International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Volume-5, Issue-1, pp.11-18, February (2018)
E-ISSN: 2348-4519

Magnetized Anti-stiff fluid Cosmological Models with Variable Cosmological Constant

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Available online at: www.isroset.org

Received 10/Jan/2018, Revised 24/Dec/2018, Accepted 22/Feb/2018, Online 28/Feb/2018

Abstract— Bianchi type-III and Kantowski Sachs cosmological model containing magnetic field with variable cosmological constant (Λ) has been studied in general theory of relativity. The general solutions of the Einstein's field equations for the cosmological models have been obtained under the assumption of anti-stiff fluid (relation between pressure and density). The behavior of the model in presence of magnetic field and singularities in the model are discussed in detail. Furthermore some physical and geometrical aspects of the model are discussed. Our derived model represents shearing, non-rotating and expanding model of the universe with big-bang starts with both scale factors is monotonically increasing function of *t*.

Keywords— *Bianchi type-III and Kantowaski Sachs model, anti-stiff fluid, variable* Λ

I. INTRODUCTION

Recent theoretical cosmological observations [1] suggested that the expansion of the universe is accelerating. In Einstein's general relativity, in order to have such acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as Dark Energy (DE). Also same observations indicate that our universe is flat and currently consists of approximately 2 /3 DE and 1/ 3 dark matter source. There are many radically different models for DE to fit the current observations such as quintessence, phantom, tachyon, chaplygin gas. In recent years, many authors [2-15] have shown keen interests in studying the DE universe with various contexts. However, the simplest and most theoretically appealing candidate for DE is vacuum energy (or the cosmological constant Λ) with a constant equation of state parameter equal to -1. Recent observations indicate that $\Lambda \approx 10^{-55} cm^{-2}$ while the particle physics prediction for Λ is greater than this value by a factor of order 10^{120} . This discrepancy is known as the cosmological constant problem. The simplest way out of this problem is to consider a varying cosmological term, which decays from a huge value at initial times to the small value observed in these days in an expanding universe [16-17].

It is interesting to note that magnetic field present in galactic and inter galactic spaces play a significant role at cosmological scale. The study of magnetic field in the matter distribution is of considerable interest as it provides an effective way to understand the initial phases of cosmic evolution. The inclusion of the magnetic field is motivated by the observational cosmology and astrophysics indicating that many subsystems of the universe possess magnetic fields. A cosmological model containing a global magnetic field is necessarily anisotropic. An understanding of the effect of a magnetic field upon the dynamics of the universe is necessary during early and late time evolution of the universe. During the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled to the gravitational field and subsequently forming neutral matter during expansion of the universe [18]. Bali and Meena [19] investigated magnetized stiff fluid tilted universe for perfect fluid distribution in general relativity. Banerjee and Banerjee [20] studied stationary distribution of dust and electromagnetic fields in general relativity. Banerjee et al. [21] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Recently, Bali and Upadhaya [22] have investigated LRS Bianchi Type I strings dust-magnetized cosmological models. [23-29] are some of the authors who have studied the cosmological models with magnetic field and have pointed out its important in the early evolution of the universe.

II. RELATED WORK

Metric and Energy Momentum Tensor

We consider a spatially homogeneous Bianchi type III and Kantowaski Sachs models of the form

$$ds^2 = dt^2 - R^2(t)dr^2 + S^2(t)\left(d\theta^2 + \Phi^2 d\phi^2\right), \tag{1}$$

where the metric potentials R and S are the functions of time *t*.

Above model (1) represents,

Bianchi type-III model if $\Phi = \sinh(\theta)$

Kantowski Sachs model if $\Phi = \sin(\theta)$

To discuss the kinematics of the universe, we need to define some kinematical parameters of the universe which has a great importance in cosmology.

Spatial volume and the scale factor for the metric (1) are

$$V = a^3 = \left(RS^2\right)\Phi . \tag{2}$$

The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, given as

$$H = \frac{1}{3}\frac{V_4}{V} = \frac{1}{3}\left(\frac{R_4}{R} + 2\frac{S_4}{S}\right). \tag{3}$$

Another important dimensionless kinematical quantity is the mean deceleration parameter q , which tells whether the universe exhibits accelerating volumetric expansion or not is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \tag{4}$$

for $-1 \le q < 0$, $q > 0$ and $q = 0$ the universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively.

To discuss whether the universe either approach isotropy or not, we define an anisotropy parameter of the expansion as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i^2}{H}\right)^2, \tag{5}$$

where $\Delta H_i = H_i - H$

The scalar expansion and shear scalar are defined as

$$\theta = \frac{R_4}{R} + 2\frac{S_4}{S} . \tag{6}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 . \tag{7}$$

The energy momentum tensor for perfect fluid in presence of magnetic field is given by

$$T_i^j = (p+\rho)v_i v^j - p g_i^j + E_i^j , \tag{8}$$

where ρ is the energy density, p be the isotropic pressure, and E_i^j is the electromagnetic field given as [30]

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{9}$$

with

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j \quad |h|^2 = h_l h^l , \tag{10}$$

where h_i be the magnetic flux vector, F_{ij} be the electromagnetic field tensor, $\in_{ijkl}$ be the Levi-Civita symbol, $\bar{\mu}$ be the magnetic permeability, and v^i be the flow velocity satisfying

$$g_{ij}v^i v^j = 1. \tag{11}$$

We assume that the coordinates to be commoving so that

$$v^1 = 0 = v^2 = v^3 , \; v^4 = 1 . \tag{12}$$

The incident magnetic field is taken along *x*-axis so that

$$h_1 \neq 0 , \; h_2 = 0 , \; h_3 = 0 , \; h_4 = 0 . \tag{13}$$

We assume that there is a magnetic field along *x*-direction. Hence, F_{23} is the only non-vanishing component of F_{ij} .

The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \text{ and } \frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 , \tag{14}$$

leads to

$$F_{23} = \text{constant} = H \text{ (say).} \tag{15}$$

where H is a constant characterizing the magnetic field intensity and $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conductivity (Roy Maartens [31]).

It follows from equation (10) that the non-zero component of magnetic flux vector is

$$h_1 = \frac{RH}{\bar{\mu}S^2 \sinh\theta} \tag{16}$$

Thus using (16) into (.9), the non-trivial components of E_i^j are given by

$$E_1^1 = -E_2^2 = \frac{H^2}{2\bar{\mu}S^4 \sinh^2\theta} = -E_3^3 = E_4^4. \tag{17}$$

The Field equation for the time dependant Λ is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} + \Lambda g_{ij} \tag{18}$$

where R_{ij} is the Ricci tensor and R is the Ricci scalar.

In the presence of magnetism and source given in equation (17) (8) and (9), the field equations (18) corresponding to the metric (1) lead to the following set of linearly independent differential equations

$$2\frac{S_{44}}{S} + \frac{S_4^2}{S^2} + \frac{m}{S^2} = -8\pi p - \Lambda + \frac{k^2}{S^4\Phi^2}, \tag{19}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4}{R}\frac{S_4}{S} = -8\pi p - \Lambda - \frac{k^2}{S^4\Phi^2}, \tag{20}$$

$$2\frac{R_4}{R}\frac{S_4}{S} + \frac{S_4^2}{S^2} + \frac{m}{S^2} + = 8\pi\rho - \Lambda + \frac{k^2}{S^4\Phi^2}, \tag{21}$$

where the suffix 4 denotes ordinary differentiation with respect to t and $k^2 = \frac{8\pi H^2}{2\bar{\mu}}$.

III. METHODOLOGY

Solution of the Field equations:

Equations (19) to (21) are the system of three independent equations in five unknowns R, S, p, ρ, Λ . Therefore, in order to fully determine the system, we use two constraining equations. To do that,

i) We assume the physically plausible conditions that the fluid is anti-stiff i.e.

$$p + \rho = 0 \tag{22}$$

ii) The comment of velocity red-shift relation for extragalactic sources recommends that Hubble expansion of the universe is isotropy within ≈ 30 percent today (Kristian and Sachs [32]).

To put more precisely, red-shift studies place the limit $(\sigma / H) \le 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Latter on Collin et al. [33] have pointed out that for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies that the condition (σ/θ) is constant. It gives

$$R = S^n, \tag{23}$$

where n is any constant.

Using equations (19), (21) and (22), we get

$$\frac{S_{44}}{S} - \frac{R_4}{R}\frac{S_4}{S} = 0 \tag{24}$$

which leads to

$$\frac{S_{44}}{S_4} - \frac{R_4}{R} = 0 \tag{25}$$

Using equation (25) and (23) we get

$$S = \left[(1-n)c_1 t + (1-n)c_2\right]^{\frac{1}{1-n}} \tag{26}$$

$$R = \left[(1-n)c_1 t + (1-n)c_2\right]^{\frac{n}{1-n}} \tag{27}$$

IV. RESULTS AND DISCUSSIONS

Bianchi type-III model ($m = -1$):

FRW models are homogeneous and isotropic. These isotropic models are unstable near the origin and fail to describe the early universe [46], but anisotropy plays a significant role in the models near t = 0, hence spatially homogeneous and anisotropic Bianchi type models are undertaken to study the universe at its early stage of evolution.

Moreover, these models help in obtaining more general cosmological models than the isotropic FRW models. Out of the different Bianchi type models, the LRS Bianchi-III model is more interesting to explain the phenomenon of the universe.

For Bianchi type-III model, above set of field equations (19) – (21) reduces to

$$2\frac{S_{44}}{S} + \frac{S_4^2}{S^2} - \frac{1}{S^2} = -8\pi p - \Lambda + \frac{k^2}{S^4\sinh^2\theta} \tag{28}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4}{R}\frac{S_4}{S} = -8\pi p - \Lambda - \frac{k^2}{S^4\sinh^2\theta} \tag{29}$$

$$2\frac{R_4}{R}\frac{S_4}{S} + \frac{S_4^2}{S^2} - \frac{1}{S^2} + = 8\pi\rho - \Lambda + \frac{k^2}{S^4\sinh^2\theta} \tag{30}$$

Using equations (26) and (27), spatially homogeneous and anisotropic magnetized Bianchi type-III the cosmological model becomes

$$ds^2 = dt^2 - [(1-n)c_1 t + (1-n)c_2]^{\frac{2n}{1-n}} dr^2 + [(1-n)c_1 t + (1-n)c_2]^{\frac{2}{1-n}} \left(d\theta^2 + \sinh^2 \theta d\phi^2\right) \quad (31)$$

Above equation represent a singular model and singularity exist at point $t = t_s = -c_2/c_1$.

Physical parameters:

We find the physical parameters which are important for describing the physical behavior of the universe, solving equations (28) - (30) and using the equations (26) and (27), we get the expressions for

Cosmological constant,

$$\Lambda = \left\{ \begin{array}{c} \dfrac{nc_1^2 - 2n^2c_1^2 - 3c_1^2}{2[(1-n)c_1 t + (1-n)c_2]^2} - \dfrac{1}{2[(1-n)c_1 t + (1-n)c_2]^{\frac{2}{1-n}}} \\ - \dfrac{k^2}{\sinh^2 \theta [(1-n)c_1 t + (1-n)c_2]^{\frac{4}{1-n}}} \end{array} \right\} \quad (32)$$

Isotropic pressure,

$$p = \frac{1}{16\pi} \left\{ \begin{array}{c} \dfrac{-4c_1^2 - 2c_1^2}{[(1-n)c_1 t + (1-n)c_2]^2} + \dfrac{4}{[(1-n)c_1 t + (1-n)c_2]^{\frac{2}{1-n}}} \\ + \dfrac{k^2}{\sinh^2 \theta [(1-n)c_1 t + (1-n)c_2]^{\frac{4}{1-n}}} \end{array} \right\} \quad (33)$$

Energy density,

$$\rho = \frac{1}{16\pi} \left\{ \begin{array}{c} \dfrac{-2n^2c_1^2 - 11nc_1^2 - 5c_1^2}{[(1-n)c_1 t + (1-n)c_2]^2} + \dfrac{5}{[(1-n)c_1 t + (1-n)c_2]^{\frac{2}{1-n}}} \\ + \dfrac{k^2}{\sinh^2 \theta [(1-n)c_1 t + (1-n)c_2]^{\frac{4}{1-n}}} \end{array} \right\} \quad (34)$$

For Kantowski Sachs model ($m = 1$):

According to the theoretical astronomic observation by Misner in 1968 and the modern experimental data, on the large scale the universe is an isotropic and homogeneous in its present state of evolution but it might not be the same in the past. Therefore the models with anisotropic background that approach to isotropy at late times are most suitable for describing the entire evolution of the universe. The spatially homogeneous and anisotropic Kantowski Sachs space-time provides such a framework and considered as possible candidates for an early era in cosmology. For Kantowski Sachs model, above set of field equations (19) – (21) reduces to

$$2\frac{S_{44}}{S} + \frac{S_4^2}{S^2} + \frac{1}{S^2} = -8\pi p - \Lambda + \frac{k^2}{S^4 \sin^2 \theta}, \quad (35)$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4}{R}\frac{S_4}{S} = -8\pi p - \Lambda - \frac{k^2}{S^4 \sin^2 \theta}, \quad (36)$$

$$2\frac{R_4}{R}\frac{S_4}{S} + \frac{S_4^2}{S^2} + \frac{1}{S^2} = 8\pi\rho - \Lambda + \frac{k^2}{S^4 \sin^2 \theta}. \quad (37)$$

Using equations (26) and (27), spatially homogeneous and anisotropic magnetized Kantowaski Sachs cosmological model becomes

$$ds^2 = dt^2 - [(1-n)c_1 t + (1-n)c_2]^{\frac{2n}{1-n}} dr^2 + [(1-n)c_1 t + (1-n)c_2]^{\frac{2}{1-n}} \left(d\theta^2 + \sin^2 \theta d\phi^2\right) \quad (38)$$

Above equation represent a singular model and singularity exist at point $t = t_s = -c_2/c_1$.

Physical parameters:

Cosmological constant,

$$\Lambda = \left\{ \begin{array}{c} \dfrac{nc_1^2 - 2n^2c_1^2 - 3c_1^2}{2[(1-n)c_1t + (1-n)c_2]^2} + \dfrac{1}{2[(1-n)c_1t+(1-n)c_2]^{\frac{2}{1-n}}} \\ - \dfrac{k^2}{[(1-n)c_1t+(1-n)c_2]^{\frac{4}{1-n}} \sin^2\theta} \end{array} \right\} \quad (39)$$

The observational and theoretical [1] features suggest that the most natural candidate for the missing energy is the vacuum energy density or the cosmological constant. Positive value corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe the value of cosmological constant is a positive to which the expansion will tends to accelerate whereas in the universe with negative value, the expansion will slow down, stop and reverse.

Isotropic pressure,

$$p = \frac{1}{16\pi G}\left\{ \begin{array}{c} \dfrac{-4nc_1^2 - 2c_1^2}{[(1-n)c_1t+(1-n)c_2]^2} - \dfrac{2}{[(1-n)c_1t+(1-n)c_2]^{\frac{2}{1-n}}} \\ + \dfrac{k^2}{[(1-n)c_1t+(1-n)c_2]^{\frac{4}{1-n}} \sin^2\theta} \end{array} \right\} \quad (40)$$

Energy density,

$$\rho = \frac{1}{16\pi G}\left\{ \begin{array}{c} \dfrac{-2n^2c_1^2 - 11nc_1^2 - 5c_1^2}{[(1-n)c_1t+(1-n)c_2]^2} - \dfrac{5}{[(1-n)c_1t+(1-n)c_2]^{\frac{2}{1-n}}} \\ + \dfrac{k^2}{[(1-n)c_1t+(1-n)c_2]^{\frac{4}{1-n}} \sin^2\theta} \end{array} \right\} \quad (41)$$

Geometrical Parameters:

Average scale factor

$$a = [(1-n)c_1t + (1-n)c_2]^{\frac{(1+n)}{3(1-n)}}(\Phi) \quad (42)$$

Spatial volume

$$V = [(1-n)c_1t + (1-n)c_2]^{\frac{1+n}{1-n}}(\Phi) \quad (43)$$

Hubble Parameter,

$$H = \frac{c_1(n+2)}{3[(1-n)c_1t + (1-n)c_2]}. \quad (44)$$

Expansion scalar,

$$\theta = \frac{c_1(n+2)}{[(1-n)c_1t + (1-n)c_2]}. \quad (45)$$

Shear scalar,

$$\sigma^2 = \frac{c_1^2(n+2)^2}{6[(1-n)c_1t + (1-n)c_2]^2} \quad (46)$$

Deceleration parameter,

$$q = \frac{(1-4n)}{n+2}. \quad (47)$$

The sign of q indicates whether the model inflates or not. The negative sign of q indicates inflation. Also, recent observations of type Ia, expose that the present universe is accelerating and the value of deceleration parameter lies on some place in the range $-1 \le q \le 0$, it follows that in our derived model the value of q resembles with recent observations i.e. -0.4.

V. GRAPHICAL INTERPRETATION

Figure (*a*) is the plot of cosmological constant term versus time *t*. From this figure, it is observe that in Bianchi type-III universe it is positive decreasing function. The observations on red-shift of type Ia supernova suggests that our universe may be an accelerating one with induced cosmological density through the cosmological term.

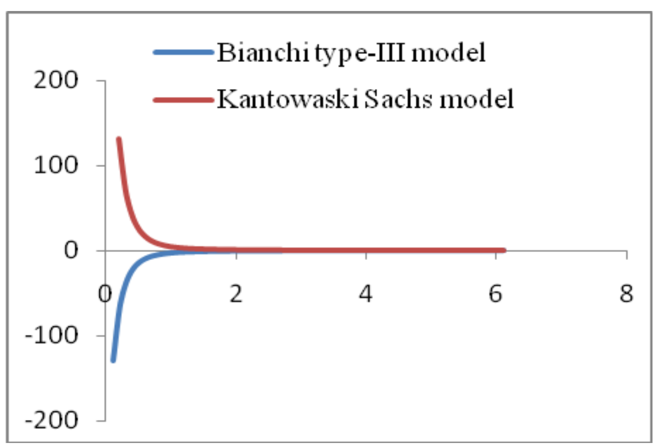

Figure (a): Cosmological constant $(\wedge)$ versus time (t) with $n = 1/2$.

Thus, the nature of in our derived model is supported by recent observations and also resembles with the work of [34] well as the recent investigation via different theoretical models and cosmography tests by Bamba et al. [35], whereas in Kantowski Sachs universe it is negative.

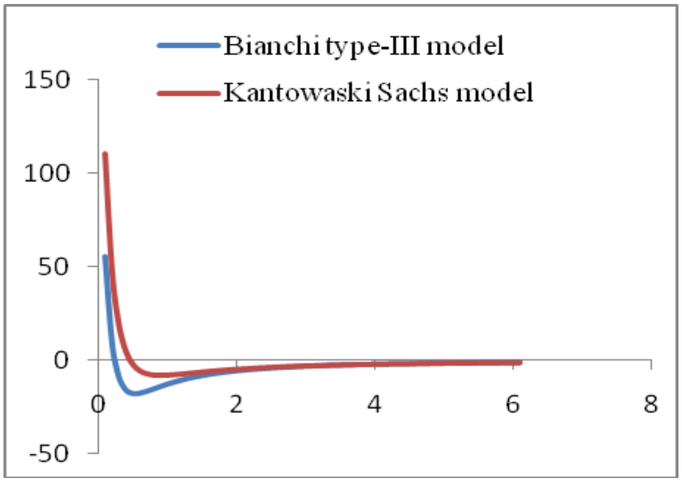

Figure (b): Energy density ((ρ) versus time (*t*) with $n = 1/2$.

In this cosmology, it is observed that in both the universe the energy density is a function of time *t* and decreasing with the expansion. At the initial stage $t \to 0$ the universe has infinitely large energy density $\rho \to \infty$ but with the expansion of the universe it declines and at large $t \to \infty$ it is null $\rho \to 0$. This behavior is shown in **figure (*b*).**The average scale factor and spatial volume of the universe starts with big bang at $t = t_s$.

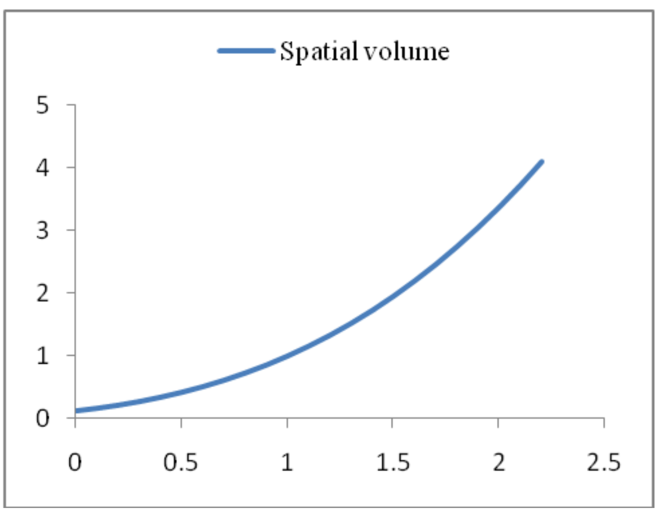

Figure (*c*): Spatial volume (*V*) versus time (*t*) with $n = 1/2$. The average scale factor and spatial volume increases with time *t* i.e. when $t \to \infty$ spatial volume $V \to \infty$ (see **figure (*c*)).** Thus inflation is possible in Bianchi type-III universe.

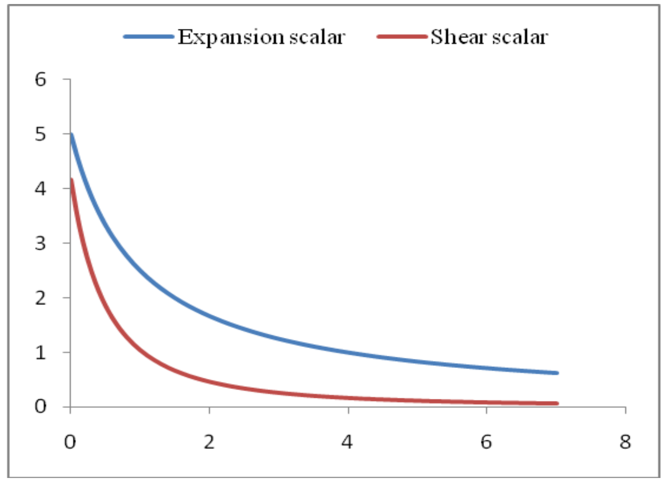

Figure (*d*): Expansion scalar (θ) and shear scalar (σ^2) versus time (*t*) with $n = 1/2$.

In this derived universe, Hubble parameter, expansion scalar, and shear scalar all are time dependent, Hubble's parameter and the scalar expansion vanishes as $t \to \infty$. Also it is a positive decreasing function of time. The shear scalar σ^2 is constant at initial epoch i.e. at $t \to 0$ while they become negligible at t→∞ (see **figure (*d*)).** Since $\left(\theta^2/\sigma^2\right)$ = constant, the model does not approach isotropy for the whole range of time *t*.

Thus, the model represents shearing, non-rotating and expanding model of the universe with big-bang starts with both scale factors are monotonically increasing function of *t*.

VI. CONCLUSIONS

In this paper, we have studied Bianchi type-III and Kantowski Sachs cosmological model containing perfect fluid with magnetic field in general theory of relativity. By assuming anti-stiff fluid condition and the relation between metric potential, we find the general solutions of the Einstein's field equations. Also, we have discussed some geometrical and physical properties of the model.It is observed that the present model exhibits point type singularity and it evolves with a zero volume at time $t = t_s$ .Hubble's parameter and the scalar expansion are the positive decreasing function of time and vanish as $t \to \infty$. The shear scalar σ^2 is constant at initial epoch i.e. at $t \rightarrow 0$ while they become negligible at $t \rightarrow \infty$ .In both models, we see that the deceleration parameter is constant and having negative value. Hence both the models are shows accelerating phase of the universe. It is seen that, in both universe the energy density of fluid decrease as the universe expands. Thus our model approaches towards a flat universe at late time. Thus our model is in good agreement with the recent observation. The cosmological constant having positive values in Bianchi type-III universe while in Kantowaski Sachs it is negative.

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