

# Inventory Replenishment Policy with Time and Reliability Varying Demand

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**Abstract:** In real life situation, demand may be increase, decrease or constant. But, in our present study demand is assumed to be an increasing function of time which depends on reliability. Shortages are allowed and excess demand is backlogged. The economic production lot size and the reliability of the production process along with the production period are the decision variables and total cost per cycle is the objective function which is to be minimized. Further the parameters involved in the business may likely to be changed due to the fast growing marketing system. Therefore; it will be more realistic and market friendly to deal with a fuzzy model rather than a crisp model. Both crisp and fuzzy models have been proposed to determine the optimal solution. The demand, shortage cost, holding cost and deterioration rate and reliability are considered as pentagonal fuzzy numbers. Defuzzification of the total cost has been carried out by Graded Mean Representation Method and Signed Distance Method. Sensitivity analysis is also incorporated to investigate the effect of different system parameters in enhancing the cost.

**Keywords:** Reliability, Pentagonal Fuzzy Number, Graded Mean Representation Method, Signed Distance Method.

**Mathematics Subject Classification:** 90B05

## I. INTRODUCTION

Inventory control is the supervision of supply, storage and accessibility of items in order to ensure an adequate supply without excessive oversupply. It can also be referred as the process of managing the timing and the quantities of goods to be ordered and stocked, so that demands can be met satisfactorily and economically. Success of inventory control depends on some important issues, i.e., uncertainty about the size of future demands, uncertainty of inventory cost, uncertainty of deterioration, reliability of the production process, etc. In the above cases, it has been assumed that the inventory parameters are crisp or precise or probabilistic but in reality they may deviate a little from their actual value without following any probability distribution. To deal with such type of uncertainty in inventory parameters, the notion of fuzziness has been initialized by Zimmermann [4]. This model has also been maneuvered by various researchers. Tripathy et al. established a fuzzy Economic Order Quantity model

with reliability where the unit cost depends on demand [3]. Dutta and Kumar developed a fuzzy inventory model without shortage where holding cost and ordering cost were taken as trapezoidal fuzzy numbers [1]. Tripathy and Pattnaik focused on optimal disposal mechanism by considering the system cost as fuzzy under flexibility and reliability criteria [5]. Tripathy and Behera formulated a fuzzy inventory model for time deteriorating items using penalty cost under the condition of infinite production rate [6]. Dutta and Kumar explored an optimal ordering policy for an inventory model for deteriorating items without shortage where demand rate, ordering cost and holding cost were taken as fuzzy in nature [2]. Jaggi et al. introduced a fuzzy inventory model for deteriorating items with time varying demand and shortages [8]. Singh and Rathore studied an inventory model for deteriorating items with reliability consideration and trade credit [7].

The present study elaborates the above model in fuzzy environment and compares the result obtained in crisp and fuzzy environment to prove the superiority of the proposed model over the crisp model in achieving the optimal

solution. Here shortage cost, Holding cost, deterioration rate and reliability are considered as pentagonal fuzzy numbers. Defuzzification of the total cost has been accomplished by Graded Mean Representation Method and Signed Distance Method. Numerical examples have been cited as a proof of the validation of the model. Sensitivity analysis has also been carried out to prove the credibility of the proposed model over crisp model.

The remainder of this paper is organized as follows. In the next section, the definition and preliminaries are described. Section III presents the assumptions and notations. Section IV describes the mathematical model and solution procedure. In section V numerical examples are presented to illustrate the model and the sensitivity analysis of the optimal solution with respect to parameter of the system is also carried out. Section VI contains the results and discussion. Section VII presents the conclusion with future directions.

## II. DEFINITIONS AND PRELIMINARIES

We first recapitulate some underlying definitions and basic results of fuzzy numbers.

**Fuzzy Set:** Fuzzy set is a vague boundary set as compared to the crisp set. It is a collection of objects in connection with expression of uncertainty of the property characterizing the objects by grades from the interval between 0 and 1.

**Membership function:** In fuzzy set each element is mapped to [0,1] by membership function.

$$\mu_A: X \rightarrow [0,1]$$

**$\alpha$ -Cut set:** The  $\alpha$ -cut of a fuzzy set  $A_\alpha$  is made up of members whose membership function is not less than  $\alpha$ .

**Fuzzy number:** Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number  $R$ . Generally a fuzzy interval is expressed by two end points and a peak point. The  $\alpha$ -cut operation can also be applied to the fuzzy number. If we denote  $\alpha$ -cut interval for fuzzy number  $A$  as  $A_\alpha$ , the obtained interval  $A_\alpha$  is defined as

$$A_\alpha = [a_1^\alpha, a_3^\alpha] \quad \text{Pentagonal fuzzy number}$$

(PFN):

$$C_{R_1}(\alpha) = d - (d - c)\alpha = R_1^{-1}(\alpha)$$

$$C_{R_2}(\alpha) = e - (e - d)\alpha = R_2^{-1}(\alpha)$$

Due to error in measuring technique, instrumental faultiness, etc some data in our observation cannot be precisely or accurately determined. Let us consider that we measure the weather temperature and humidity simultaneously. The temperature is approximately  $35^\circ C$  with normal humidity i.e. the temperature is not perfect either more or less than  $35^\circ C$ , which affects normal humidity in the atmosphere. Thus, variation in temperature also affects the percentages of humidity. This phenomenon happens in general. This concept of variation leads to a new type of fuzzy number called the pentagonal fuzzy number is a 5-tuple subset of a real number having 5 parameters.

A pentagonal fuzzy number  $C = (c_1, c_2, c_3, c_4, c_5)$ , where  $c_3$  is the middle point and  $(c_1, c_2)$  and  $(c_4, c_5)$  are the left and right side points of  $c_3$ , respectively. Now, we consider the mathematical definition of a pentagonal fuzzy number.

**Definition 1: Pentagonal fuzzy number:** A fuzzy number  $C = (c_1, c_2, c_3, c_4, c_5)$  is called a pentagonal fuzzy number when the membership function has the form

$$\mu_C(x; w_1, w_2) = \begin{cases} L_1(x) = \frac{x - c_1}{c_2 - c_1}, & c_1 \leq x \leq c_2 \\ L_2(x) = \frac{x - c_2}{c_3 - c_2}, & c_2 \leq x \leq c_3 \\ 1 & x = c_3 \\ R_1(x) = \frac{x - c_3}{c_4 - c_3}, & c_3 \leq x \leq c_4 \\ R_2(x) = \frac{x - c_4}{c_5 - c_4}, & c_4 \leq x \leq c_5 \\ 0, & x > c_5 \end{cases}$$

The  $\alpha$ -cut of  $C = (c_1, c_2, c_3, c_4, c_5)$ ,  $0 \leq \alpha \leq 1$  is  $(\alpha) = [C_L(\alpha), C_R(\alpha)]$

Where,  $C_{L_1}(\alpha) = a + (b - a)\alpha = L_1^{-1}(\alpha)$

$C_{L_2}(\alpha) = b + (c - d)\alpha = L_2^{-1}(\alpha)$

So,  $L^{-1}(\alpha) = \frac{1}{2} [L_1^{-1}(\alpha) + L_2^{-1}(\alpha)] = \frac{1}{2} [a + b + (c - a)\alpha]$

$R^{-1}(\alpha) = \frac{1}{2} [R_1^{-1}(\alpha) + R_2^{-1}(\alpha)] = \frac{1}{2} [d + e - (e - c)\alpha]$

**Definition 2:** If  $C = (c_1, c_2, c_3, c_4, c_5)$  is a pentagonal fuzzy number then the graded mean representation method

of  $\tilde{C}$  is defined a  $P(\tilde{C}) = \frac{\int_0^w C \left( \frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh}{\int_0^{w_c} h dh}$ ,

$0 \leq h \leq w_C$  and  $0 \leq w_C \leq 1$ .

$$P(\tilde{C}) = \frac{c_1 + 3c_2 + 4c_3 + 3c_4 + c_5}{12}$$

**Definition 3:** Let  $C = (c_1, c_2, c_3, c_4, c_5)$  is a pentagonal fuzzy number then the signed distance of  $\tilde{C}$  is defined as

$$d(C, 0) = \int_0^1 d([C_L(\alpha), C_R(\alpha)], 0) = \frac{1}{8}(c_1 + 2c_2 + 2c_3 + 2c_4 + c_5).$$

**Arithmetic Operations of PFN:**

Formation of an arithmetic operation is crucial in the study of fuzzy numbers; the author tries to establish some basic arithmetic operations of PFN.

(1) **Addition:** Let  $C = (c_1, c_2, c_3, c_4, c_5)$  and

$D = (d_1, d_2, d_3, d_4, d_5)$  be two PFNs: then,

$$C + D = (c_1 + d_1, c_2 + d_2, c_3 + d_3, c_4 + d_4, c_5 + d_5)$$

(2) **Subtraction:** we define the subtraction of two

PFNs  $C = (c_1, c_2, c_3, c_4, c_5)$  and

$D = (d_1, d_2, d_3, d_4, d_5)$ , we get

$$C - D = (c_1 - d_1, c_2 - d_2, c_3 - d_3, c_4 - d_4, c_5 - d_5). \quad (3)$$

**Scalar Multiplication:** Let  $C = (c_1, c_2, c_3, c_4, c_5)$  be a PFN

and  $k \in R$  be any scalar. If

$$k \geq 0, kC = (kc_1, kc_2, kc_3, kc_4, kc_5).$$

(4) **Multiplication:** Let  $C = (c_1, c_2, c_3, c_4, c_5)$  and

$D = (d_1, d_2, d_3, d_4, d_5)$  be two PFNs: then,

$$C * D = (c_1 * d_1, c_2 * d_2, c_3 * d_3, c_4 * d_4, c_5 * d_5).$$

(5) **Inverse:** we define the inverse of a PFN when all its components are non zero. Suppose

$C = (c_1, c_2, c_3, c_4, c_5)$  is a PFN: then,

$$C^{-1} \approx \frac{1}{C} \approx \left( \frac{1}{c_5}, \frac{1}{c_4}, \frac{1}{c_3}, \frac{1}{c_2}, \frac{1}{c_1} \right).$$

If one of the components of a PFN becomes zero; then we cannot find its inverse.

(6) **Division:** the division of two PFNs  $C = (c_1, c_2, c_3, c_4, c_5)$  and  $D = (d_1, d_2, d_3, d_4, d_5)$  is approximated as the multiplication with inverse.

$$\frac{C}{D} \approx CD^{-1} \approx \left( \frac{c_1}{d_5}, \frac{c_2}{d_4}, \frac{c_3}{d_3}, \frac{c_4}{d_2}, \frac{c_5}{d_1} \right)$$

(7) **Exponent:** The exponent of a PFN  $C = (c_1, c_2, c_3, c_4, c_5)$  is defined as the power of its components.  $C^n \approx (c_1^n, c_2^n, c_3^n, c_4^n, c_5^n)$

**III. ASSUMPTIONS AND NOTATIONS**

We adopt the following assumptions and notations for the models to be discussed.

**Assumptions:**

(i) Demand  $R(t) = a(1+r)^t$  is assumed to be an increasing function of time and reliability i.e  $a$  is the positive constant and  $a > 0$ .

(ii) Replenishment rate is infinite.

(iii) Lead time is zero.

(iv) Shortages are allowed and fully backlogged.

**Notations:**

(i)  $R(t)$  : Demand rate for any time per unit time. (ii)  $A_0$  : Ordering cost per order.

(iii)  $\theta_0$  : Deterioration rate.

(iv)  $T$  : Length of the cycle.

(v)  $Q_0$  : Ordering quantity per unit time.

(vi)  $h_0$  : Holding cost.

(vii)  $S_0$  : Shortages cost.

(viii)  $C_0$  : Unit cost per unit time.

(ix)  $r$  : Reliability of the product.

(x)  $\phi(t_1, T)$  : Total inventory cost per unit time.

(xi)  $\tilde{R}$  : Fuzzy demand.

- (xii)  $\tilde{\theta}_0$  : Fuzzy deterioration rate.
- (xiii)  $\tilde{h}_0$  : Fuzzy holding cost per unit time.
- (xiv)  $\tilde{S}_0$  : Fuzzy shortages cost per unit time.
- (xv)  $\tilde{C}_0$  : Fuzzy unit cost per unit time.
- (xvi)  $\tilde{\phi}(t_1, T)$  : Total fuzzy inventory cost per unit time.
- (xvii)  $\tilde{\phi}_{GM}(t_1, T)$  : Defuzzified value of  $\tilde{\phi}(t_1, T)$  by applying Graded mean representation method.
- (xviii)  $\tilde{\phi}_{SD}(t_1, T)$  : Defuzzified value of  $\tilde{\phi}(t_1, T)$  by applying Signed distance method.
- (xix)  $t_1$  : Optimal time period of the positive stock.

**IV. MATHEMATICAL MODEL**

Let  $Q_1$  be the inventory level at the beginning of the cycle. Amid the period  $(0, t_1)$ , the depletion in the inventory occurs due to demand and deterioration. Ultimately, inventory reaches zero level at time  $t_1$ . The period  $(t_1, T)$  is the period of shortages, which are fully backlogged. Then the behavior of the inventory level is governed by the following differential equations.

**Crisp Model:**

$$\frac{dq(t)}{dt} + \theta_0 q(t) = -R(t) \quad 0 \leq t \leq t_1 \quad (1)$$

Deterioration cost ( $q_{dc}$ ) in the cycle is given by

$$q_{dc} = Q_0 - \int_0^{t_1} q(t) dt = a \left[ \frac{\theta_0 t_1^2}{2} + \frac{\theta_0 r t_1^3}{3} \right] \quad (7)$$

Shortage cost ( $q_{sc}$ ) in the cycle is given by

$$q_{sc} = - \left[ a \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right] \quad (8)$$

Thus, the total cost of the system per unit time is given by

$$\phi(t_1, T) = \frac{1}{T} [A_0 + h_0 q_{hc} + C_0 q_{dc} + S_0 q_{sc}]$$

With the boundary conditions  $q(0) = Q_0$  and  $q(t_1) = 0$

$$\frac{dq(t)}{dt} = -R(t) \quad t_1 \leq t \leq T \quad (2)$$

Solution of equation (1) and (2) yields

$$q(t) = a \left[ (1 - \theta_0 t)(t_1 - t) + \left( \frac{\theta_0}{2} - \frac{\theta_0^2 t}{2} \right) (t_1^2 - t^2) + \left( \frac{r}{2} - \frac{\theta_0 r t}{2} \right) (t_1^2 - t^2) + \left( \frac{\theta_0 r}{3} - \frac{\theta_0^2 r t}{3} \right) (t_1^3 - t^3) \right] \quad (3)$$

and

$$q(t) = a \left[ (t_1 - t) + r \left( \frac{t_1^2}{2} - \frac{t^2}{2} \right) \right] \quad (4)$$

By using  $I(t_1) = 0$ , we have

$$Q_0 = a \left[ t_1 + \frac{\theta_0 t_1^2}{2} + \frac{r t_1^2}{2} + \frac{\theta_0 r t_1^3}{3} \right] \quad (5)$$

Holding cost ( $q_{hc}$ ) in the cycle is given by

$$q_{hc} = \int_0^{t_1} q(t) dt = a \left[ \frac{3t_1^2}{2} + \frac{2\theta_0 t_1^3}{3} + \frac{2r t_1^3}{3} + \frac{5\theta_0 r t_1^4}{12} - \frac{\theta_0 t_1^3}{6} - \frac{\theta_0^2 t_1^4}{8} - \frac{\theta_0 r t_1^4}{8} - \frac{\theta_0^2 r t_1^5}{10} \right] \quad (6)$$

$$\phi(t_1, T) = \frac{1}{T} \left[ A_0 + h_0 a \left( \frac{3t_1^2}{2} + \frac{2\theta_0 t_1^3}{3} + \frac{2r t_1^3}{3} + \frac{5\theta_0 r t_1^4}{12} - \frac{\theta_0 t_1^3}{6} - \frac{\theta_0^2 t_1^4}{8} - \frac{\theta_0 r t_1^4}{8} - \frac{\theta_0^2 r t_1^5}{10} \right) + C_0 \left( \frac{\theta_0 t_1^2}{2} + \frac{\theta_0 r t_1^3}{3} \right) - S_0 \left( a \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \quad (9)$$

**Fuzzy Model:**

Due to uncertainty in the environment, we assume some of the parameters of the inventory system like  $a, h_0, S_0, r, \theta_0$  and  $C_0$  may change within certain limits. Here we have considered  $\tilde{a}, \tilde{h}_0, \tilde{S}_0, \tilde{r}, \tilde{\theta}_0$  and  $\tilde{C}_0$  are pentagonal fuzzy number. The fuzzy total cost is given by

$$\tilde{\phi}(t_1, T) = \frac{1}{T} \left[ \tilde{A}_0 + \tilde{h}_0 \tilde{a} \left( \frac{3\tilde{t}_1^2}{2} + \frac{2\tilde{\theta}_0 \tilde{t}_1^3}{3} + \frac{2\tilde{r} \tilde{t}_1^3}{3} + \frac{5\tilde{\theta}_0 \tilde{r} \tilde{t}_1^4}{12} - \frac{\tilde{\theta}_0 \tilde{t}_1^3}{6} - \frac{\tilde{\theta}_0^2 \tilde{t}_1^4}{8} - \frac{\tilde{\theta}_0 \tilde{r} \tilde{t}_1^4}{8} - \frac{\tilde{\theta}_0^2 \tilde{r} \tilde{t}_1^5}{10} \right) + \tilde{C}_0 \left( \frac{\tilde{\theta}_0 \tilde{t}_1^2}{2} + \frac{\tilde{\theta}_0 \tilde{r} \tilde{t}_1^3}{3} \right) - \tilde{S}_0 \left( a \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \quad (10)$$

The total profit  $\tilde{\phi}(t_1, T)$  has been defuzzified by employing signed distance method and graded mean representation method. When  $\tilde{a} = (a_1, a_2, a_3, a_4, a_5)$ ,  $\tilde{h}_0 = (h_{01}, h_{02}, h_{03}, h_{04}, h_{05})$ ,  $\tilde{S}_0 = (S_{01}, S_{02}, S_{03}, S_{04}, S_{05})$ ,  $\tilde{r} = (r_1, r_2, r_3, r_4, r_5)$ ,  $\tilde{\theta}_0 = (\theta_{01}, \theta_{02}, \theta_{03}, \theta_{04}, \theta_{05})$  and  $\tilde{C}_0 = (C_{01}, C_{02}, C_{03}, C_{04}, C_{05})$  are considered as pentagonal fuzzy numbers, by using graded mean representation method for defuzzification, we have

$$\phi_{GM}(t_1, T) = \frac{1}{12} \left[ \phi_{GM_1}(t_1, T) + 3\phi_{GM_2}(t_1, T) + 4\phi_{GM_3}(t_1, T) + 3\phi_{GM_4}(t_1, T) + \phi_{GM_5}(t_1, T) \right] \quad (11)$$

Where,

$$\phi_{GM_1}(t_1, T) = \frac{1}{T} \left[ A_0 + h_{01} a_1 \left( \frac{3t_1^2}{2} + \frac{2\theta_{02} t_1^3}{3} + \frac{2r_2 t_1^3}{3} + \frac{5\theta_{02} r_2 t_1^4}{12} - \frac{\theta_{02} t_1^3}{6} - \frac{\theta_{02}^2 t_1^4}{8} - \frac{\theta_{02} r_2 t_1^4}{8} - \frac{\theta_{02}^2 r_2 t_1^5}{10} \right) + C_{02} \left( \frac{\theta_{02} t_1^2}{2} + \frac{\theta_{02} r_2 t_1^3}{3} \right) - S_{02} \left( a_2 \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_2 \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right]$$

$$\phi_{GM_2}(t_1, T) = \frac{1}{T} \left[ A_0 + h_{02} a_2 \left( \frac{3t_1^2}{2} + \frac{2\theta_{02} t_1^3}{3} + \frac{2r_2 t_1^3}{3} + \frac{5\theta_{02} r_2 t_1^4}{12} - \frac{\theta_{02} t_1^3}{6} - \frac{\theta_{02}^2 t_1^4}{8} - \frac{\theta_{02} r_2 t_1^4}{8} - \frac{\theta_{02}^2 r_2 t_1^5}{10} \right) + C_{02} \left( \frac{\theta_{02} t_1^2}{2} + \frac{\theta_{02} r_2 t_1^3}{3} \right) - S_{02} \left( a_2 \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_2 \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right]$$

$$\phi_{GM_3}(t_1, T) = \frac{1}{T} \left[ A_0 + h_{03} a_3 \left( \frac{3t_1^2}{2} + \frac{2\theta_{03} t_1^3}{3} + \frac{2r_3 t_1^3}{3} + \frac{5\theta_{03} r_3 t_1^4}{12} - \frac{\theta_{03} t_1^3}{6} - \frac{\theta_{03}^2 t_1^4}{8} - \frac{\theta_{03} r_3 t_1^4}{8} - \frac{\theta_{03}^2 r_3 t_1^5}{10} \right) + C_{03} \left( \frac{\theta_{03} t_1^2}{2} + \frac{\theta_{03} r_3 t_1^3}{3} \right) - S_{03} \left( a_3 \left( t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_3 \left( t_1^2 T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right]$$

$$\phi_{GM_4}(t_1, T) = \frac{1}{T} \left[ A_0 + h_{04}a_4 \left( \frac{3t_1^2}{2} + \frac{2\theta_{04}t_1^3}{3} + \frac{2r_4t_1^3}{3} + \frac{5\theta_{04}r_4t_1^4}{12} - \frac{\theta_{04}t_1^3}{6} - \frac{\theta_{04}^2t_1^4}{8} - \frac{\theta_{04}r_4t_1^4}{8} - \frac{\theta_{04}^2r_4t_1^5}{10} \right) + C_{04} \left( \frac{\theta_{04}t_1^2}{2} + \frac{\theta_{04}r_4t_1^3}{3} \right) - S_{04} \left( a_4 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_4 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right]$$

$$\phi_{GM_5}(t_1, T) = \frac{1}{T} \left[ A_0 + h_{05}a_5 \left( \frac{3t_1^2}{2} + \frac{2\theta_{05}t_1^3}{3} + \frac{2r_5t_1^3}{3} + \frac{5\theta_{05}r_5t_1^4}{12} - \frac{\theta_{05}t_1^3}{6} - \frac{\theta_{05}^2t_1^4}{8} - \frac{\theta_{05}r_5t_1^4}{8} - \frac{\theta_{05}^2r_5t_1^5}{10} \right) + C_{05} \left( \frac{\theta_{05}t_1^2}{2} + \frac{\theta_{05}r_5t_1^3}{3} \right) - S_{05} \left( a_5 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_5 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right]$$

To minimize total cost function per unit time  $\phi_{dG}(t_1, T)$ , the optimal value of  $t_1$  and  $T$  can be obtained by solving the following equations:

$$\frac{d\phi_{GM}(t_1, T)}{dt_1} = 0 \text{ and } \frac{d\phi_{GM}(t_1, T)}{dT} = 0 \tag{12}$$

Equation (12) is equivalent to

$$\frac{1}{12T} \left[ h_{01}a_1 \left( 3t_1 + 2\theta_{01}t_1^2 + 2r_1t_1^2 + \frac{5\theta_{01}r_1t_1^3}{3} - \frac{\theta_{01}t_1^2}{2} - \frac{\theta_{01}^2t_1^3}{2} - \frac{\theta_{01}r_1t_1^3}{2} - \frac{\theta_{01}^2r_1t_1^4}{2} \right) + C_{01}a_1(\theta_{01}t_1 + \theta_{01}r_1t_1^2) - S_{01}(a_1(T - t_1) + r_1(2t_1T - 2t_1^2)) + 3h_{02}a_2 \left( \frac{3t_1 + 2\theta_{02}t_1^2 + 2r_2t_1^2 + \frac{5\theta_{02}r_2t_1^3}{3} - \frac{\theta_{02}t_1^2}{2} - \frac{\theta_{02}^2t_1^3}{2} - \frac{\theta_{02}r_2t_1^3}{2} - \frac{\theta_{02}^2r_2t_1^4}{2} \right) + C_{02}a_2(\theta_{02}t_1 + \theta_{02}r_2t_1^2) - S_{02}(a_2(T - t_1) + r_2(2t_1T - 2t_1^2)) + 4h_{03}a_3 \left( \frac{3t_1 + 2\theta_{03}t_1^2 + 2r_3t_1^2 + \frac{5\theta_{03}r_3t_1^3}{3} - \frac{\theta_{03}t_1^2}{2} - \frac{\theta_{03}^2t_1^3}{2} - \frac{\theta_{03}r_3t_1^3}{2} - \frac{\theta_{03}^2r_3t_1^4}{2} \right) + C_{03}a_3(\theta_{03}t_1 + \theta_{03}r_3t_1^2) - S_{03}(a_3(T - t_1) + r_3(2t_1T - 2t_1^2)) + 3h_{04}a_4 \left( \frac{3t_1 + 2\theta_{04}t_1^2 + 2r_4t_1^2 + \frac{5\theta_{04}r_4t_1^3}{3} - \frac{\theta_{04}t_1^2}{2} - \frac{\theta_{04}^2t_1^3}{2} - \frac{\theta_{04}r_4t_1^3}{2} - \frac{\theta_{04}^2r_4t_1^4}{2} \right) + C_{04}a_4(\theta_{04}t_1 + \theta_{04}r_4t_1^2) - S_{04}(a_4(T - t_1) + r_4(2t_1T - 2t_1^2)) + h_{05}a_5 \left( \frac{3t_1 + 2\theta_{05}t_1^2 + 2r_5t_1^2 + \frac{5\theta_{05}r_5t_1^3}{3} - \frac{\theta_{05}t_1^2}{2} - \frac{\theta_{05}^2t_1^3}{2} - \frac{\theta_{05}r_5t_1^3}{2} - \frac{\theta_{05}^2r_5t_1^4}{2} \right) + C_{05}a_5(\theta_{05}t_1 + \theta_{05}r_5t_1^2) - S_{05}(a_5(T - t_1) + r_5(2t_1T - 2t_1^2)) \right] \tag{13}$$

and

$$\left[ \frac{1}{12T} \left\{ s_{01} \left\{ -a_1(t_1 - T) + r_1(t_1^2 - T^2) \right\} + 3s_{02} \left\{ -a_2(t_1 - T) + r_2(t_1^2 - T^2) \right\} + 4s_{03} \left\{ -a_3(t_1 - T) + r_3(t_1^2 - T^2) \right\} + \right. \right. \\
 \left. \left. 3s_{04} \left\{ -a_4(t_1 - T) + r_4(t_1^2 - T^2) \right\} + s_{05} \left\{ -a_5(t_1 - T) + r_5(t_1^2 - T^2) \right\} \right\} - \right. \\
 \left. \left[ 12A_0 + h_{01}a_1 \left( \frac{3t_1^2}{2} + \frac{2\theta_{01}t_1^3}{3} + \frac{2r_1t_1^3}{3} + \frac{5\theta_{01}r_1t_1^4}{12} - \frac{\theta_{01}t_1^4}{3} - \frac{\theta_{01}^2t_1^4}{8} - \frac{\theta_{01}^2r_1t_1^5}{10} \right) + C_{01}a_1 \left( \frac{\theta_{01}t_1^2}{2} + \frac{\theta_{01}r_1t_1^3}{3} \right) \right] \right. \\
 \left. + S_{01} \left( -a_1 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_1 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 3h_{02}a_2 \left( \frac{3t_1^2}{2} + \frac{2\theta_{02}t_1^3}{3} + \frac{2r_2t_1^3}{3} + \frac{5\theta_{02}r_2t_1^4}{12} - \frac{\theta_{02}t_1^4}{3} - \frac{\theta_{02}^2t_1^4}{8} - \frac{\theta_{02}^2r_2t_1^5}{10} \right) + \right. \\
 \left. C_{02}a_2 \left( \frac{\theta_{02}t_1^2}{2} + \frac{\theta_{02}r_2t_1^3}{3} \right) + S_{02} \left( -a_2 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_2 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 4h_{03}a_3 \left( \frac{3t_1^2}{2} + \frac{2\theta_{03}t_1^3}{3} + \frac{2r_3t_1^3}{3} + \frac{5\theta_{03}r_3t_1^4}{12} - \frac{\theta_{03}t_1^4}{3} - \frac{\theta_{03}^2t_1^4}{8} - \frac{\theta_{03}^2r_3t_1^5}{10} \right) + \right. \\
 \left. C_{03}a_3 \left( \frac{\theta_{03}t_1^2}{2} + \frac{\theta_{03}r_3t_1^3}{3} \right) + S_{03} \left( -a_3 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_3 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 3h_{04}a_4 \left( \frac{3t_1^2}{2} + \frac{2\theta_{04}t_1^3}{3} + \frac{2r_4t_1^3}{3} + \frac{5\theta_{04}r_4t_1^4}{12} - \frac{\theta_{04}t_1^4}{3} - \frac{\theta_{04}^2t_1^4}{8} - \frac{\theta_{04}^2r_4t_1^5}{10} \right) + \right. \\
 \left. C_{04}a_4 \left( \frac{\theta_{04}t_1^2}{2} + \frac{\theta_{04}r_4t_1^3}{3} \right) + S_{04} \left( -a_4 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_4 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + h_{05}a_5 \left( \frac{3t_1^2}{2} + \frac{2\theta_{05}t_1^3}{3} + \frac{2r_5t_1^3}{3} + \frac{5\theta_{05}r_5t_1^4}{12} - \frac{\theta_{05}t_1^4}{3} - \frac{\theta_{05}^2t_1^4}{8} - \frac{\theta_{05}^2r_5t_1^5}{10} \right) + \right. \\
 \left. C_{05}a_5 \left( \frac{\theta_{05}t_1^2}{2} + \frac{\theta_{05}r_5t_1^3}{3} \right) + S_{05} \left( -a_5 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_5 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \quad (14)$$

Further, for the total cost function  $\phi_{dG}(t_1, T)$  to be convex, the following conditions must be satisfied

$$\frac{d^2\phi_{GM}(t_1, T)}{dt_1^2} > 0, \quad \frac{d^2\phi_{GM}(t_1, T)}{dT^2} > 0, \quad (15)$$

$$\text{and} \left( \frac{d^2\phi_{GM}(t_1, T)}{dt_1^2} \right) \left( \frac{d^2\phi_{GM}(t_1, T)}{dT^2} \right) - \left( \frac{d^2\phi_{GM}(t_1, T)}{dt_1dT} \right)^2 > 0 \quad (16)$$

The second derivative of the total cost function  $\phi_{dG}(t_1, T)$  is complicated and it is very difficult to prove its convexity mathematically. Thus, the convexity of total cost function has been established graphically (figure 1).

Using Signed Distance Method the, total cost is given by

$$\phi_{SD}(t_1, T) = \frac{1}{8} [\phi_{SD_1}(t_1, T) + 2\phi_{SD_2}(t_1, T) + 2\phi_{SD_3}(t_1, T) + 2\phi_{SD_4}(t_1, T) + \phi_{SD_5}(t_1, T)] \quad (17)$$

Where,

$$\begin{aligned} \phi_{SD_1}(t_1, T) &= \frac{1}{T} \left[ A_0 + h_{01}a_1 \left( \frac{3t_1^2}{2} + \frac{2\theta_{01}t_1^3}{3} + \frac{2r_1t_1^3}{3} + \frac{5\theta_{01}r_1t_1^4}{12} - \frac{\theta_{01}t_1^3}{6} - \frac{\theta_{01}^2t_1^4}{8} - \frac{\theta_{01}r_1t_1^4}{8} - \frac{\theta_{01}^2r_1t_1^5}{10} \right) + \right. \\ &\quad \left. C_{01} \left( \frac{\theta_{01}t_1^2}{2} + \frac{\theta_{01}r_1t_1^3}{3} \right) - S_{01} \left( a_1 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_1 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \\ \phi_{SD_2}(t_1, T) &= \frac{1}{T} \left[ A_0 + h_{02}a_2 \left( \frac{3t_1^2}{2} + \frac{2\theta_{02}t_1^3}{3} + \frac{2r_2t_1^3}{3} + \frac{5\theta_{02}r_2t_1^4}{12} - \frac{\theta_{02}t_1^3}{6} - \frac{\theta_{02}^2t_1^4}{8} - \frac{\theta_{02}r_2t_1^4}{8} - \frac{\theta_{02}^2r_2t_1^5}{10} \right) + \right. \\ &\quad \left. C_{02} \left( \frac{\theta_{02}t_1^2}{2} + \frac{\theta_{02}r_2t_1^3}{3} \right) - S_{02} \left( a_2 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_2 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \\ \phi_{SD_3}(t_1, T) &= \frac{1}{T} \left[ A_0 + h_{03}a_3 \left( \frac{3t_1^2}{2} + \frac{2\theta_{03}t_1^3}{3} + \frac{2r_3t_1^3}{3} + \frac{5\theta_{03}r_3t_1^4}{12} - \frac{\theta_{03}t_1^3}{6} - \frac{\theta_{03}^2t_1^4}{8} - \frac{\theta_{03}r_3t_1^4}{8} - \frac{\theta_{03}^2r_3t_1^5}{10} \right) + \right. \\ &\quad \left. C_{03} \left( \frac{\theta_{03}t_1^2}{2} + \frac{\theta_{03}r_3t_1^3}{3} \right) - S_{03} \left( a_3 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_3 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \\ \phi_{SD_4}(t_1, T) &= \frac{1}{T} \left[ A_0 + h_{04}a_4 \left( \frac{3t_1^2}{2} + \frac{2\theta_{04}t_1^3}{3} + \frac{2r_4t_1^3}{3} + \frac{5\theta_{04}r_4t_1^4}{12} - \frac{\theta_{04}t_1^3}{6} - \frac{\theta_{04}^2t_1^4}{8} - \frac{\theta_{04}r_4t_1^4}{8} - \frac{\theta_{04}^2r_4t_1^5}{10} \right) + \right. \\ &\quad \left. C_{04} \left( \frac{\theta_{04}t_1^2}{2} + \frac{\theta_{04}r_4t_1^3}{3} \right) - S_{04} \left( a_4 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_4 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \\ \phi_{SD_5}(t_1, T) &= \frac{1}{T} \left[ A_0 + h_{05}a_5 \left( \frac{3t_1^2}{2} + \frac{2\theta_{05}t_1^3}{3} + \frac{2r_5t_1^3}{3} + \frac{5\theta_{05}r_5t_1^4}{12} - \frac{\theta_{05}t_1^3}{6} - \frac{\theta_{05}^2t_1^4}{8} - \frac{\theta_{05}r_5t_1^4}{8} - \frac{\theta_{05}^2r_5t_1^5}{10} \right) + \right. \\ &\quad \left. C_{05} \left( \frac{\theta_{05}t_1^2}{2} + \frac{\theta_{05}r_5t_1^3}{3} \right) - S_{05} \left( a_5 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_5 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \end{aligned}$$

The total cost function  $\phi_{SD}(t_1, T)$  has been minimized following the same process as has been stated above. To minimize total cost function per unit time  $\phi_{SD}(t_1, T)$ , the optimal value of  $t_1$  and  $T$  can be obtained by solving the following equations:

$$\frac{d\phi_{SD}(t_1, T)}{dt_1} = 0 \text{ and } \frac{d\phi_{SD}(t_1, T)}{dT} = 0 \tag{18}$$

$$\begin{aligned} &\left[ h_{01}a_1 \left( 3t_1 + 2\theta_{01}t_1^2 + 2r_1t_1^2 + \frac{5\theta_{01}r_1t_1^3}{3} - \frac{\theta_{01}t_1^2}{2} - \frac{\theta_{01}^2t_1^3}{2} - \frac{\theta_{01}r_1t_1^3}{2} - \frac{\theta_{01}^2r_1t_1^4}{2} \right) + C_{01}a_1(\theta_{01}t_1 + \theta_{01}r_1t_1^2) \right. \\ &\quad \left. - S_{01} \left( a_1(T - t_1) + r_1(2t_1T - 2t_1^2) \right) + 2h_{02}a_2 \left( \frac{3t_1 + 2\theta_{02}t_1^2 + 2r_2t_1^2 + \frac{5\theta_{02}r_2t_1^3}{3} - \frac{\theta_{02}t_1^2}{2} - \frac{\theta_{02}^2t_1^3}{2} - \frac{\theta_{02}r_2t_1^3}{2} - \frac{\theta_{02}^2r_2t_1^4}{2} \right) \right. \\ &\quad \left. + C_{02}a_2(\theta_{02}t_1 + \theta_{02}r_2t_1^2) - S_{02} \left( a_2(T - t_1) + r_2(2t_1T - 2t_1^2) \right) + 2h_{03}a_3 \left( \frac{3t_1 + 2\theta_{03}t_1^2 + 2r_3t_1^2 + \frac{5\theta_{03}r_3t_1^3}{3} - \frac{\theta_{03}t_1^2}{2} - \frac{\theta_{03}^2t_1^3}{2} - \frac{\theta_{03}r_3t_1^3}{2} - \frac{\theta_{03}^2r_3t_1^4}{2} \right) \right. \\ &\quad \left. + C_{03}a_3(\theta_{03}t_1 + \theta_{03}r_3t_1^2) - S_{03} \left( a_3(T - t_1) + r_3(2t_1T - 2t_1^2) \right) + 2h_{04}a_4 \left( \frac{3t_1 + 2\theta_{04}t_1^2 + 2r_4t_1^2 + \frac{5\theta_{04}r_4t_1^3}{3} - \frac{\theta_{04}t_1^2}{2} - \frac{\theta_{04}^2t_1^3}{2} - \frac{\theta_{04}r_4t_1^3}{2} - \frac{\theta_{04}^2r_4t_1^4}{2} \right) \right. \\ &\quad \left. + C_{04}a_4(\theta_{04}t_1 + \theta_{04}r_4t_1^2) - S_{04} \left( a_4(T - t_1) + r_4(2t_1T - 2t_1^2) \right) + h_{05}a_5 \left( \frac{3t_1 + 2\theta_{05}t_1^2 + 2r_5t_1^2 + \frac{5\theta_{05}r_5t_1^3}{3} - \frac{\theta_{05}t_1^2}{2} - \frac{\theta_{05}^2t_1^3}{2} - \frac{\theta_{05}r_5t_1^3}{2} - \frac{\theta_{05}^2r_5t_1^4}{2} \right) \right. \\ &\quad \left. + C_{05}a_5(\theta_{05}t_1 + \theta_{05}r_5t_1^2) - S_{05} \left( a_5(T - t_1) + r_5(2t_1T - 2t_1^2) \right) \right] \tag{19} \end{aligned}$$



$$\left[ \frac{1}{8T} \left\{ s_{01} \left( -a_1(t_1 - T) + r_1(t_1^2 - T^2) \right) + 3s_{02} \left( -a_2(t_1 - T) + r_2(t_1^2 - T^2) \right) + 4s_{03} \left( -a_3(t_1 - T) + r_3(t_1^2 - T^2) \right) + \right. \right. \\
 \left. \left. 3s_{04} \left( -a_4(t_1 - T) + r_4(t_1^2 - T^2) \right) + s_{05} \left( -a_5(t_1 - T) + r_5(t_1^2 - T^2) \right) \right\} - \right. \\
 \left. \left[ 8A + h_{01}a_1 \left( \frac{3t_1^2}{2} + \frac{2\theta_{01}t_1^3}{3} + \frac{2r_1t_1^3}{3} + \frac{5\theta_{01}r_1t_1^4}{12} - \frac{\theta_{01}t_1^3}{3} - \frac{\theta_{01}^2t_1^4}{8} - \frac{\theta_{01}r_1t_1^4}{8} - \frac{\theta_{01}^2r_1t_1^5}{10} \right) + C_{01}a_1 \left( \frac{\theta_{01}t_1^2}{2} + \frac{\theta_{01}r_1t_1^3}{3} \right) \right] \right. \\
 \left. + S_{01} \left( -a_1 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_1 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 2h_{02}a_2 \left( \frac{3t_1^2}{2} + \frac{2\theta_{02}t_1^3}{3} + \frac{2r_2t_1^3}{3} + \frac{5\theta_{02}r_2t_1^4}{12} \right. \right. \\
 \left. \left. - \frac{\theta_{02}t_1^3}{3} - \frac{\theta_{02}^2t_1^4}{8} - \frac{\theta_{02}r_2t_1^4}{8} - \frac{\theta_{02}^2r_2t_1^5}{10} \right) + \right. \\
 \left. C_{02}a_2 \left( \frac{\theta_{02}t_1^2}{2} + \frac{\theta_{02}r_2t_1^3}{3} \right) + S_{02} \left( -a_2 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_2 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 2h_{03}a_3 \left( \frac{3t_1^2}{2} + \frac{2\theta_{03}t_1^3}{3} + \frac{2r_3t_1^3}{3} + \right. \right. \\
 \left. \left. \frac{5\theta_{03}r_3t_1^4}{12} - \frac{\theta_{03}t_1^3}{3} - \frac{\theta_{03}^2t_1^4}{8} - \frac{\theta_{03}r_3t_1^4}{8} - \frac{\theta_{03}^2r_3t_1^5}{10} \right) + \right. \\
 \left. \frac{1}{8T^2} \left[ C_{03}a_3 \left( \frac{\theta_{03}t_1^2}{2} + \frac{\theta_{03}r_3t_1^3}{3} \right) + S_{03} \left( -a_3 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_3 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + 2h_{04}a_4 \left( \frac{3t_1^2}{2} + \frac{2\theta_{04}t_1^3}{3} + \frac{2r_4t_1^3}{3} \right. \right. \right. \\
 \left. \left. + \frac{5\theta_{04}r_4t_1^4}{12} - \frac{\theta_{04}t_1^3}{3} - \frac{\theta_{04}^2t_1^4}{8} - \frac{\theta_{04}r_4t_1^4}{8} - \frac{\theta_{04}^2r_4t_1^5}{10} \right) - \right. \\
 \left. + C_{04}a_4 \left( \frac{\theta_{04}t_1^2}{2} + \frac{\theta_{04}r_4t_1^3}{3} \right) + S_{04} \left( -a_4 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_4 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) + h_{05}a_5 \left( \frac{3t_1^2}{2} + \frac{2\theta_{05}t_1^3}{3} + \frac{2r_5t_1^3}{3} \right. \right. \\
 \left. \left. + \frac{5\theta_{05}r_5t_1^4}{12} - \frac{\theta_{05}t_1^3}{3} - \frac{\theta_{05}^2t_1^4}{8} - \frac{\theta_{05}r_5t_1^4}{8} - \frac{\theta_{05}^2r_5t_1^5}{10} \right) \right. \\
 \left. + C_{05}a_5 \left( \frac{\theta_{05}t_1^2}{2} + \frac{\theta_{05}r_5t_1^3}{3} \right) + S_{05} \left( -a_5 \left( t_1T - \frac{t_1^2}{2} - \frac{T^2}{2} \right) + r_5 \left( t_1^2T - \frac{2t_1^3}{3} - \frac{T^3}{3} \right) \right) \right] \right] \tag{20}$$

Further, for the total cost function  $\phi_{SD}(t_1, T)$  to be convex, the following conditions must be satisfied

$$\frac{d^2\phi_{SD}(t_1, T)}{dt_1^2} > 0, \frac{d^2\phi_{SD}(t_1, T)}{dT^2} > 0 \tag{21}$$

$$\left( \frac{d^2\phi_{SD}(t_1, T)}{dt_1^2} \right) \left( \frac{d^2\phi_{SD}(t_1, T)}{dT^2} \right) - \left( \frac{d^2\phi_{SD}(t_1, T)}{dt_1dT} \right)^2 > 0 \tag{22}$$

The second derivative of the total cost function  $\phi_{GM}(t_1, T)$  is complicated and it is very difficult to prove its convexity mathematically. Thus, the convexity of total cost function has been established graphically, (Figure 2).

**V. NUMERICAL EXAMPLES**

For the illustration of proposed model we consider following inventory system in which values of different parameters in proper units.

**Crisp Model:**

$A_0 = 1500 / \text{order}$ ,  $h_0 = Rs.7/\text{order}$ ,  $a = 100 \text{ units /order}$ ,  
 $\theta_0 = 0.006 / \text{year}$ ,  $S_0 = Rs. 4 \text{ umits /year}$ , The  
 $C_0 = Rs.6 /\text{unit}$ ,  $r = 0.6 \text{ units}$ .  
 solution of the crisp model is

$\phi(t_1, T) = Rs. 385.8$ ,  $t_1 = 0.7820 \text{ year}$ ,  $T = 5.7310 \text{ year}$ .

**Fuzzy Model:**

$\tilde{h}_0 = (3, 5, 7, 9, 11)$ ,  $\tilde{a} = (3, 6, 10, 13, 16)$ ,  
 $\tilde{\theta}_0 = (0.002, 0.004, 0.006, 0.008, 0.01)$ ,  
 $\tilde{r} = (0.2, 0.4, 0.6, 0.8, 1)$ ,  $\tilde{C}_0 = (2, 4, 6, 8, 10)$ .

The Solution of the fuzzy model can be explored by the following two methods.

Table 1. Optimum values of  $t_1$ , T and  $\phi(t_1, T)$  when  $\tilde{a}, \tilde{h}_0, \tilde{\theta}_0, \tilde{r}, \tilde{C}_0, \tilde{S}_0$  are pentagonal fuzzy numbers.

Methods employed	$t_1$	T	$\phi(t_1, T)$
GMRM	2.3202	21.0859	410.583
SDM	2.2541	19.996	378.744

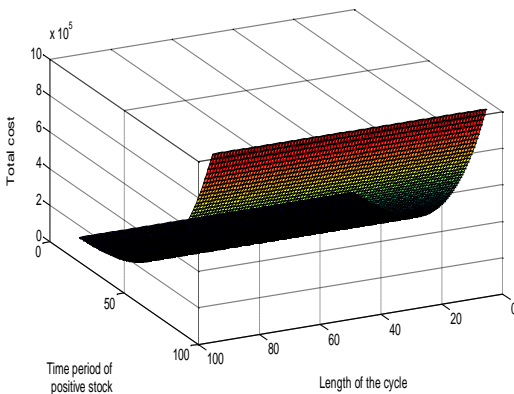


Figure 1. Total fuzzy cost  $\phi_{GM}(t_1, T)$  vs. Time period of positive stock ( $t_1$ ) and Length of the cycle ( $T$ )

Table 2. Optimum values of  $t_1$ , T and  $\phi(t_1, T)$  when  $\tilde{a}, \tilde{\theta}_0, \tilde{r}, \tilde{C}_0, \tilde{S}_0$  are pentagonal fuzzy numbers.

Method employed	$t_1$	T	$\phi(t_1, T)$
GMRM	2.4959	20.9685	384.577
SDM	2.4586	20.8406	369.883

Table 3. Optimum values of  $t_1$ , T and  $\phi(t_1, T)$  when  $\tilde{a}, \tilde{\theta}_0, \tilde{r}, \tilde{C}_0$  are pentagonal fuzzy numbers.

Method employed	$t_1$	T	$\phi(t_1, T)$
GMRM	2.2720	20.1058	337.881
SDM	2.1975	18.9656	333.612

Table 4. Optimum values of  $t_1$ , T and  $\phi(t_1, T)$  when  $\tilde{a}, \tilde{\theta}_0, \tilde{r}$  are pentagonal fuzzy numbers.

Method employed	$t_1$	T	$\phi(t_1, T)$
GMRM	2.2727	20.1053	337.854
SDM	2.1984	18.965	333.600

Table 5. Optimum values of  $t_1$ , T and  $\phi(t_1, T)$  when  $\tilde{a}$  and  $\tilde{\theta}_0$  are pentagonal fuzzy numbers.

Method employed	$t_1$	T	$\phi(t_1, T)$
GMRM	2.0563	17.8861	299.86
SDM	2.0237	17.5304	298.545

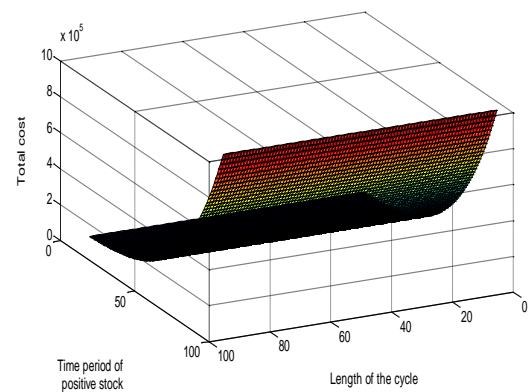


Figure 2. Total fuzzy Cost  $\phi_{SD}(t_1, T)$  vs. Time period of positive stock ( $t_1$ ) and Length of the cycle ( $T$ )

**Sensitivity Analysis:**

A sensitivity analysis is performed to examine the impact and effect of changes in fuzzy parameter  $\tilde{r}$ ,  $\tilde{\theta}$  and  $\tilde{h}$  on the optimal solution by taking the defuzzified values of these parameters. The results are shown in below tables.

**By using Graded Mean Representation Method:**

Table 6. Sensitivity Analysis of reliability ( $r$ ) with time period of positive stock, length of the cycle and total cost

$r$	$t_1$	$T$	$\phi_{GM}(t_1, T)$
0.2	6.0597	54.7609	1017.71
0.4	3.1145	28.0512	520.718
0.6	2.1424	19.3751	359.674
0.8	1.6637	15.1476	281.307
1	1.3801	12.664	235.312

Table 7. Sensitivity Analysis of deterioration rate ( $\theta_0$ ) with time period of positive stock, length of the cycle and total cost

$\theta_0$	$t_1$	$T$	$\phi_{GM}(t_1, T)$
0.002	2.3334	21.0791	391.131
0.004	2.3283	21.0817	391.356
0.006	2.3233	21.0843	391.578
0.008	2.3183	21.0869	391.796
0.01	2.3133	21.0894	392.01

Table 8. Sensitivity Analysis of holding cost ( $h_0$ ) with time period of positive stock, length of the cycle and total cost

$h_0$	$t_1$	$T$	$\phi_{GM}(t_1, T)$
2	5.2957	19.5849	283.51
3	4.2181	20.0062	319.17
4	3.5567	20.3341	343.17
5	3.0994	20.5915	360.644
6	2.7600	20.7984	373.993

**By using Signed Distance Method:**

Table 9. Sensitivity Analysis of reliability with time period of positive stock, length of the cycle and total cost

$r$	$t_1$	$T$	$\phi_{SD}(t_1, T)$
0.2	5.761	51.7766	961.15
0.4	2.9558	26.4661	490.764
0.6	2.0291	18.2336	338.117
0.8	1.5263	14.2191	263.771
1	1.3622	11.8608	220.290

Table 10. Sensitivity Analysis of deterioration rate with time period of positive stock, length of the cycle and total cost

$\theta_0$	$t_1$	$T$	$\phi_{SD}(t_1, T)$
0.002	2.3515	20.9076	387.955
0.004	2.3463	20.9106	388.188
0.006	2.3413	20.9135	388.417
0.008	2.3362	20.9164	388.343
0.01	2.3313	20.9193	388.865

Table 11. Sensitivity Analysis of holding cost with time period of positive stock, length of the cycle and total cost

$h_0$	$t_1$	$T$	$\phi_{SD}(t_1, T)$
2	5.3432	19.2274	281.722
3	4.2803	19.7206	319.093
4	3.6200	20.0854	344.25
5	3.1603	20.3649	362.506
6	2.8176	20.5864	376.452

Behavior of reliability, deterioration rate and holding cost in both the methods has been presented in the following figures.

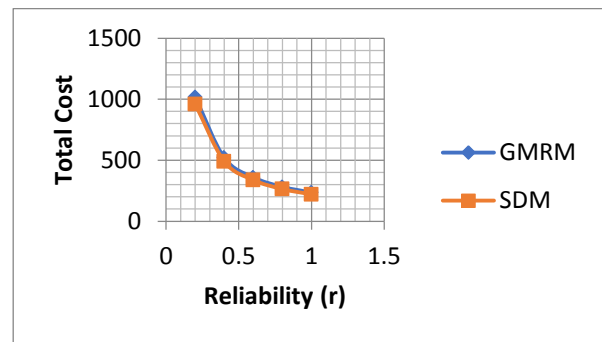


Figure 3. Behavior of reliability in GMRM and SDM

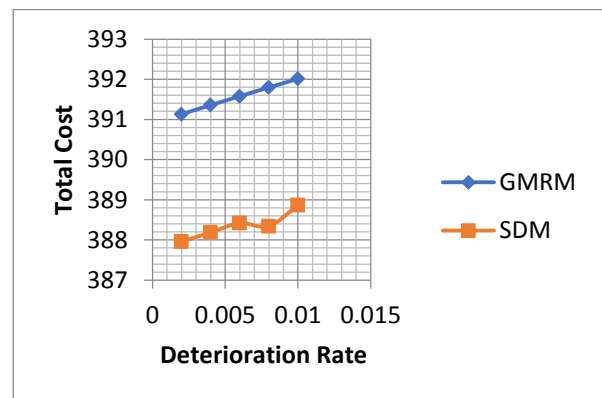


Figure 4. Behavior of deterioration rate in GMRM and SDM

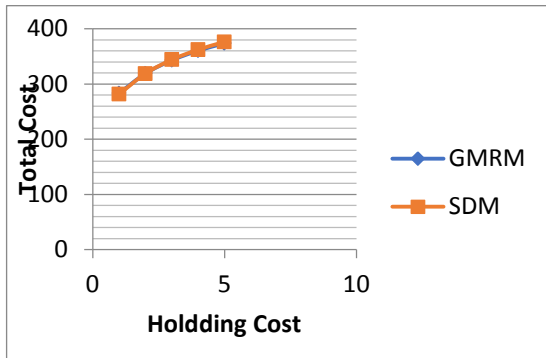


Figure 5. Behavior of holding cost in GMRM and SDM

## VI. RESULT AND DISCUSSION

- It is indicated in Table 6 and Table 9 that as the value of  $r$  increases, total cost, total cycle length and time period of positive stock increase substantially.
- It is indicated in Table 7 and Table 10 that as the value of  $\theta_0$  increases, total cost and total cycle length increase and time period of positive stock decreases gradually.
- It is indicated in Table 8 and Table 11 that as the value of  $h_0$  increases, total cost and total cycle length increase and time period of positive stock decreases drastically.

## VII. CONCLUSION

In this paper, we have used signed distance method and graded mean representation method for defuzzifying holding cost, setup cost, reliability, deterioration rate and shortage cost. These costs are taken as pentagonal fuzzy number. Demand is assumed to be an increasing function of time and reliability. It is seen from our analysis that increase reliability of the item gives rise to decrease cost as it is associated with quality assurances. So, the management has to fix up the reliability at certain level depending on the cost of investment.

The proposed model can be extended by introducing trade credit, weibull deterioration and partial backlogging.

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Note: Utkal University is an A+ grade University as per NAAC and tier 1 University by the UGC. It is the oldest university of the state and the mother University of the State.