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Some Fixed Point Theorems in $\varphi - \psi$ weak contraction on Fuzzy Metric Space

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Abstract— In this paper, we discuss some results on fixed point theorems in $\varphi - \psi$ weak contraction on fuzzy metric spaces, which are study of generalisation of some existing results are also given in the form of corollary.

Keywords— *fuzzy metric space, continuous t-norm, $\varphi - \psi$ weak contraction*

I. INTRODUCTION

In 1965, Zadeh[17] initiated the concept of fuzzy sets theory. In 1975, Kramosil and Michalek [12] introduced concept of fuzzy metric space. In 1988, Grabiec [6] propounded of the Banach contraction theorem in fuzzy metric spaces. Here after George and Veeramani [5] modified the definition of fuzzy metric spaces from [12]. Many researchers using [5, 6, 13, 14] and developing fuzzy metric spaces see,[1, 9, 10]. In 1984, M.S.Khan, M.Swaleh and S.Sessa[8] discussed Banach fixed point theorem in metric spaces via altering distance function. In 2012, Y.Shen, Dong Qiu and Wei Chen [16] proved fuzzy metric space using altering distance function as follows

$$\varphi(M(Tx,Ty,t)) \geq k(t).\varphi(M(x,y,t)), \forall x,y \in X, t > 0, (1.1)$$

obtained fixed point result for self-mapping of T. Recently, many authors using altering distance function and give their contribution in various metric spaces [3, 4].

In this paper, we proved some fixed point theorems in $\varphi - \psi$ weak contraction on fuzzy metric spaces, which are our study of generalisation of some existing results.

Definition 1.1 A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = (x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]$ .In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A , the second element $\mu_A(x)$ belongs to the interval $[0,1]$, is called the membership function.

Definition 1.2 A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0,1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Example 1.3

1. Lukasievicz t -norm: $a * b = \max(a + b - 1, 0)$
2. Product t -norm: $a * b = a.b$
3. Minimum t -norm: $a * b = min(a, b)$

Definition 1.4 A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a nonempty set, $*$ is a continuous t -norm and M is a fuzzy set on $X \times X \times (0, \infty) \to [0,1]$ satisfies the following conditions: $\forall x, y, z \in X$ and $s, t > 0$

1. $M(x, y, 0) = 0, t > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y, t > 0$
3. $M(x, y, t) = M(y, x, t)$

4. $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s)$
5. $M(x,y,.)\colon [0,\infty) \to [0,1]$ is left-continuous.

Then M is called a fuzzy metric on X .

Definition 1.5 A fuzzy metric space is an ordered triple such that X is a non-empty set, $*$ is a continuous t -norm and M is a fuzzy set on $X \times X \times (0,\infty) \to [0,1]$ satisfies the following
conditions: $\forall x,y,z \in X$ *and* $s,t > 0$

1. $M(x,y,t) > 0$, $t > 0$
2. $M(x,y,t) = 1$ if and only if $x = y, t > 0$
3. $M(x,y,t) = M(y,x,t)$
4. $M(x,z,t+s) * M(y,z,s) \leq M(x,z,t+s)$
5. $M(x,y,.)\colon (0,\infty) \to [0,1]$ is continuous.

Then M is called a fuzzy metric on X .

Definition 1.6 Let $(X,M,*)$ be a fuzzy metric space, for $t > 0$ the open ball $B(x,r,t)$ with a centre $x \in X$ and a radius $0 < r < 1$ is defined by
$B(x,r,t) = y \in X\colon M(x,y,t) > 1-r$.

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x,r,t) \subset A$. Let τ denote the family of all open subsets of X . Then τ is topology on X , called the topology induced by the fuzzy metric M .

Definition 1.7 Let $(X,M,*)$ be a fuzzy metric space

1. A sequence x_n in X is said to be convergent to a point x in $(X,M,*)$ if $\lim_{t\to\infty} M(x,y,t) = 1$ for all $t > 0$.
2. sequence x_n in X is called a Cauchy sequence in $(X,M,*)$, if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that $M(x_n,x_m,t) > 1-\varepsilon$ for each $n,m \geq n_0$
3. A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.
4. A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact

Lemma 1.8 Let $(X,M,*)$ be a fuzzy metric space. For all $u,v \in X$, $M(u,v,.)$ is non-decreasing function.
Proof. If $M(u,v,t) > M(u,v,s)$ for some $0 < t < s$.
Then $M(u,v,t) * M(v,v,s-t) \leq M(u,v,s) < M(u,v,t)$,
Thus $M(u,v,t) < M(u,v,t) < M(u,v,t)$,
(since $M(v,v,s-t) = 1$)
which is a contradiction

Definition 1.9 A function $\varphi\colon [0,1] \to [0,1]$ is called control function or an altering distance function if it satisfies the following properties:
(CF1). φ is strictly decreasing and continuous;
(CF2). $\varphi(\lambda) \geq 0$, $\forall \lambda \neq 1$ if $\varphi(\lambda) = 0$ if and only if $\lambda = 1$. It is obvious that $\lim_{\lambda\to 1^-} \varphi(\lambda) = \varphi(1) = 0$.
where φ in class of function Φ.

II. Main Results

Theorem 2.1 Let $(X,M,*)$ be a complete strong fuzzy metric space with continuous t-norm $*$ and let T is a self-mapping in X such that

$$\begin{aligned}\varphi(M(Tu,Tv,t)) \leq\ & \varphi(M(u,Tu,t)) + M(v,Tv,t) + M(Tu,v,t) \\ & + M(u,Tv,t) + M(u,v,t) + \max(M(u,Tu,t), M(v,Tv,t)) \\ & - \psi(M(u,Tu,t) + M(v,Tv,t) + M(Tu,v,t) + \\ & M(u,Tv,t) + M(u,v,t) + \max(M(u,Tu,t), M(v,Tv,t)),\end{aligned} \tag{2.1}$$

where φ and ψ are altering distance function and ultra altering distance function respectively, $\varphi(t+s) \leq \varphi(t) + \varphi(s)$ for all $t,s \in S$ Then T has a unique fixed point in X.

Proof. Let u be any arbitrary point in X and define a sequence $u_n \in X$ such that $u_{n+1} = Tu_n$.
Assume that $u_{n+1} = Tu_n = u_n$ for some $n \in \mathbb{N}$, then u_n is a fixed point of T.

Suppose $u_{n+1} \neq u_n$, put $u = u_{n-1}$ and $v = u_n$ in equation (2.1) we get

$$\varphi(M(Tu_{n-1}, Tu_n, t)) \leq \varphi(M(u_{n-1}, Tu_{n-1}, t) + M(u_n, Tu_n, t) + M(Tu_{n-1}, u_n, t)(M(u_{n-1}, Tu_n, t) + M(u_{n-1}, u_n, t) + \max(M(u_{n-1}, Tu_{n-1}, t), M(u_n, Tu_n, t)) - \psi(M(u_{n-1}, Tu_{n-1}, t) + M(u_n, Tu_n, t) + M(Tu_{n-1}, u_n, t)M(u_{n-1}, Tu_n, t) + M(u_{n-1}, u_n, t) + \max(M(u_{n-1}, Tu_{n-1}, t), M(u_n, Tu_n, t)) \qquad (2.2)$$

$$\leq \varphi(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + (M(u_n, u_n, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) - \psi(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + (M(u_n, u_n, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))$$

$$\leq \varphi(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + (M(u_n, u_n, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) - \psi(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + (M(u_n, u_n, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))$$

$$\leq \varphi(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) - \psi M(u_{n-1}, u_n, t) + (M(u_n, u_{n+1}, t)) + (M(u_{n-1}, u_{n+1}, t)) + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))$$

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_n, t)) + \varphi\max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) \qquad (2.3)$$

Here $(X, M,*)$ is a strong fuzzy metric space then we have

$M(u_{n-1}, u_{n+1}, t) \geq M(u_{n-1}, u_n, t) * M(u_n, u_{n+1}, t)$ (by using (GV4'))

$$\varphi(M(u_{n-1}, u_{n+1}, t)) \geq \varphi((M(u_{n-1}, u_n, t)) * (M(u_n, u_{n+1}, t)))$$

$$\varphi(M(u_{n-1}, u_{n+1}, t)) \geq \varphi((M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t))) \qquad (2.4)$$

Using above inequalities in (2.3) we get

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi((M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + [(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t))] + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))) - \psi((M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t)) + [(M(u_{n-1}, u_n, t)) + (M(u_n, u_{n+1}, t))] + (M(u_{n-1}, u_n, t)) + \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)))$$

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_n, t)) + \varphi\max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) \qquad (2.5)$$

If $\max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) = M(u_{n-1}, u_n, t)$(2.6)

Then the above inequality (2.4) becomes

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t)) + \varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_{n-1}, u_n, t))$$

Continuing this process, we get,

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi(M(u_{n-1}, u_n, t)) < \varphi(M(u_{n-1}, u_n, t)) \qquad (2.7)$$

Similarly,

$$\text{If} \quad \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) = M(u_n, u_{n+1}, t) \qquad (2.8)$$

Then the inequality (2.5) becomes

$$\varphi(M(u_n, u_{n+1}, t)) \leq \varphi(M(u_{n-1}, u_n, t)) < \varphi(M(u_{n-1}, u_n, t)) \qquad (2.9)$$

Hence $\varphi(M(u_n, u_{n+1}, t)) < \varphi(M(u_{n-1}, u_n, t))$

This gives $(M(u_n, u_{n+1}, t)) > (M(u_{n-1}, u_n, t))$

Since the sequence $M(u_n, u_{n+1}, t)$ is non decreasing
Taking limit $n \to \infty$, we get

$$\lim_{n\to\infty} M(u_n, u_{n+1}, t) = q(r), \text{for} q: (0, \infty) \to [0,1] \tag{2.10}$$

Suppose that $q(r) \neq 1$ for some $r > 0$ as $n \to \infty$,
Now (2.7) becomes,

$$\varphi(q(r)) \leq \varphi(q(r)) < \varphi(q(r)) \tag{2.11}$$

which is a contradiction.
Hence $\lim_{n\to\infty} M(u_n, u_{n+1}, t) = 1, t > 0$
Next we prove that the sequence u_n is a Cauchy's sequence.
Assume that u_n is not a Cauchy's sequence then for any $0 < \varepsilon < 1, t > 0$ then there exists sequence u_{n_k} and u_{m_k} where $n_k, m_k \geq n$ and $n_k, m_k \in \mathbb{N}(n_k > m_k)$

$$\text{suchthat} \quad M(u_{n_k}, u_{m_k}, t) \leq 1 - \varepsilon \tag{2.12}$$

Let n_k be least integer exceeding m_k satisfying the above property

$$\therefore \quad M(u_{n_k-1}, u_{m_k}, t) > 1 - \varepsilon, \quad n_k, m_k \in \mathbb{N} \quad \text{and} \quad t > 0 \tag{2.13}$$

Put $u = u_{n_k-1}$ and $v = u_{m_k-1}$

$$\begin{aligned}\varphi(M(Tu_{n_k-1}, Tu_{m_k-1}, t)) \leq\ & \varphi((M(u_{n_k-1}, Tu_{n_k-1}, t)) + (M(u_{m_k-1}, Tu_{m_k-1}, t)) \\ & +(M(Tu_{n_k-1}, u_{m_k-1}, t))(M(u_{n_k-1}, Tu_{m_k-1}, t)) \\ & +(M(u_{n_k-1}, u_{m_k-1}, t)) + \\ & \max(M(u_{n_k-1}, Tu_{n_k-1}, t), M(u_{m_k-1}, Tu_{m_k-1}, t))) \\ & -\psi((M(u_{n_k-1}, Tu_{n_k-1}, t)) + (M(u_{m_k-1}, Tu_{m_k-1}, t)) \\ & +(M(Tu_{n_k-1}, u_{m_k-1}, t))(M(u_{n_k-1}, Tu_{m_k-1}, t)) \\ & +(M(u_{n_k-1}, u_{m_k-1}, t)) + \\ & \max(M(u_{n_k-1}, Tu_{n_k-1}, t), M(u_{m_k-1}, Tu_{m_k-1}, t)))\end{aligned}$$

$$\begin{aligned}\varphi(M(u_{n_k}, u_{m_k}, t)) \leq\ & \varphi((M(u_{n_k-1}, u_{n_k}, t)) + (M(u_{m_k-1}, u_{m_k}, t)) \\ & +(M(u_{n_k}, u_{m_k-1}, t))(M(u_{n_k-1}, u_{m_k}, t)) \\ & +(M(u_{n_k-1}, u_{m_k-1}, t)) \\ & +\max(M(u_{n_k-1}, u_{n_k}, t), M(u_{m_k-1}, u_{m_k}, t))) \\ & -\psi((M(u_{n_k-1}, u_{n_k}, t)) + (M(u_{m_k-1}, u_{m_k}, t)) \\ & +(M(u_{n_k}, u_{m_k-1}, t))(M(u_{n_k-1}, u_{m_k}, t)) \\ & +(M(u_{n_k-1}, u_{m_k-1}, t)) \\ & +\max(M(u_{n_k-1}, u_{n_k}, t), M(u_{m_k-1}, u_{m_k}, t)))\end{aligned} \tag{2.14}$$

If $\max(M(u_{n_k-1}, u_{n_k}, t), M(u_{m_k-1}, u_{m_k}, t)) = M(u_{n_k-1}, u_{n_k}, t)$

$$\begin{aligned}\varphi(M(u_{n_k},u_{m_k},t)) &\le \varphi((M(u_{n_k-1},u_{n_k},t))+(M(u_{m_k-1},u_{m_k},t))\\&+(M(u_{n_k},u_{m_k-1},t))(M(u_{n_k-1},u_{m_k},t))+\\&+(M(u_{n_k-1},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\\&-\psi((M(u_{n_k-1},u_{n_k},t))+(M(u_{m_k-1},u_{m_k},t))\\&(M(u_{n_k},u_{m_k-1},t))(M(u_{n_k-1},u_{m_k},t))\\&+(M(u_{n_k-1},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\\&\le \varphi((M(u_{n_k-1},u_{n_k},t))+(M(u_{m_k-1},u_{m_k},t))\\&+(M(u_{n_k},u_{m_k-1},t))(M(u_{n_k-1},u_{m_k},t))\\&+(M(u_{n_k-1},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\\&-\psi((M(u_{n_k-1},u_{n_k},t))+(M(u_{m_k-1},u_{m_k},t))\\&+(M(u_{n_k},u_{m_k-1},t))(M(u_{n_k-1},u_{m_k},t))\\&+(M(u_{n_k-1},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\\&\le \varphi(M(u_{n_k-1},u_{n_k},t))+\varphi(M(u_{m_k-1},u_{m_k},t))\\&+\varphi(M(u_{n_k},u_{m_k-1},t))\varphi(M(u_{n_k-1},u_{m_k},t))\\&+\varphi(M(u_{n_k-1},u_{m_k-1},t))+\varphi(M(u_{n_k-1},u_{n_k},t))\end{aligned} \tag{2.15}$$

then we get,

$$\varphi(M(u_{n_k},u_{m_k-1},t)) \le \varphi(M(u_{n_k},u_{m_k},t))+\varphi(M(u_{m_k},u_{m_k-1},t)) \tag{2.16}$$

$$\text{and}\varphi(M(u_{n_k-1},u_{m_k-1},t)) \le \varphi(M(u_{n_k-1},u_{n_k},t))+\varphi(M(u_{n_k},u_{m_k-1},t))$$

Applying the previous inequalities we get

$$\begin{aligned}\varphi(M(u_{n_k-1},u_{m_k-1},t)) &\le \varphi(M(u_{n_k-1},u_{n_k},t))+\varphi(M(u_{n_k},u_{m_k},t))+\\&\varphi(M(u_{m_k},u_{m_k-1},t))\end{aligned} \tag{2.17}$$

Also (2.13) and (CF1) we get

$$\varphi(M(u_{n_k-1},u_{m_k},t)) \le \varphi(1-\varepsilon). \tag{2.18}$$

Substituting (2.16), (2.17), and (2.18) in (2.15) we have

$$\begin{aligned}\varphi(M(u_{n_k},u_{m_k},t)) &\le \varphi(M(u_{n_k-1},u_{n_k},t)+(M(u_{m_k-1},u_{m_k},t))\\&+(M(u_{n_k},u_{m_k},t))+(M(u_{m_k},u_{m_k-1},t))(1-\varepsilon)\\&+(M(u_{n_k-1},u_{n_k},t))+(M(u_{n_k},u_{m_k},t))\\&+(M(u_{m_k},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\\&-\psi(M(u_{n_k-1},u_{n_k},t)+(M(u_{m_k-1},u_{m_k},t))\\&+(M(u_{n_k},u_{m_k},t))+(M(u_{m_k},u_{m_k-1},t))(1-\varepsilon)\\&+(M(u_{n_k-1},u_{n_k},t))+(M(u_{n_k},u_{m_k},t))\\&+(M(u_{m_k},u_{m_k-1},t))+(M(u_{n_k-1},u_{n_k},t)))\end{aligned}$$

$$\begin{aligned}\varphi(M(u_{n_k},u_{m_k},t)) &\le \varphi(M(u_{n_k-1},u_{n_k},t)\\&+(M(u_{m_k},u_{m_k-1},t))\varphi(1-\varepsilon))\end{aligned} \tag{2.19}$$

Using (2.12) we obtain,

$$\varphi(M(u_{n_k},u_{m_k},t)) > \varphi(1-\varepsilon) \tag{2.20}$$

$$\begin{aligned}\varphi(1-\varepsilon) &\le \varphi(M(u_{n_k-1},u_{n_k},t)\\&+(M(u_{m_k},u_{m_k-1},t))\varphi(1-\varepsilon))\end{aligned} \tag{2.21}$$

Taking $k \to \infty$ in above inequality we obtain

$$\varphi(1-\varepsilon) \le \varphi(1-\varepsilon) \tag{2.22}$$

That is $\varphi(1-\varepsilon) \le 0,$

Which is a contradiction, $\varepsilon > 0$
Hence u_n is a Cauchy's sequence.
Since X is complete and there exist $z \in X$ such that $\lim_{n\to\infty} u_n = z$
That is $M(u_n, z, t) = 1$ as $n \to \infty$
Put $u = u_{n-1}$ and $v = z$ in equation (2.1) we get

$$\begin{aligned}\varphi(M(u_n, Tz, t)) \quad &\le \varphi(M(u_{n-1}, u_n, t) + (M(z, Tz, t)) \\ &+(M(u_n, z, t)) + (M(u_{n-1}, Tz, t)) \\ &+(M(u_{n-1}, z, t)) + \max(M(u_{n-1}, u_n, t), M(z, Tz, t)))\end{aligned} \quad (2.23)$$

Taking $n \to \infty$ in (2.23)

$$\varphi(M(z, Tz, t)) \le 0, t > 0 \quad (2.24)$$

Therefore, $M(z, Tz, t) = 1$, and $z = Tz$.
To prove Uniqueness,
Suppose that w is another fixed point of T , that is $Tw = w$ where $q \ne z$

$$\varphi(M(z, w, t)) \le 0, t > 0 \quad (2.25)$$

Hence $z = w$ is the unique fixed point of T.

Corollary 2.2 *Let $(X, M,*)$ be a complete strong fuzzy metric space with continuous t-norm $*$ and let T is a self-mapping in X . If there exists a control function φ and $= (t)$, such that*

$$\begin{aligned}\varphi(M(Tu, Tv, t)) \quad &\le \varphi(M(u, Tu, t)) + (M(v, Tv, t)) + (M(Tu, v, t)) + (M(u, Tv, t)) + (M(u, v, t)) \\ &-\psi(M(u, Tu, t)) + (M(v, Tv, t)) + (M(Tu, v, t)) + (M(u, Tv, t)) + (M(u, v, t))\end{aligned} \quad (2.26)$$

Then T has a unique fixed point in X.

Proof. The proof of the above theorem (2.1) considering the fuzzy contraction on the fuzzy metric space $(X, M,*)$,

$$\begin{aligned}\varphi(M(Tu, Tv, t)) \quad &\le \varphi(M(u, Tu, t) + (M(v, Tv, t)) \\ &+(M(Tu, v, t)) + (M(u, Tv, t)) + (M(u, v, t))) \\ &-\psi(M(u, Tu, t) + (M(v, Tv, t)) \\ &+(M(Tu, v, t)) + (M(u, Tv, t)) + (M(u, v, t))).\end{aligned}$$

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