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An SIRS Model with Disease Induced Death and Non- Linear Incidence Rate

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***Abstract*—** In this paper, we consider an SIRS model with non linear incidence and disease induced death rate in which we consider the incidence rate KI^pS^q for p = 2 and q = 1. We discussed about the stability for the system of differential equations and found that the model is stable by the Routh-Hurwitz criterion.

Keywords- Mathematical Modelling, SIRS Model, Non-linear incidence rate, Stability.

I. INTRODUCTION

In Standard epidemiological models, the incidence rate is bilinear in the infective fraction I and the susceptible fraction S [1]. Depending on parameter values, such models usually have an asymptotically stable trivial equilibrium corresponding to the disease-free state, or an asymptotically stable nontrivial equilibrium corresponding to the endemic state. When the restriction to bilinear incidence rates is dropped, the system can have a much wider range of dynamical behaviors.

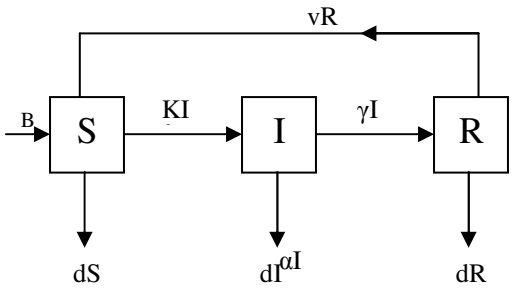
There are a variety of reasons that the standard bilinear form may require modification [5]. The first is that the underlying assumption of homogeneous mixing may be invalid; in this case, as personal communication has argued, it is best to introduce the necessary population structure and represent heterogeneous mixing directly. However, incidence rates that increase more gradually than linear in I and S can also arise from saturation effects. In contrast, a rate of increase faster than linear would be observed under various conditions.

A detailed analysis of co dimension 1 bifurcations for the SEIRS and SIRS models with the incidence rate $\beta I^P S^q$ was given by liu et al.[6] A co dimension 2 diffraction analysis of the SIRS model was presented by Lizana and Riivero [7]. Homoclinic bifurcation in an SIQR model for childhood diseases was studied by Wu and Feng [8]. Bakcward bifurcations of epidemic models with or without time delays were investigated by van den Driessche and Watmouth [3], Hadeler and van den Driessche [2], Dushoff et al. [4], etc.

In this paper, we consider an SIRS model with non linear incidence and disease induced death rate in which we consider the incidence rate KI^pS^q which is supposed by Liu et.al and discussed about the stability for the system of differential equations.

II. MATHEMATICAL FORMULATION

In this paper we generalized the model of Ruan and Wang by considering the non-linear incidence rate KI^pS^q. Let S(t) be the number of susceptible individual, I(t) be the number of infective individuals and R(t) be the number of removed individuals at time t.



The differential equations of the above model is

$$\frac{dS}{dt} = B - ds - KI^2S + vR$$
$$\frac{dI}{dt} = KI^2S - (d + \gamma + \alpha)I \qquad (2.1)$$
$$\frac{dR}{dt} = \gamma I - (d + v)R$$

where

B= is the recruitment rate of the population
d = is the death rate of the population
α = is the disease induced death rate of the population
γ = is the recovery rate of infective individuals
v = is the rate of removed individuals who lose immunity and return to susceptible parameters

Before going into detail let us simplify the model .Summing up the three equations in (2.1) and denoting the number of total population by N(t) we obtain

$$N = S + I + R$$

Therefore,

$$\frac{dN}{dt} = B - \alpha I - dN \qquad (2.2)$$

since N(t) tends to a constant as t tends to infinity we assume that the population is in equilibrium and investigate of the system on the plane

$$S + I + R = N_{\circ} > 0 .$$

Substitute value of $S = N_o - I - R$ in above equations, we get

$$\begin{aligned} \frac{dI}{dt} &= KI^2 \left(N_o - I - R \right) - \left(d + \gamma + \alpha \right) I \\ \frac{dR}{dt} &= \gamma I - \left(d + v \right) R \end{aligned} \qquad (2.3)$$

to be concise in notations, rescale (2.3) by

$$X = \left(\sqrt{\frac{K}{d+v}} \right) I$$

$$Y = \left(\sqrt{\frac{K}{d+v}} \right) R$$

and $\theta = (d+v)t$

for simplicity we still use variable I, R, t instead of X,Y,θ the equations becomes

$$\frac{dI}{dt} = \frac{dI}{d\theta} \times \frac{d\theta}{dt}$$

$$\frac{dI}{dt} = \left(A - I - R \right) - mI$$

where $A = N_o \frac{\sqrt{K}}{d+v}, \; m = \left(\frac{d+\gamma+\alpha}{d+v} \right)$

and $\frac{dR}{dt} = \frac{dR}{d\theta} \times \frac{d\theta}{dt}$

$$\frac{dR}{dt} = qI - R$$

where $q = \frac{\gamma}{d+v}$.

Therefore after simplification we obtain the following equations

$$\frac{dI}{dt} = \left(A - I - R \right) - mI$$

$$\frac{dR}{dt} = qI - R \qquad (2.4)$$

where $A = N_o \frac{\sqrt{K}}{d+v}, \; m = \left(\frac{d+\gamma+\alpha}{d+v} \right) and \;\; q = \frac{\gamma}{d+v}$

III. STABILITY ANALYSIS.

The objective of this section is to perform a quantitative analysis of system (2.4). We start determine the stability.

$$F_1 = \left(A - I - R \right) - mI$$

$$F_2 = qI - R$$

$$\frac{\partial F_1}{\partial I} = -1 - m$$

$$\frac{\partial F_2}{\partial I} = q$$

$$\frac{\partial F_1}{\partial R} = -1$$

$$\frac{\partial F_2}{\partial R} = -1 .$$

The Jacobian matrix

$$\mathrm{J} = \begin{pmatrix} \frac{\partial F_1}{\partial I} & \frac{\partial F_1}{\partial R} \\ \frac{\partial F_2}{\partial I} & \frac{\partial F_2}{\partial R} \end{pmatrix}$$

$$J = \begin{bmatrix} -1-m & -1 \\ q & -1 \end{bmatrix}$$

$$\Rightarrow \det(J) = 1 + m + q .$$

The characteristic equation of Jacobian matrix is

$$| J - \lambda I | = 0$$

i.e. $\begin{vmatrix} -1-m-\lambda & -1 \\ q & -1-\lambda \end{vmatrix} = 0 .$

On simplification the determinant, we have

$$(-1-m-\lambda)(-1-\lambda)+q=0$$

or $\lambda^2 + a_1 \lambda + a_2 = 0$

where $a_1 = m + 2$

$a_2 = m + q + 1$

Clearly, $a_1 > 0$ and $a_2 > 0$.

Therefore, By Routh-Hurwitz criteria, the model is stable.

IV. CONCLUSION.

In this paper, we studied an epidemic model with non-linear incidence rate $KI^p s^q$, where p = 2, q = 1. The

model provides learning about the behavior of disease and we observe that the model is stable by the Routh-Hurwitz stability criteria. The result helps to develop social consciousness about the disease among susceptible.

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Dr. Pradeep Porwal was born in 1984 at Ujjain (M.P.), India. He obtained his Ph.d. degree under the supervision of renowned Mathematician Professor V. H. Badshah, Professor & Head, School of Studies in Mathematics, Vikram University, Ujjain. He served Lokmanya Tilak Science and Commerce College, Ujjain and joined Department of Mathematics, Vikram University, Ujjain as a Guest Faculty in 2010. In the field of Mathematical Modeling (Bio-Mathematics) he has more than 21 research papers published in national and international peer-reviewed reputed journals.

