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Design of Attribute Control Chart based on Inverse Rayleigh Distribution under Type-I Censoring

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Abstract - Control charts are one of the most common techniques that have been used to observe and control the process deviations in the industry. A control chart provides information about the performance or capability of a process. Lifetime considered as a quality characteristics for some products. Industries are routinely performing suitable life tests for their products. In life testing, censoring schemes are adopted to save time and cost of life test. Time censoring (type-I), product censoring (type-II) and hybrid censoring (Mixture of type-I and type-II censoring) are some of the censoring schemes employed in the life tests. The main objective of this paper is to design an life test based attribute control chart for inverse Rayleigh distribution under type-I censoring scheme. The performance of the control chart is studied using average run length (*ARL*). Designed control chart is illustrated through numerical examples.

Keywords - *Inverse Rayleigh distribution, attribute control chart, type-I censoring, average run length.*

I. INTRODUCTION

The control charts are one of the most important statistical tool used in industries to monitor the production process for high quality of the products. Many control chart schemes have been developed to monitor the process for various situations. The control chart is a graph used to study how a process changes over time. A control chart always has a central line (*CL*) for the average, an upper line for the upper control limit (*UCL*) and a lower line for the lower control limit (*LCL*). When the control statistic is plotted beyond the *LCL* or *UCL* limits, the process is declared as out of control. The process is said to be in control if the control statistic is inside the control limits. There are two type of control charts are available namely attribute and variable control charts. The attribute control charts are used to classify the non-conforming items from the conforming items. The variable control charts are used when the industrial data obtained from the measurement process.

Life time considered as a quality characteristics for some products. Monitoring the production process for such products is carried out by suitable life tests. Based on the information obtained from the life test the product may be classified as conforming or nonconforming. The testing of this type of products is time consuming as one has to wait for a long time until the test duration ends. In this situation, censoring scheme is an important and non-ignorable one. Type-I censoring, Type-II censoring and hybrid censoring are some of the censoring schemes employed in life testing. Under Type-I censoring, the life test is terminated at predetermined time *t*. During the life test, if the product failed before test termination time *t*, then the product may be classified as nonconforming product.

The attribute control chart such as *np* control chart is designed on the basis of the fraction nonconforming, which is obtained by assuming that the quality of interest follows the normal distribution. It may happen in the real life that the distribution of the quality of interest is not normal. In this case, the application of the existing control chart may mislead the industrial engineers and can cause the increment of the non-conforming items. Many authors in the literature have worked on attribute control charts including [1] – [9]. By exploring the literature, we note that there have been no studies on a control chart to incorporate a life test for a non normal distribution such as Inverse Rayleigh distribution under type-I censoring scheme.

Inverse Rayleigh distribution is one of the statistical distributions which attracted the attention of the researchers due to its applications in scientific and engineering related studies. This distribution has been applied widely as a life time distribution in reliability and life testing. It is a special case of inverse Weibull distribution. This distribution is used in situation of products which have initially more probable failures.

In this paper, we will develop an attribute control chart based on inverse Rayleigh distribution under type-I censoring scheme. We will find the control chart co-efficient and discuss the behaviour of average run length of the proposed control chart. The designing of the proposed control chart is presented when the scale parameter is shifted. The application of the proposed chart is given with the help of simulated data. A brief introduction about the Inverse Rayleigh distribution is given in Section 2. Designing of the control chart for this distribution is discussed in Section 3. A numerical illustration is given in Section 4. Finally the conclusions are given in Section 5.

II. INVERSE RAYLEIGH DISTRIBUTION

[10] Defined inverse Rayleigh distribution for applying in reliability and survived analysis since it has monotone failure rate. [11] Mentioned that the distribution of lifetimes of several types of experimental units can be approximated by the inverse Rayleigh distribution.

Let T be the lifetime of the product. Which is distributed according to the inverse Rayleigh distribution with scale parameter (θ), then the probability density function of T is given by

$$f(t;\theta)=\left(\frac{2\theta}{t^3}\right)e^{\left(-\frac{\theta}{t^2}\right)};t>0,\theta>0 \tag{1}$$

The cumulative distribution function of the inverse Rayleigh distribution (θ) is given by

$$F(t;\theta)=e^{\left(-\frac{\theta}{t^2}\right)};t>0,\theta>0 \tag{2}$$

The mean life of a product for this distribution is given as

$$\mu=\sqrt{\pi\theta} \tag{3}$$

[11] – [14] studied more on the inverse Rayleigh distribution.

III. DESIGN OF CONTROL CHART BASED ON INVERSE RAYLEIGH DISTRIBUTION

We propose the following *np* control chart for inverse Rayleigh distribution under type-I censoring scheme based on the number of products for each subgroup:

Step 1 Select a set of n products randomly from the production process.

Step 2 Conduct the life test on the selected items considering t as the test termination time. Observe the number of failed items (D, say).

Step 3 Terminate the life test after reached at time t.

Step 4 Declare the process as out of control if D > UCL or D < LCL. Declare the process as in control if LCL $\le D \le$ UCL.

Note that the above is called as *np* control chart because the number of failures (D) rather than the fraction nonconforming (p) is plotted. The random variable D follows a binomial distribution with parameters n and p_0 when process is in control, where p_0 is the probability that an item is failed before time t. Therefore, the control limits for in control process are given as follows:

$$UCL=np_0+k\sqrt{np_0(1-p_0)} \tag{4a}$$

$$LCL=\max[0,np_0-k\sqrt{np_0(1-p_0)}] \tag{4b}$$

Where, k is the coefficient of the control limits. The fraction nonconforming in control process (p_0) may be obtained from equation (2) by

$$p_0=e^{\left(-\frac{\theta_0}{t^2}\right)}=e^{\left(-\frac{1}{a^2\pi}\right)} \tag{5}$$

In practice, probability p_0 is usually unknown, therefore, the control limit for the practical application are given as

$$UCL=\overline{D}+k\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)} \tag{6a}$$

$$LCL=\max\left[0,\overline{D}-k\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}\right] \tag{6b}$$

Where $\overline{D}$ is the average number of failures over the subgroups.

The probability of declaring as in control for the proposed control chart is given as follows:

$$p_{in}^0=P\left(LCL\le D\le UCL\mid p_0\right)$$

$$p_{in}^0=\sum_{d=[LCL]+1}^{[UCL]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} \tag{7}$$

The performance of the control chart is usually evaluated by average run length. The *ARL* for in control process as follows:

$$ARL_0 = \frac{1}{1-\left[\sum_{d=[LCL]+1}^{UCL}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\right]} \tag{8}$$

3.1. *ARL* WHEN PROCESS MEAN IS SHIFTED

Suppose now that the process mean is shifted from μ_0 to μ_1. Then the probability in (5) becomes

$$p_1 = e^{\left(-\frac{\theta_1}{t^2}\right)} = e^{-\frac{1}{a^2\pi}\left(\frac{\mu_1}{\mu_0}\right)^2} \tag{9}$$

If we consider the mean shift such as $\mu_1 = f\,\mu_0$ for a constant *f*, then equation (9) is rewritten as

$$p_1 = e^{-\left(\frac{f^2}{a^2\pi}\right)} \tag{10}$$

Now, the probability that the process is declared to be in control when the process has shifted to μ_1 is given by

$$p_{in}^1 = P\left(LCL \le D \le UCL \mid p_1\right)$$

$$p_{in}^1 = \sum_{d=[LCL]+1}^{[UCL]}\binom{n}{d}p_1^d\left(1-p_1\right)^{n-d} \tag{11}$$

The *ARL* for the shifted process is given as follows:

$$ARL_1 = \frac{1}{1-\left[\sum_{d=[LCL]+1}^{UCL}\binom{n}{d}p_1^d\left(1-p_1\right)^{n-d}\right]} \tag{12}$$

We used the following algorithm to complete the tables for the proposed control chart.

(1) Specify the values of *ARL*, say r_0 and sample size *n*.

(2) Determine the values of control chart parameters and truncated time constant *a* for which the ARL from equation (8) is close to r_0.

(3) Use the values of control chart parameters obtained in step 2 to find the ARL_1 according to shift constant *f* using equation (12).

We determine the control chart parameters and ARL_1 for various values of r_0, and *n* from Tables 1 & 2, we note that the decreasing tend in ARL_S as the shift constant *f* decreases.

Table 1 ARLs for the proposed chart for n = 20 when the mean shifted

	r_0 = 200	r_0 = 400	r_0 = 500
a	0.595	0.5231	0.5109
k	2.567	2.774	2.742
UCL	14	12	12
LCL	2	0	0
Shift (*f*)	*ARL*	*ARL*	*ARL*
1.00	200.01	400.04	500.29
0.90	69.04	60.01	91.11
0.80	14.51	11.19	14.88
0.70	4.40	3.35	3.96
0.60	1.96	1.59	1.72
0.50	1.25	1.12	1.15
0.40	1.04	1.01	1.02
0.30	1.00	1.00	1.00
0.20	1.00	1.00	1.00
0.10	1.00	1.00	1.00

Table 2 ARLs for the proposed chart for r_0 = 300 when the mean shifted

	n = 25	*n* = 39	*n* = 43
a	0.5134	0.6498	0.4446
k	2.634	2.84	2.705
UCL	14	27	16
LCL	1	9	1
Shift (*f*)	*ARL*	*ARL*	*ARL*
1.00	300.04	300.02	300.15
0.90	52.06	50.85	19.06
0.80	8.49	7.78	2.85
0.70	2.52	2.31	1.21
0.60	1.31	1.25	1.01
0.50	1.04	1.03	1.00
0.40	1.00	1.00	1.00
0.30	1.00	1.00	1.00
0.20	1.00	1.00	1.00
0.10	1.00	1.00	1.00

3.2. *ARL* WHEN SCALE PARAMETER IS SHIFTED

Suppose now that the scale parameter is shifted from θ_0 to θ_1. Then the probability in (5) becomes

$$p_2 = e^{\left(-\frac{\theta_1}{t^2}\right)} \tag{13}$$

If we consider the scale parameter shift such as $\theta_1 = c\,\theta_0$ for a constant *c*, then equation (13) is rewritten as

$$p_2 = e^{\left(-\frac{c}{a^2\pi}\right)} \tag{14}$$

Now, the probability that the process is declared to be in control when the process has shifted to θ_1 is given by

$$p_{in}^{2} = P\left(LCL \le D \le UCL \mid p_2\right) \tag{15}$$

The *ARL* for the shifted process is given as follows:

$$ARL_1 = \frac{1}{1-\left[\sum_{d=[LCL]+1}^{UCL}\binom{n}{d}p_2^d\left(1-p_2\right)^{n-d}\right]} \tag{16}$$

We used the following algorithm to complete the tables for the proposed control chart.

(1) Specify the values of *ARL*, say r_0 and sample size n.

(2) Determine the values of control chart parameters and truncated time constant a for which the *ARL* from equation (8) is close to r_0.

(3) Use the values of control chart parameters obtained in step 2 to find the ARL_1 according to shift constant c using equation (16).

We determine the control chart parameters and ARL_1 for various values of r_0, and n from Tables 3 & 4, we note that the decreasing tend in ARL_S as the shift constant c decreases.

Table 3 *ARLs* for the proposed chart for n = 20 when the scale parameter shifted

	r_0 = 200	r_0 = 400	r_0 = 500
a	0.595	0.5231	0.5109
k	2.567	2.774	2.742
UCL	14	12	12
LCL	2	0	0
Shift (c)	*ARL*	*ARL*	*ARL*
1.00	200.01	400.04	500.29
0.90	148.04	157.20	246.19
0.80	62.89	54.00	81.42
0.70	24.76	19.63	27.38
0.60	10.33	7.87	10.14
0.50	4.73	3.59	4.28
0.40	2.45	1.93	2.14
0.30	1.49	1.27	1.33
0.20	1.11	1.04	1.05
0.10	1.01	1.00	1.00

Table 4 *ARLs* for the proposed chart for r_0 = 300 when the scale parameter shifted

	n = 25	n = 39	n = 43
a	0.5134	0.6498	0.4446
k	2.634	2.84	2.705
UCL	14	27	16
LCL	1	9	1
Shift (c)	*ARL*	*ARL*	*ARL*
1.00	300.04	300.02	300.15
0.90	146.92	152.82	71.90
0.80	46.37	45.04	16.64
0.70	15.42	14.34	5.01
0.60	5.89	5.37	2.09
0.50	2.69	2.46	1.25
0.40	1.53	1.44	1.03
0.30	1.12	1.09	1.00
0.20	1.01	1.01	1.00
0.10	1.00	1.00	1.00

ILLUSTRATION

Suppose that the lifetime of the products follows the inverse Rayleigh distribution. Suppose that the target average life of the product is μ_0 = 1000 hours, a = 0.5231, r_0 = 400 and n = 20. Then from equation (5), we get p_0 = 0.3125. Also, from Table 1 we obtain the k = 2.774, LCL = 0 and UCL = 12. Therefore the designed control chart as follows:

Step 1: Take a sample of size 20 at each subgroup and put them on the life test during t = 523 hours. Count the number of failed items (D) during the test.

Step 2: Declare the process as in control if $0 \le D \le 12$ and as out of control, otherwise.

IV. SIMULATION STUDY

In this section, the application of the proposed chart is discussed with the help of simulated data. The data are generated using the inverse Rayleigh distribution with an average lifetime μ_0 = 1000 hours. Let n = 20 and r_0 = 200. It is declared that the process is in control when μ_0 = 1000 hours and f = 1. The first 15 observations of subgroup size 20 are generated using in control parameters. Now, suppose that the process has shifted due to the shift in the mean of inverse Rayleigh distribution. The shift constant f is taken as 0.7. The next 15 observations are generated with shifted mean when f = 0.7.

Let life test termination time t = 595 hours. The number of failures D are counted and reported in Table 5 for each subgroup. The average number of failures $\overline{D}$ is computed. The UCL = 14 and LCL = 2 is for simulated data. The data are plotted in Figure 1.

Table 5 Simulated Data

S. No.	**Sample**																				**D**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
1	834	2706	444	699	432	710	615	323	627	863	751	608	448	355	2596	1994	365	414	749	569	8
2	915	1277	1328	634	317	653	460	588	715	686	640	469	2461	1460	466	564	1131	491	697	710	7
3	1077	697	715	375	341	325	517	585	627	627	275	889	2870	6221	832	803	1689	510	1850	1381	7
4	1472	1036	894	831	403	391	582	1812	282	1884	606	425	1401	946	688	845	453	214	761	451	8
5	1099	564	517	413	689	1131	381	2668	1331	407	874	653	301	412	1083	929	542	763	489	637	9
6	602	373	580	588	429	507	441	2457	650	810	597	369	671	3709	695	510	1069	431	927	546	10
7	605	525	557	662	404	361	562	812	634	1527	2703	1598	727	882	771	701	305	950	564	574	8
8	746	2558	480	297	736	521	628	1851	1585	241	449	899	1131	353	760	686	349	356	1386	724	8
9	731	508	583	794	472	332	614	374	406	519	707	2231	565	852	352	1214	1062	637	1434	303	10
10	463	447	705	1125	463	578	1968	473	1253	872	471	392	578	491	2612	651	1285	444	220	1674	11
11	957	637	4584	576	690	356	1154	1443	621	1123	388	967	725	989	1489	371	1119	987	458	317	6
12	1654	1324	4355	1439	596	605	3664	299	403	3495	5609	715	536	451	2065	1454	413	1081	678	1376	5
13	561	510	759	712	396	496	696	655	595	1664	1010	471	1745	614	1394	852	403	1607	536	338	7
14	709	2076	451	1591	2615	660	1011	2951	936	978	2712	489	491	989	523	618	1016	1077	507	646	5
15	323	1123	1937	580	1377	531	444	1381	280	536	779	644	945	917	2840	592	1084	856	534	405	9
16	928	340	298	385	292	684	365	348	287	756	513	618	871	510	885	1125	658	767	478	344	11
17	300	697	403	325	444	452	905	1411	384	254	6589	1076	278	235	485	368	292	349	230	667	14
18	283	550	856	262	512	714	523	686	261	552	188	1161	649	480	458	1799	377	319	414	334	14
19	306	1802	212	561	1584	379	439	803	1005	423	886	458	226	589	727	750	657	373	1600	819	11
20	421	384	222	435	264	1165	515	544	281	560	1207	821	495	532	455	429	448	310	419	483	17
21	604	630	1563	163	759	269	434	180	1068	506	253	702	680	256	1509	246	454	269	730	877	10
22	511	545	738	393	599	836	1578	220	930	694	836	4942	230	802	192	1660	381	363	286	332	10
23	272	730	287	1622	681	450	636	784	256	277	405	563	652	342	3690	360	462	304	359	1460	12
24	320	501	449	437	372	284	433	641	1367	777	262	1307	248	396	218	1073	403	5206	263	912	13
25	447	601	530	262	384	286	1082	890	190	435	814	350	360	317	368	253	504	1053	410	1226	14
26	4496	1826	583	170	329	547	831	340	1249	1750	333	745	499	201	1075	205	641	777	460	410	11
27	290	956	464	327	4838	1274	903	258	366	410	5776	480	720	720	393	8549	686	517	570	1216	10
28	412	726	731	270	1375	509	537	252	850	490	384	594	474	315	754	444	630	235	307	355	14
29	452	601	486	216	477	891	293	559	304	867	803	745	417	277	961	6601	576	2279	699	449	11
30	968	500	629	930	538	2093	389	483	713	774	543	627	369	939	340	554	1358	257	286	622	10

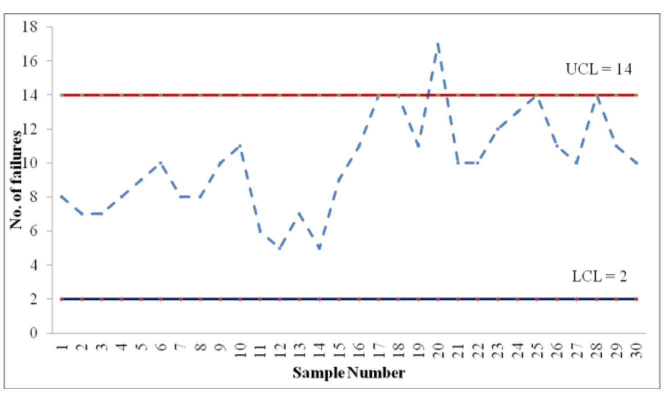

Figure 1. Control chart for simulation data.

From Figure 1, it can be observed that the proposed chart shows that shift at 20th (5th observation after the shift) observation while tabulated ARL is 4.40. So, the proposed chart efficiently detects the shift in the process.

V. CONCLUSION

In this paper, a new attribute control chart is developed for the inverse Rayleigh distribution under type-I censoring. The new control chart is quite flexible and can be used to monitor the lifetimes of quality products. The tables are presented for the industrial use and explained with the help of simulated data. The performance of the new control chart is reported in terms of *ARLs* according to various shift parameters. It may be noted that if the type-I censoring scheme is employed to carry out the life test, implementation of the sampling inspection will reduce the time and cost of conducting the life test. The proposed attribute control chart can be extended for some other distributions as a future research.

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