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On the General Product-connectivity Index of Transformation Graphs

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Abstract—The general product-connectivity index is a molecular structure descriptor of a molecular graph, which generalizes both Randić index and second Zagreb index. In this paper, we obtain general product-connectivity index of subdivision graph, partial complement of subdivision graph, semitotal-point graph, semitotal-line graph and total graph. Also bounds for general product-connectivity index of some transformation graphs.

Keywords—*General product-connectivity index, Semitotal-point graph, Semitotal-line graph, Total transformation graphs, First and second Zagreb indices.*

I. INTRODUCTION

Simple, finite and undirected graphs are considered throughout this paper. Let *G* be a graph with the vertex set $V(G)$ and edge set $E(G)$ such that $|V(G)|=n$ is the *order* and $|E(G)|=m$ is the *size* of *G*. The *degree of a vertex* $v\in V(G)$ is denoted by $d_G(v)$. The *degree of an edge* $e=uv$ in *G* is denoted by $d_G(e)$, and is defined by $d_G(e)=d_G(u)+d_G(v)-2$. The *maximum* and *minimum* vertex degree of *G* are denoted by $\Delta(G)=\Delta$ and $\delta(G)=\delta$ respectively. The vertices and edges of *G* are called its *elements*. Here, we denote the adjacency (or incidence) of elements by the symbol $\sim$ and nonadjacency (or nonincidence) by $\nsim$. As usual $S(G)$ is subdivision graph of *G*, $L(G)$ is line graph and $T(G)$ is total graph. The *jump graph* $J(G)$ of a graph *G* is complement of line graph.

The *partial complement of subdivision graph* $\overline{S}(G)$ [9] is a graph with the vertex set $V(G)\cup E(G)$ such that two vertices of $\overline{S}(G)$ are adjacent if and only if one corresponds to a vertex v of *G* and other to an edge e of *G* and v is not incident to e in *G*.

The *semitotal-point graph* $T_2(G)$ [15] is a graph whose vertex set is $V(G)\cup E(G)$ in which two vertices are adjacent if and only if (i) they are adjacent vertices in *G* or (ii) one is a vertex of *G* and the other is an edge of *G* incident to it.

The *semitotal-line graph* $T_1(G)$ [15] is a graph whose vertex set is $V(G)\cup E(G)$ in which two vertices are adjacent if and only if (i) they are adjacent edges in *G* or (ii) one is a vertex of *G* and the other is an edge of *G* incident to it.

The *tadpole graph* $T_{n,k}$ [7] is the graph formed by joining the end point of a path of length *k* to a *n*-cycle. The sum C_n+K_1 of a cycle C_n and a single vertex is referred to as a *wheel graph* W_{n+1} of order $n+1$. The *ladder graph* L_n [7] is the product $K_2\times P_n$. For notations and undefined terminologies we follow [6], [11].

Some Results on Contra Harmonic Mean Labeling of Graphs obtained in [13] and Anti-magic labeling for Boolean graph of path $BG(P_n)$, $(n\geq 4)$ is obtained in [16].

A molecular graph is a simple graph representing the carbon-atom skeleton of an organic molecule (usually, of a hydrocarbons). Thus, the vertices of a molecular graph represent the carbon atoms, and its edges the carbon-carbon bonds. According to the IUPAC definition, a *topological index* (or *molecular structure descriptor*) is a numerical value associated with chemical constitution for correlation of chemical structure with various physical properties, chemical reactivity or biological activity. The significance of topological indices is usually associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) [10].

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The *first and second Zagreb indices* are defined in [8] as

$$M_1 = M_1(G) = \sum_{v \in V(G)} d_G(v)^2 \text{ and}$$

$$M_2 = M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v) \text{ respectively.}$$

The *Randić index* of a graph *G* is defined in [14] as

$$R(G) = \sum_{uv \in E(G)} [d_G(u) d_G(v)]^{-\frac{1}{2}}.$$

It has been extended to the *general product-connectivity index* defined in [5] as

$$R_\alpha(G) = \sum_{uv \in E(G)} [d_G(u)\, d_G(v)]^\alpha,$$

where α is any real number.

The *first general Zagreb index* of a graph *G* is defined in [12] as

$$M_1^\alpha(G) = \sum_{u \in V(G)} [d_G(u)]^\alpha,$$

where α is any real number.

The *general sum-connectivity index* defined in [19] as

$$\chi_\alpha(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^\alpha,$$

where α is any real number.

The general sum-connectivity index of some transformation graphs is obtained in [4], [17]. In section II, we obtain general product-connectivity index of subdivision graph, partial complement of subdivision graph, semitotal-point graph, semitotal-line graph and total graph. Also bounds for general product-connectivity index of some transformation graphs.

II. General product-connectivity index of transformation graphs

Theorem 2.1 *If* G *is a graph and* $\alpha \in \mathbb{R}$, *Then*

$$R_\alpha(S(G)) = 2^\alpha M_1^{\alpha+1}(G).$$

Proof. Since $S(G)$ has $n+m$ vertices and $2m$ edges,

$$R_\alpha(S(G)) = \sum_{ue \in E(S(G))} [d_{S(G)}(u)\, d_{S(G)}(e)]^\alpha.$$

Note that $d_{S(G)}(u) = d_G(u)$ for $u \in V(G)$ and $d_{S(G)}(e) = 2$ for $e = uv \in E(G)$.

Therefore,

$$\begin{aligned} R_\alpha(S(G)) &= \sum_{ue \in E(S(G))} [d_G(u) \cdot 2]^\alpha \\ &= 2^\alpha \sum_{ue \in E(S(G))} [d_G(u)]^\alpha \\ &= 2^\alpha \sum_{u \in V(G)} d_G(u) [d_G(u)]^\alpha \\ &= 2^\alpha M_1^{\alpha+1}(G). \end{aligned}$$

Corollary 2.2 *If* P_n *is a path and* $\alpha \in \mathbb{R}$, *then*

$$R_\alpha(S(P_n)) = 2^{\alpha+1}[1 + (n-2)2^\alpha].$$

Proof. From Theorem 2.1, we have

$$R_\alpha(S(P_n)) = 2^\alpha M_1^{\alpha+1}(P_n).$$

Since the path P_n contains two vertices of degree one and $(n-2)$ vertices of degree two,

$$M_1^{\alpha+1}(P_n) = \sum_{u \in V(P_n)} [d_{P_n}(u)]^{\alpha+1} = 2 + (n-2) \cdot 2^{\alpha+1}.$$

Hence $R_\alpha(S(P_n)) = 2^{\alpha+1}[1 + (n-2)2^\alpha]$.

Corollary 2.3 *If* W_{n+1} *is the wheel graph and* $\alpha \in \mathbb{R}$, *then*

$$R_\alpha(S(W_{n+1})) = 2^\alpha n[3^{\alpha+1} + n^\alpha].$$

Proof. From Theorem 2.1, we have

$$R_\alpha(S(W_{n+1})) = 2^\alpha M_1^{\alpha+1}(W_{n+1})$$

Since the wheel W_{n+1} contains n vertices of degree three and one vertex of degree n,

$$M_1^{\alpha+1}(W_{n+1}) = \sum_{u \in V(W_{n+1})} [d_{W_{n+1}}(u)]^{\alpha+1} = n[3^{\alpha+1} + n^\alpha].$$

Thus $R_\alpha(S(W_{n+1})) = 2^\alpha n[3^{\alpha+1} + n^\alpha]$.

Corollary 2.4 *If* $T_{n,k}$ *is the tadpole graph and* $\alpha \in \mathbb{R}$, *then*

$$R_\alpha(S(T_{n,k})) = 2^\alpha\{3^{\alpha+1} + (n+k-2) \cdot 2^{\alpha+1} + 1\}.$$

Proof. From Theorem 2.1, we have

$$R_\alpha(S(T_{n,k})) = 2^\alpha M_1^{\alpha+1}(T_{n,k})$$

The tadpole $T_{n,k}$ has $n+k-2$ vertices of degree two, one vertex of degree three and one vertex of degree one.

$$\begin{aligned} M_1^{\alpha+1}(T_{n,k}) &= \sum_{u \in V(T_{n,k})} [d_{T_{n,k}}(u)]^{\alpha+1} \\ &= 3^{\alpha+1} + (n+k-2) \cdot 2^{\alpha+1} + 1 \end{aligned}$$

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$$R_\alpha(S(T_{n,k}))=2^\alpha\{3^{\alpha+1}+(n+k-2)\cdot 2^{\alpha+1}+1\}.$$

Corollary 2.5 *If* L_n *is a ladder and* $\alpha\in\mathbb{R}$, *then*

$$R_\alpha(S(L_n))=2^{\alpha+1}\{2^{\alpha+2}+(n-2)\cdot 3^{\alpha+1}\}.$$

Proof. From Theorem 2.1, we have

$$R_\alpha(S(L_n))=2^\alpha M_1^{\alpha+1}(L_n).$$

Since the ladder L_n has four vertices of degree two and $2(n-2)$ vertices of degree three,

$$M_1^{\alpha+1}(L_n)=\sum_{u\in V(L_n)}[d_{L_n}(u)]^{\alpha+1}=2^{\alpha+3}+2(n-2)\cdot 3^{\alpha+1}.$$

Hence $R_\alpha(S(L_n))=2^{\alpha+1}\{2^{\alpha+2}+(n-2)\cdot 3^{\alpha+1}\}$.

Corollary 2.6 ([2]) *If* G *is a graph and* $\alpha\in\mathbb{R}$, *then*

$$M_2(S(G))=2M_1(G).$$

Theorem 2.7 *Let* G *be any graph and* $\alpha\in\mathbb{R}$. *Then*

$$R_\alpha(\overline{S}(G))=(n-2)^\alpha\left[\sum_{u\in V(G)}[m-d_G(u)]^{\alpha+1}\right].$$

Proof. Since $\overline{S}(G)$ has $n+m$ vertices and $m(n-2)$ edges, we have

$$R_\alpha(\overline{S}(G))=\sum_{ue\in E(\overline{S}(G))}[d_{\overline{S}(G)}(u)\ d_{\overline{S}(G)}(e)]^\alpha$$

Note that $d_{\overline{S}(G)}(u)=m-d_G(u)$ for $u\in V(G)$ and $d_{\overline{S}(G)}(e)=n-2$ for $e=uv\in E(G)$. Hence

$$R_\alpha(\overline{S}(G))=\sum_{ue\in E(\overline{S}(G))}[(m-d_G(u))(n-2)]^\alpha$$

$$=(n-2)^\alpha\sum_{ue\in E(\overline{S}(G))}[m-d_G(u)]^\alpha$$

$$=(n-2)^\alpha\sum_{u\in V(G)}(m-d_G(u))[m-d_G(u)]^\alpha$$

Corollary 2.8 *If* P_n *is a path and* $\alpha\in\mathbb{R}$, *then*

$$R_\alpha(\overline{S}(P_n))=(n-2)^\alpha[2(m-1)^{\alpha+1}+(n-2)(m-2)^{\alpha+1}].$$

Proof. From Theorem 2.7, we have

$$R_\alpha(\overline{S}(P_n))=(n-2)^\alpha\left[\sum_{u\in V(P_n)}[m-d_{P_n}(u)]^{\alpha+1}\right].$$

Since the path P_n contains two vertices of degree one and $(n-2)$ vertices of degree two,

$$\sum_{u\in V(P_n)}[m-d_{P_n}(u)]^{\alpha+1}=2(m-1)^{\alpha+1}+(n-2)(m-2)^{\alpha+1}.$$

Hence

$$R_\alpha(\overline{S}(P_n))=(n-2)^\alpha\left[2(m-1)^{\alpha+1}+(n-2)(m-2)^{\alpha+1}\right].$$

Corollary 2.9 *If* W_{n+1} *is the wheel graph and* $\alpha\in\mathbb{R}$, *then*

$$R_\alpha(\overline{S}(W_{n+1}))=(n-2)^\alpha[n(m-3)^{\alpha+1}+(m-n)^{\alpha+1}].$$

Proof. From Theorem 2.7, we have

$$R_\alpha(\overline{S}(W_{n+1}))=(n-2)^\alpha\left[\sum_{u\in V(W_{n+1})}[m-d_{W_{n+1}}(u)]^{\alpha+1}\right].$$

Since the wheel W_{n+1} contains n vertices of degree three and one vertex of degree n,

$$\sum_{u\in V(W_{n+1})}[m-d_{W_{n+1}}(u)]^{\alpha+1}=n(m-3)^{\alpha+1}+(m-n)^{\alpha+1}.$$

Hence

$$R_\alpha(\overline{S}(W_{n+1}))=(n-2)^\alpha[n(m-3)^{\alpha+1}+(m-n)^{\alpha+1}].$$

Corollary 2.10 *If* $T_{n,k}$ *is the tadpole graph and* $\alpha\in\mathbb{R}$, *then*

$$R_\alpha(\overline{S}(T_{n,k}))=(n-2)^\alpha[(m-1)^{\alpha+1}+(m-3)^{\alpha+1}+(n+k-2)(m-2)^{\alpha+1}].$$

Proof. From Theorem 2.7, we have

$$R_\alpha(\overline{S}(T_{n,k}))=(n-2)^\alpha\left[\sum_{u\in V(T_{n,k})}[m-d_{T_{n,k}}(u)]^{\alpha+1}\right].$$

Since the tadpole $T_{n,k}$ has $n+k-2$ vertices of degree two, one vertex of degree three and one vertex of degree one,

$$\sum_{u\in V(T_{n,k})}[m-d_{T_{n,k}}(u)]^{\alpha+1}=(m-1)^{\alpha+1}+(m-3)^{\alpha+1}+(n+k-2)(m-2)^{\alpha+1}.$$

Corollary 2.11 *If* L_n *is the ladder and* $\alpha\in\mathbb{R}$, *then*

$$R_\alpha(\overline{S}(L_n))=(n-2)^\alpha[4(m-2)^{\alpha+1}+2(n-2)(m-3)^{\alpha+1}].$$

Proof. From Theorem 2.7, we have

$$R_\alpha(\overline{S}(L_n))=(n-2)^\alpha\left[\sum_{u\in V(L_n)}[m-d_{L_n}(u)]^{\alpha+1}\right].$$

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Since the ladder L_n has four vertices of degree two and $2(n-2)$ vertices of degree three,

$$\sum_{u\in V(L_n)}[m-d_{L_n}(u)]^{\alpha+1}=4(m-2)^{\alpha+1}+2(n-2)(m-3)^{\alpha+1}.$$

Corollary 2.12 ([3]) *If G is an (n, m) graph and $\alpha\in\mathbb{R}$, then*

$$M_2(\overline{S}(G))=(n-2)\Big[m^2(n-4)+M_1(G)\Big].$$

Theorem 2.13 *If G is any (n, m) graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_2(G))=4^\alpha\Big[R_\alpha(G)+M_1^{\alpha+1}(G)\Big].$$

Proof. Since $T_2(G)$ has $n+m$ vertices and $3m$ edges,

$$R_\alpha(T_2(G))=\sum_{xy\in E(T_2(G))}[d_{T_2(G)}(x)d_{T_2(G)}(y)]^\alpha$$
$$=\sum_{uv\in E(T_2(G))\cap E(G)}[d_{T_2(G)}(u)d_{T_2(G)}(v)]^\alpha+\sum_{ue\in E(T_2(G)\cap E(S(G))}[d_{T_2(G)}(u)\,d_{T_2(G)}(e)]^\alpha.$$

Note that $d_{T_2(G)}(u)=2d_G(u)$ for $u\in V(G)$ and $d_{T_2(G)}(e)=2$ for $e=uv\in E(G)$. Hence

$$R_\alpha(T_2(G))=\sum_{uv\in E(G)}[2d_G(u)2d_G(v)]^\alpha+\sum_{u\sim e}[2d_G(u)\cdot 2]^\alpha$$
$$=4^\alpha\sum_{uv\in E(G)}[d_G(u)d_G(v)]^\alpha+4^\alpha\sum_{u\sim e}[d_G(u)]^\alpha$$
$$=4^\alpha\left(R_\alpha(G)+\sum_{u\in V(G)}d_G(u)[d_G(u)]^\alpha\right).$$

Corollary 2.14 *If P_n is a path and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_2(P_n))=4^\alpha[(n-1)\cdot 2^{\alpha+1}+(n-3)\cdot 4^\alpha+2].$$

Proof. From Theorem 2.13, we have

$$R_\alpha(T_2(P_n))=4^\alpha\left\{R_\alpha(P_n)+M_1^{\alpha+1}(P_n)\right\}.$$

Since the path P_n contains two vertices of degree one and $(n-2)$ vertices of degree two,

$R_\alpha(P_n)=2^{\alpha+1}+(n-3)\cdot 4^\alpha$ and

$$M_1^{\alpha+1}(P_n)=\sum_{u\in V(P_n)}[d_{P_n}(u)]^{\alpha+1}=2+(n-2)\cdot 2^{\alpha+1}.$$

Hence

$$R_\alpha(T_2(P_n))=4^\alpha[(n-1)\cdot 2^{\alpha+1}+(n-3)\cdot 4^\alpha+2].$$

Corollary 2.15 *If W_{n+1} is a wheel graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_2(W_{n+1}))=4^\alpha n[9^\alpha+(3n)^\alpha+3^{\alpha+1}+n^\alpha].$$

Proof. From Theorem 2.13, we have

$$R_\alpha(T_2(W_{n+1}))=4^\alpha\{R_\alpha(W_{n+1})+M_1^{\alpha+1}(W_{n+1})\}$$

Since the wheel W_{n+1} contains n vertices of degree three and one vertex of degree n,

$R_\alpha(W_{n+1})=n[9^\alpha+(3n)^\alpha]$ and

$$M_1^{\alpha+1}(W_{n+1})=\sum_{u\in V(W_{n+1})}[d_{W_{n+1}}(u)]^{\alpha+1}=n[3^{\alpha+1}+n^\alpha].$$

Hence $R_\alpha(T_2(W_{n+1}))=4^\alpha n[9^\alpha+(3n)^\alpha+3^{\alpha+1}+n^\alpha]$.

Corollary 2.16 *If $T_{n,k}$ is the tadpole graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_2(T_{n,k}))=\begin{cases}4^\alpha[3\cdot 6^\alpha+(n+k-4)\cdot 4^\alpha+3^{\alpha+1}\\ \quad+[2(n+k)-3]\cdot 2^\alpha+1] & if\ k>1\\ 4^\alpha[2\cdot 6^\alpha+(n-2)\cdot 4^\alpha+4\cdot 3^\alpha\\ \quad+(n-1)\cdot 2^{\alpha+1}+1] & if\ k=1.\end{cases}$$

Proof. From Theorem 2.13, we have

$$R_\alpha(T_2(T_{n,k}))=4^\alpha\{R_\alpha(T_{n,k})+M_1^{\alpha+1}(T_{n,k})\}$$

Since the tadpole $T_{n,k}$ has $n+k-2$ vertices of degree two, one vertex of degree three and one vertex of degree one,

$$R_\alpha(T_{n,k})=\begin{cases}3\cdot 6^\alpha+(n+k-4)\cdot 4^\alpha+2^\alpha & if\quad k>1\\ 2\cdot 6^\alpha+(n-2)\cdot 4^\alpha+3^\alpha & if\quad k=1\end{cases}$$

and

$$M_1^{\alpha+1}(T_{n,k})=3^{\alpha+1}+(n+k-2)\cdot 2^{\alpha+1}+1.$$

On substituting, we obtain the result.

Corollary 2.17 *If L_n, $n>2$ is the ladder and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_2(L_n))=\begin{cases}4^\alpha\{(3n-8)\cdot 9^\alpha+4\cdot 6^\alpha+2^{2\alpha+1}\\ \quad+2^{\alpha+3}+2(n-2)\cdot 3^{\alpha+1}\} & if\quad n>2\\ 4^{\alpha+1}\{2^{\alpha+1}+4^\alpha\} & if\quad n=2.\end{cases}$$

Proof. From Theorem 2.13, we have

$$R_\alpha(T_2(L_n))=4^\alpha\{R_\alpha(L_n)+M_1^{\alpha+1}(L_n)\}.$$

The ladder L_n has four vertices of degree two and $2(n-2)$ vertices of degree three.

Therefore,

$$R_\alpha(L_n)=\begin{cases}2^{2\alpha+1}+4\cdot 6^\alpha+(3n-8)\cdot 9^\alpha & if\quad n>2\\ 4^{\alpha+1} & if\quad n=2\end{cases}$$

and

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$$M_1^{\alpha+1}(L_n)=\sum_{u\in V(L_n)}[d_{L_n}(u)]^{\alpha+1}=2^{\alpha+3}+2(n-2)\cdot 3^{\alpha+1}.$$

On substituting we obtain the result.

Theorem 2.18 *If G is an (n, m) graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T_1(G))=\sum_{uv\in E(G)}\left([d_G(u)^\alpha+d_G(v)^\alpha][d_G(u)+d_G(v)]^\alpha\right)$$
$$+\sum_{uvw\in E_2(G)}\{[d_G(u)+d_G(v)]^\alpha\,[d_G(v)+d_G(w)]^\alpha\},$$

where $E_2(G)$ is set of all pairs of adjacent edges.

Proof. Since $T_1(G)$ has $n+m$ vertices and $m+\frac{1}{2}M_1$ edges,

$$R_\alpha(T_1(G))=\sum_{xy\in E(T_1(G))}[d_{T_1(G)}(x)\,d_{T_1(G)}(y)]^\alpha$$
$$=\sum_{ue\in E(T_1(G))\cap E(S(G))}[d_{T_1(G)}(u)d_{T_1(G)}(v)]^\alpha$$
$$+\sum_{e_ie_j\in E(T_1(G))\cap E(L(G))}[d_{T_1(G)}(e_i)d_{T_1(G)}(e_j)]^\alpha$$

Note that $d_{T_1(G)}(u)=d_G(u)$ for $u\in V(G)$ and $d_{T_1(G)}(e)=d_G(u)+d_G(v)$ for $e=uv\in E(G)$.

$$R_\alpha(T_1(G))=\sum_{u\sim e,\,e=uv}[d_G(u)\{d_G(u)+d_G(v)\}]^\alpha$$
$$+\sum_{e_i\sim e_j,\,e_i=uv,\,e_j=vw}\left\{[d_G(u)+d_G(v)][d_G(v)+d_G(w)]\right\}^\alpha$$
$$=\sum_{uv\in E(G)}([d_G(u)\{d_G(u)+d_G(v)\}]^\alpha$$
$$+[d_G(v)\{d_G(u)+d_G(v)\}]^\alpha)$$
$$+\sum_{uv,vw\in E(G)}\{[d_G(u)+d_G(v)][d_G(v)+d_G(w)]\}^\alpha.$$

Theorem 2.19 *If G is an (n, m) graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(T(G))=4^\alpha R_\alpha(G)$$
$$+2^\alpha\sum_{uv\in E(G)}\left([d_G(u)^\alpha+d_G(v)^\alpha][d_G(u)+d_G(v)]^\alpha\right)$$
$$+\sum_{uvw\in E_2(G)}\{[d_G(u)+d_G(v)]^\alpha\,[d_G(v)+d_G(w)]^\alpha\},$$

where $E_2(G)$ is set of all pairs of adjacent edges.

Proof. Since $T(G)$ has $n+m$ vertices and $2m+\frac{1}{2}M_1$ edges,

$$R_\alpha(T(G))=\sum_{uv\in E(T(G))\cap E(G)}[d_{T(G)}(u)\,d_{T(G)}(v)]^\alpha$$
$$+\sum_{ue\in E(T(G))\cap E(S(G))}[d_{T(G)}(u)\,d_{T(G)}(e)]^\alpha$$
$$+\sum_{e_ie_j\in E(T(G))\cap E(L(G))}[d_{T(G)}(e_i)\,d_{T(G)}(e_j)]^\alpha$$

Note that $d_{T(G)}(u)=2d_G(u)$ for $u\in V(G)$ and $d_{T(G)}(e)=d_G(u)+d_G(v)$ for $e=uv\in E(G)$.

$$R_\alpha(T(G))=\sum_{uv\in E(G)}[2d_G(u)\cdot 2d_G(v)]^\alpha$$
$$+\sum_{u\sim e,e=uv}[2d_G(u)\{d_G(u)+d_G(v)\}]^\alpha$$
$$+\sum_{e_i\sim e_j,\,e_i=uv,\,e_j=vw}\left\{[d_G(u)+d_G(v)][d_G(v)+d_G(w)]\right\}^\alpha$$
$$=4^\alpha R_\alpha(G)+2^\alpha\sum_{uv\in E(G)}([d_G(u)\{d_G(u)+d_G(v)\}]^\alpha$$
$$+[d_G(v)\{d_G(u)+d_G(v)\}]^\alpha)$$
$$+\sum_{uv,vw\in E(G)}\{[d_G(u)+d_G(v)][d_G(v)+d_G(w)]\}^\alpha.$$

Theorem 2.20 *If G is an (n, m) graph and $\alpha<0$, then*

$$R_\alpha(L(G))\le 4^\alpha(\delta-1)^{2\alpha}\left(\frac{M_1}{2}-m\right)$$
$$R_\alpha(L(G))\ge 4^\alpha(\Delta-1)^{2\alpha}\left(\frac{M_1}{2}-m\right)$$

the equalities hold if and only if G is a regular graph.

Proof. The line graph $L(G)$ has m vertices, $\frac{M_1}{2}-m$ edges and $d_{L(G)}(e)=d_G(u)+d_G(v)-2$ for $e=uv\in E(G)$. Therefore,

$$R_\alpha(L(G))=\sum_{e_ie_j\in E(L(G))}[d_{L(G)}(e_i)d_{L(G)}(e_j)]^\alpha$$
$$=\sum_{e_i\sim e_j,\,e_i=uv,\,e_j=vw}\{[d_G(u)+d_G(v)-2]$$
$$[d_G(v)+d_G(w)-2]\}^\alpha$$

Note that $d_G(u)\le\Delta$ and $d_G(u)\ge\delta$ for any vertex $u\in V(G)$. The equalities hold if and only if G is a regular graph. Also given that $\alpha<0$, we have

$$R_\alpha(L(G))\le\sum_{e_i\sim e_j}[2\delta-2]^{2\alpha}=4^\alpha(\delta-1)^{2\alpha}\left(\frac{M_1}{2}-m\right).$$

Similarly, we can prove the other side inequality.

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Theorem 2.21 *If G is an (n, m) graph and $\alpha<0$, then*

$$R_\alpha(J(G)) \le (m+1-2\delta)^{2\alpha}\left[\binom{m+1}{2}-\frac{1}{2}M_1\right]$$

$$R_\alpha(J(G)) \ge (m+1-2\Delta)^{2\alpha}\left[\binom{m+1}{2}-\frac{1}{2}M_1\right]$$

the equalities hold if and only if G is a regular graph.

Proof. The jump graph has m vertices, $\binom{m+1}{2}-\frac{1}{2}M_1$ edges and $d_{J(G)}(e)=m+1-d_G(u)-d_G(v)$ for $e=uv\in E(G)$.

Therefore, by using definition of $R_\alpha(G)$ and above information, we obtain the inequalities.

Wu and Meng in [15] defined total transformation graphs G^{xyz}.

Definition 2.1 Let $G=(V(G),E(G))$ be a graph and x, y, z be three variables taking values $+$ or $-$. The *total transformation graph* G^{xyz} is the graph having $V(G)\cup E(G)$ as the vertex set, and α, $\beta\in V(G)\cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\alpha,\beta\in V(G)$, α, β are adjacent in *G* if $x=+$ and α, β are not adjacent in *G* if $x=-$.

(ii) $\alpha,\beta\in E(G)$, α, β are adjacent in *G* if $y=+$ and α, β are not adjacent in *G* if $y=-$.

(iii) $\alpha\in V(G)$ and $\beta\in E(G)$, α, β are incident in *G* if $z=+$ and α, β are not incident in *G* if $z=-$.

Since there are eight distinct $3-$permutatoins of $\{+,-\}$, there are eight different transformations of a given graph *G*. It is interesting to see that G^{+++} is just the total graph $T(G)$, G^{---} is the complement of $T(G)$ and G^{-++} is quasi-total graph [1]. Also note that $\overline{G^{++-}}=G^{--+}$, $\overline{G^{+-+}}=G^{-+-}$, and $\overline{G^{-++}}=G^{+--}$.

Theorem 2.22 *If G is an (n, m) graph and $\alpha\in\mathbb{R}$, then*

$$R_\alpha(G^{-++})=(n-1)^{2\alpha}\left[\binom{n}{2}-m\right]+2(n-1)^\alpha\chi_\alpha(G)$$
$$+\sum_{uvw\in E_2(G)}\{[d_G(u)+d_G(v)]^\alpha\,[d_G(v)+d_G(w)]^\alpha\},$$

where $E_2(G)$ is set of all pairs of adjacent edges.

Proof. The graph G^{-++} has $n+m$ vertices, $\binom{n}{2}+\frac{1}{2}M_1$ edges, $d_{G^{-++}}(u)=n-1$ for $u\in V(G)$ and $d_{G^{-++}}(e)=d_G(u)+d_G(v)$ for $e=uv\in E(G)$. Therefore,

$$R_\alpha(G^{-++})=\sum_{uv\in E(\overline{G})}\{(n-1)(n-1)\}^\alpha$$
$$+\sum_{u\sim e,\, e=uv}[(n-1)\{d_G(u)+d_G(v)\}]^\alpha$$
$$+\sum_{e_i\sim e_j,\, e_i=uv,\, e_j=vw}\{[d_G(u)+d_G(v)][d_G(v)+d_G(w)]\}^\alpha.$$

Theorem 2.23 *If G is an (n, m) graph and $\alpha<0$, then $\gamma_1\le R_\alpha(G^{++-})\le\gamma_2$, where*

$$\gamma_1=m^{2\alpha+1}+m^{\alpha+1}(n-2)\left[n-4+2\Delta\right]^\alpha+\left[\frac{1}{2}M_1-m\right][n-4+2\Delta]^{2\alpha}$$

$$\gamma_2=m^{2\alpha+1}+m^{\alpha+1}(n-2)\left[n-4+2\delta\right]^\alpha+\left[\frac{1}{2}M_1-m\right][n-4+2\delta]^{2\alpha}$$

the equalities hold if and only if G is a regular graph.

Proof. The graph G^{++-} has $n+m$ vertices, $m(n-2)+\frac{1}{2}M_1$ edges, $d_{G^{++-}}(u)=m$ for $u\in V(G)$ and $d_{G^{++-}}(e)=n-4+d_G(u)+d_G(v)$ for $e=uv\in E(G)$. Therefore,

$$R_\alpha(G^{++-})=\sum_{u\sim v}[m\cdot m]^\alpha$$
$$+\sum_{u\nsim e,e=vw}[m(n-4+d_G(v)+d_G(w))]^\alpha$$
$$+\sum_{e_i\sim e_j,\, e_i=uv,\, e_j=vw}\{[n-4+d_G(u)+d_G(v)][n-4+d_G(v)+d_G(w)]\}^\alpha$$

Note that $d_G(u)\le\Delta$ and $d_G(u)\ge\delta$ for any vertex $u\in V(G)$. The equalities hold if and only if G is a regular graph.

$$R_\alpha(G^{++-})\ge\sum_{u\sim v}(m)^{2\alpha}+\sum_{u\nsim e}\{m[n-4+2\Delta]\}^\alpha+\sum_{e_i\sim e_j}[n-4+2\Delta]^{2\alpha}\text{ as }\alpha<0$$

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$$R_\alpha(G^{++-}) \geq m^{2\alpha+1} + m^{\alpha+1}(n-2)\left[n-4+2\Delta\right]^\alpha + \left[\frac{1}{2}M_1 - m\right][n-4+2\Delta]^{2\alpha}.$$

Similarly, we can compute the other side inquality.

Theorem 2.24 *If G is an* (n, m) *graph and* $\alpha < 0$ *, then* $\gamma_1 \leq R_\alpha(G^{-+-}) \leq \gamma_2$ *, where*

$$\gamma_1 = (n+m-1-2\Delta)^{2\alpha}\left[\binom{n}{2} - m\right] + (n-4+2\Delta)^{2\alpha}\left[\frac{1}{2}M_1 - m\right] + m(n-2)(n+m-1-2\Delta)^\alpha(n-4+2\Delta)^\alpha$$

$$\gamma_2 = (n+m-1-2\delta)^{2\alpha}\left[\binom{n}{2} - m\right] + (n-4+2\delta)^{2\alpha}\left[\frac{1}{2}M_1 - m\right] + m(n-2)(n+m-1-2\delta)^\alpha(n-4+2\delta)^\alpha;$$

the equalities hold if and only if G is a regular graph.

Proof. The graph G^{-+-} has $\binom{n}{2} + \frac{1}{2}M_1 + m(n-4)$ edges, $d_{G^{-+-}}(u) = n+m-1-2d_G(u)$ for $u \in V(G)$ $d_{G^{-+-}}(e) = n-4+d_G(u)+d_G(v)$ for $e = uv \in E(G)$. Therefore,

$$R_\alpha(G^{-+-}) = \sum_{uv \in E(\overline{G})} \{[n+m-1-2d_G(u)][n+m-1-2d_G(v)]\}^\alpha + \sum_{e_i \sim e_j,\, e_i=uv,\, e_j=vw} \{[n-4+d_G(u)+d_G(v)][n-4+d_G(v)+d_G(w)]\}^\alpha + \sum_{u \nsim e, e=vw} \{[n+m-1-2d_G(u)][n-4+d_G(v)+d_G(w)]\}^\alpha.$$

Note that $d_G(u) \leq \Delta$ and $d_G(u) \geq \delta$ for any vertex $u \in V(G)$. The equalities hold if and only if *G* is a regular graph. On simlifying we get the desired result.

Theorem 2.25 *If G is an* (n, m) *graph and* $\alpha < 0$*, then*

(1) $\gamma_1 \leq R_\alpha(G^{+--}) \leq \gamma_2$ *, where*

$$\gamma_1 = m^{2\alpha+1} + m^{\alpha+1}(n-2)\left[n+m-1-2\Delta\right]^\alpha + (n+m-1-2\Delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right],$$

$$\gamma_2 = m^{2\alpha+1} + m^{\alpha+1}(n-2)\left[n+m-1-2\delta\right]^\alpha + (n+m-1-2\delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right].$$

(2) $\gamma_1 \leq R_\alpha(G^{--+}) \leq \gamma_2$ *, where*

$$\gamma_1 = (m+3-2\Delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right] + (n-1)^{2\alpha}\left[\binom{n}{2} - m\right] + 2m(n-1)^\alpha(m+3-2\Delta)^\alpha,$$

$$\gamma_2 = (m+3-2\delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right] + (n-1)^{2\alpha}\left[\binom{n}{2} - m\right] + 2m(n-1)^\alpha(m+3-2\delta)^\alpha.$$

.

(3) $\gamma_1 \leq R_\alpha(G^{+-+}) \leq \gamma_2 \leq$*, where*

$$\gamma_1 = 4^\alpha R_\alpha(G) + (m+3-2\Delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right] + 2^{\alpha+1}m[\Delta]^\alpha(m+3-2\Delta)^\alpha,$$

$$\gamma_2 = 4^\alpha R_\alpha(G) + (m+3-2\delta)^{2\alpha}\left[\binom{m+1}{2} - \frac{1}{2}M_1\right] + 2^{\alpha+1}m[\delta]^\alpha(m+3-2\delta)^\alpha.$$

(4) $\gamma_1 \leq R_\alpha(G^{---}) \leq \gamma_2$ *, where*

$$\gamma_1 = \left[n+m-1-2\Delta\right]^{2\alpha}\left[\binom{n}{2} + \binom{m}{2} + m(n-2) - \frac{1}{2}M_1\right],$$

$$\gamma_2 = \left[n+m-1-2\delta\right]^{2\alpha}\left[\binom{n}{2} + \binom{m}{2} + m(n-2) - \frac{1}{2}M_1\right].$$

The equalities hold if and only if G *is a regular graph.*

III. Conclusion

In this paper, we have obtained the closed formulae for general product-connectivity index of subdivision graph, partial complement of subdivision graph, semitotal-point graph,

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semitotal-line graph and total graph. Also bounds for general product-connectivity index of line graph, jump graph, and total transformation graphs. Note that, if $\alpha > 0$, then the opposite inequality is valid for all graphs. However obtaining the closed formulae for some G^{xyz} graphs is difficult.

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