c_3 = Rs.$ 30				
	k_{opt}	n'_{opt} (approx.)	n_{opt} (approx.)	RE (in %)	MSE
$\hat{P}$	1.96	---	17	100.00	6955.03
t_{P1}	1.62	83	14	157.77	4408.44
t_{P2}	1.10	81	12	151.69	4584.92

Table 6. Expected cost (in Rs.) of the estimators for the specified variance $V_0 = 4550$

Estimators	$c_1' = Rs.\ 0.50,\quad c_1 = Rs.\ 3,\quad c_2 = Rs.\ 10,\quad c_3 = Rs.\ 30$			
	k_{opt}	n'_{opt} (approx.)	n_{opt} (approx.)	Expected cost (in Rs.)
$\hat{P}$	1.96	---	27	382.14
t_{P1}	1.62	80	13	242.22
t_{P2}	1.10	82	12	251.92

From table 4, It has been observed that for the fixed values of N, n', n and different values of k, the proposed estimator's t_{P1} and t_{P2} are more efficient in comparison to the usual estimator $\hat{P}$. The estimator t_{P1} is more efficient in comparison to the estimator t_{P2} for different values of k. The values of mean square error of all estimators decrease as the values of k decrease.

From table 5, it has been observed that for the fixed cost, the proposed estimators (t_{P1}, t_{P2}) have less MSE in comparison to the usual estimator $\hat{P}$ and t_{P1} have less MSE in comparison to the estimator t_{P2}.

From table 6, it has been observed that for the specified variance V_0, the proposed estimators (t_{P1}, t_{P2}) have less total cost in comparison to the usual estimator $\hat{P}$ and t_{P1} have less total cost in comparison to the estimator t_{P2}.

VIII. CONCLUSION

From tables 1, 2, 3, we conclude that when linear relationship between study character and auxiliary character is not very strong, the proposed estimators for ratio of two population means in the presence of non-response are more efficient in comprision to the usual estimator in case of the fixed sample sizes and also in case of the fixed cost. The proposed estimators have less total cost in comprision to the usual estimator in case of the specified variance.

From tables 4, 5, 6, we conclude that when linear relationship between study character and auxiliary character is not very strong, the proposed estimators for product of two population means in the presence of non-response are more efficient in comprision to the usual estimator in case of the fixed sample sizes and also in case of the fixed cost. The proposed estimators have less total cost in comprision to the usual estimator in case of the specified variance.

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