International Journal of Scientific Research in Mathematical and Statistical Sciences
Volume-5, Issue-1, pp.1-10, February (2018)

Research Paper

E-ISSN: 2348-4519

Improved GMM Estimation of AR(1) Time Series with a Root near 1

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Available online at: www.isroset.org

Received 16/Jan/2018, Revised 30/Jan/2018, Accepted 22/Feb/2018, Online 28/Feb/2018

Abstract—In this paper, to estimate AR(1) time series model First-difference GMM and Level GMM estimation methods have been considered, which have already performed well for estimation of AR(1) panel data model. A Monte Carlo simulation is carried out in order to study the performances of the above mentioned estimators and OLS estimator. Further, comparison among these estimators have been done in terms of bias and RMSE. Study reveals that, in many cases the OLS and First difference GMM estimators behave same in terms of Bias and RMSE. For all the negative values of autoregressive parameter the RMSE and bias of Level GMM estimator is larger than the remaining estimators. But in the case of positive values of autoregressive parameter Level GMM estimator performs better than First-difference GMM and OLS estimators especially, when sample size is small and autoregressive parameter is close to one.

Keywords— AR(1), First-difference GMM estimation, Level GMM estimation, OLS, Monte Carlo simulation

I. INTRODUCTION

There are numerous time series models available in the literature. The most widely used models are the Autoregressive (AR) models, the Integrated (I) models and the Moving Average (MA) models. A common approach for modelling time series is AR model. The first order autoregression (AR(1)) is simple time series model, which can be analyzed through various standard methods. One of them is Ordinary Least Squares(OLS). The pioneers who worked in the area of OLS estimation of AR(1) time series model are [1] to [4] and more recent contributions include [5] to [7]. Their findings show that, the bias of the OLS estimator becomes large when sample size is small and an autoregressive parameter is near to unity.

The OLS method requires the assumption of orthogonality between the error term and regressor, which is often not satisfied in various applications. In such cases, the OLS estimator becomes inconsistent. To overcome this problem many estimation methods emerged in the study of estimation of AR(1) model. GMM being one of them relaxes the assumption of orthogonality and is used for estimation of AR(1) model, see [8] to [17]. Recent studies show an estimation and inference of a panel AR(1) model with small T.

In the context of panel data, to estimate AR(1) model many estimation methods are proposed. Two consistent estimation methods among them are First-difference GMM (Dif) Proposed by [18] and Level GMM (Lev) introduced by [19]. In this paper, the above mentioned two estimation methods to estimate AR(1) time series model have been considered. In First-difference GMM method, the constant is removed from the AR(1) model and then instruments from the differenced AR(1) model are considered, where as in Level GMM estimation method the constant is removed directly from the instruments and GMM estimation is performed. The bias and RMSE of the above two estimators are compared along with OLS estimator through simulation results.

The paper is organized as follows; Section 2 provides model, assumptions and model estimators. Section 3 presents Monte Carlo simulation to investigate the performances of the considered estimators. Section 4 contains results and discussion. Finally, section 5 concludes the paper.

II. THE MODEL AND ESTIMATORS

First order autoregressive time series model

$$y_t = \alpha + \rho y_{t-1} + u_t \quad t = 2,3,\ldots,T. \qquad (1)$$

where α is constant, ρ is the parameter of interest with $|\rho|<1$.

Following are the assumptions:

Assumption 1

$\{u_t\}$ $(t = 2,3,\ldots,T)$ are i.i.d across time and independent of y_1 with $E(u_t) = 0, \ Var(u_t) = \sigma_u^2.$

Assumption 2

The autoregressive process y_t is initialized at some random quantity y_1 with $Ey_1^2 < \infty$ allowing for stationarity by setting $y_1 : N\left(0, \frac{\sigma^2}{1-\rho^2}\right)$

where, $y_1 = \sum_{j=0}^{\infty} \rho^j u_{1-j}$

Based on the above assumptions two types of estimation methods are considered viz. First-difference GMM estimation method and Level GMM estimation method.

1. First-Difference GMM Estimation:

In the model (1), the constant α causes a correlation between the lagged endogeneous variable y_{t-1} and error term u_t. First differences of model (1) is performed to remove the constant and one-step First-difference GMM estimator is proposed based on the following $(T-2)$ moment conditions

$$E(Z_d^{'} \Delta u_t) = 0 \qquad (2)$$

where Z_d is a $(T-2)\times(T-2)$ instrumental matrix employed by [20] and Δu_t is a $(T-2)\times 1$ vector.

$$Z_d = \begin{bmatrix} y_1 & 0 & \ldots & 0 \\ 0 & y_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & y_{T-2} \end{bmatrix}; \quad \Delta u_t = \begin{bmatrix} \Delta u_3 \\ \Delta u_4 \\ \vdots \\ \Delta u_T \end{bmatrix}$$

Based on the moment conditions (2) the one-step first-difference GMM estimator is obtained and is given by,

$$\hat{\rho}_{dif} = (\Delta y_{t-1}' Z_d W_d^{-1} Z_d' \Delta y_{t-1})^{-1} (\Delta y_{t-1}' Z_d W_d^{-1} Z_d' \Delta y_t) \qquad (3)$$

where $\Delta y_{t-1}'$ is the $1\times(T-2)$ vector $(\Delta y_2, \Delta y_3, \ldots, \Delta y_{T-1})$, $\Delta y_t'$ is the $1\times(T-2)$ vector $(\Delta y_3, \Delta y_4, \ldots, \Delta y_T)$, Z_d is a $(T-2)\times(T-2)$ matrix and $W_d = Z_d' G_d Z_d$ is a $(T-2)\times(T-2)$ weight matrix with G_d is same as H in the estimator proposed by [18].

2. Level GMM Estimation:

On the basis of Arellano and Bover (1995) Level GMM estimator is proposed for AR(1) time series model. Here the constant α is wiped out from the instrumental variable. The one-step Level GMM estimator is based on the moment conditions

$$E(Z_l' u) = 0 \qquad (4)$$

where Z_l is a $(T-2)\times(T-2)$ instrumental matrix employed by [20] and u_t is a $(T-2)\times 1$ vector.

$$Z_l = \begin{bmatrix} \Delta y_2 & 0 & \ldots & 0 \\ 0 & \Delta y_3 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \Delta y_{T-1} \end{bmatrix}; \quad u_t = \begin{bmatrix} u_3 \\ u_4 \\ \vdots \\ u_T \end{bmatrix}$$

Based on the moment conditions (4) the one-step Level GMM estimator is obtained and is given by,

$$\hat{\rho}_{lev} = (y_{t-1}' Z_l W_l^{-1} Z_l' y_{t-1})^{-1} (y_{t-1}' Z_l W_l^{-1} Z_l' y_t) \qquad (5)$$

where y_{t-1}' is the $1\times(T-2)$ vector $(y_2, y_3, \ldots, y_{T-1})$, y_t' is the $1\times(T-2)$ vector $(y_3, y_4, \ldots, y_T)$, Z_l is a $(T-2)\times(T-2)$ matrix $(Z_{l1}, Z_{l2}, \ldots, Z_{lN})$ and $W_l = Z_l' G_l Z_l$, is a $(T-2)\times(T-2)$ weight matrix with G_l is a $(T-2)\times(T-2)$ diagonal matrix.

III. MONTE CARLO SIMULATION STUDY

In this Monte Carlo simulation study, the data is generated from the following AR(1) model to investigate the finite sample performance of the above mentioned estimators.

$$y_t = \alpha + \rho y_{t-1} + u_t$$

where u_t is iid $N(0,\sigma^2)$. The initial condition is $y_1 : N\left(0, \frac{\sigma^2}{1-\rho^2}\right)$ which is similar to the assumption in [21]. For the parameters, $\alpha = 1$, $\sigma^2 = 1$ and $\rho \in (-1,1)$ is considered, in particular $\rho \in [0.5, 0.98]$ is considered. The sample size $T = 5,10,20,40,50,100$ and 200 is chosen. The number of replications is 10000.

IV. RESULTS AND DISCUSSION

The results of the study are discussed through the tables and graphs respectively. When $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) are compared, it is found that, in the case of T=5 and 10 bias of $\hat{\rho}$ (lev) is the smallest and bias of $\hat{\rho}$ (ols) is the largest among mentioned estimators. $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) have almost equal bias for all the values of ρ over the range T=20 to 200. When T=20, $\hat{\rho}$ (lev) has less bias than $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) except at the value of $\rho = 0.5$. When the sample sizes are 40,50,100 and 200, $\hat{\rho}$ (lev) has lower bias than $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) except at the values of $\rho < 0.7$, $\rho < 0.75$, $\rho < 0.8$ and $\rho < 0.85$ respectively. In other words, when ρ is near to unity, $\hat{\rho}$ (lev) is more preferable to the other two with regard to the bias.

Pertaining to the RMSE, for T=5, the RMSE of $\hat{\rho}$ (lev) is the smallest and bias of $\hat{\rho}$ (ols) is the largest among three estimators. For the cases of $T \geq 10$, $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) have almost identical RMSE for all considered ρ values. When T=10 and 20, the RMSE of $\hat{\rho}$ (lev) is lesser than the RMSE of $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols). When the sample sizes are 40, 50, 100 and 200, the RMSE of $\hat{\rho}$ (lev) is smaller than $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) except at the values of $\rho < 0.65$, $\rho < 0.75$, $\rho < 0.85$ and $\rho < 0.9$ respectively.

The graphs are plotted in figures 1-4 for the simulation results. Figure 1 depicts the comparison of means of $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) with reference to true line over the entire range of ρ. From all graphs in figure 1, it is observed that, in terms of bias, $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) perform almost same in all the cases except at T=5. When $\rho < 0$, $\hat{\rho}$ (lev) has greater bias than other two estimators, but when ρ is near to unity $\hat{\rho}$ (lev) has small bias than other two. To understand clearly, in figure 2, the means of above three estimators have been plotted only for $\rho > 0.5$. $\hat{\rho}$ (lev) has small bias when T is not so large. As T increases, bias of $\hat{\rho}$ (lev) also increases, although $\hat{\rho}$ (lev) has smallest bias for $\rho > 0.80$ for all values of T.

Figure 3 shows the distinction of RMSE of $\hat{\rho}$ (lev), $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) for entire range of ρ. From figure 3, it is noticed that $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) behave almost same in all the cases except at T=5. RMSE of $\hat{\rho}$ (lev) is largest in negative range of ρ. Next, it starts decreasing from -0.5 and performs better than other two estimators as ρ approaches unity. For more clarity, the RMSE of above considered estimators is plotted in figure 4 for $\rho > 0.5$. From figure 4, it is observed that, $\hat{\rho}$ (lev) has less RMSE, especially when T is too small. As T increases $\hat{\rho}$ (dif) and $\hat{\rho}$ (ols) perform better than $\hat{\rho}$ (lev) but as ρ approaches unity $\hat{\rho}$ (lev) performs excellent than remaining two estimators.

V. CONCLUSION

In this study, an estimation of AR(1) time series model is done by using First-difference GMM (Arellano and Bond (1991))and Level GMM (Arellano and Bover (1995)) estimation methods. Monte Carlo simulation is carried out to investigate the performances of the considered estimators. Based on the simulation results, it is observed that, for all the negative values of ρ the RMSE and bias of Level GMM estimator is larger than the remaining estimators. In the case of $\rho < 0$ in terms of bias and RMSE, Level GMM estimator does not perform best. But in the case of $\rho > 0$, especially when ρ is close to unity, the Level GMM estimator has small bias and is more efficient than First-difference GMM and OLS estimators.

Table 1: Simulation results of comparison of the Bias and RMSE of $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) (T = 5,10,20 and 40).

T		ρ										
		0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	0.98
5	$\hat{\rho}$ (dif)	0.018	0.023	0.051	0.053	0.084	0.114	0.121	0.139	0.174	0.208	0.232
	$\hat{\rho}$ (lev)	0.593	0.592	0.61	0.623	0.648	0.674	0.7	0.718	0.749	0.783	0.798
	$\hat{\rho}$ (ols)	-0.034	-0.03	-0.01	-0.009	0.01	0.029	0.039	0.052	0.071	0.095	0.112
	Bias(dif)	-0.482	-0.527	-0.549	-0.597	-0.616	-0.636	-0.679	-0.711	-0.726	-0.742	-0.748
	Bias(lev)	0.093	0.042	0.01	-0.027	-0.052	-0.076	-0.1	-0.132	-0.151	-0.167	-0.182
	Bias(ols)	-0.534	-0.58	-0.61	-0.659	-0.69	-0.721	-0.761	-0.798	-0.829	-0.855	-0.868
	RMSE(dif)	0.842	0.907	0.928	0.958	0.939	0.982	0.975	1.01	1.031	1.055	1.066
	RMSE(lev)	0.622	0.6	0.68	0.563	0.586	0.573	0.575	0.572	0.587	0.594	0.596
	RMSE(ols)	0.682	0.721	0.743	0.786	0.816	0.84	0.873	0.907	0.936	0.958	0.972
10	$\hat{\rho}$ (dif)	0.24	0.277	0.3	0.338	0.362	0.391	0.419	0.448	0.469	0.509	0.529
	$\hat{\rho}$ (lev)	0.618	0.642	0.647	0.67	0.687	0.711	0.736	0.761	0.788	0.827	0.852
	$\hat{\rho}$ (ols)	0.232	0.268	0.29	0.327	0.35	0.378	0.403	0.432	0.451	0.486	0.503
	Bias(dif)	-0.26	-0.273	-0.3	-0.312	-0.338	-0.359	-0.381	-0.402	-0.431	-0.441	-0.451
	Bias(lev)	0.118	0.092	0.047	0.02	-0.013	-0.039	-0.064	-0.089	-0.112	-0.123	-0.128
	Bias(ols)	-0.268	-0.282	-0.31	-0.323	-0.35	-0.372	-0.397	-0.418	-0.449	-0.464	-0.477
	RMSE(dif)	0.418	0.424	0.446	0.454	0.474	0.487	0.506	0.521	0.543	0.554	0.566
	RMSE(lev)	0.304	0.293	0.287	0.282	0.277	0.279	0.278	0.277	0.286	0.289	0.289
	RMSE(ols)	0.413	0.421	0.442	0.452	0.472	0.487	0.506	0.522	0.546	0.56	0.573
20	$\hat{\rho}$ (dif)	0.372	0.411	0.455	0.489	0.53	0.568	0.607	0.638	0.672	0.707	0.733
	$\hat{\rho}$ (lev)	0.668	0.68	0.701	0.716	0.736	0.757	0.781	0.808	0.842	0.88	0.904
	$\hat{\rho}$ (ols)	0.371	0.41	0.454	0.488	0.528	0.566	0.605	0.635	0.669	0.703	0.728
	Bias(dif)	-0.128	-0.139	-0.145	-0.161	-0.17	-0.182	-0.193	-0.212	-0.228	-0.243	-0.247
	Bias(lev)	0.168	0.13	0.101	0.066	0.036	0.007	-0.019	-0.042	-0.058	-0.07	-0.076
	Bias(ols)	-0.129	-0.14	-0.146	-0.162	-0.172	-0.184	-0.195	-0.215	-0.231	-0.247	-0.252
	RMSE(dif)	0.249	0.254	0.256	0.266	0.269	0.277	0.277	0.292	0.3	0.31	0.312
	RMSE(lev)	0.242	0.217	0.199	0.187	0.172	0.168	0.166	0.164	0.162	0.16	0.159
	RMSE(ols)	0.249	0.254	0.256	0.265	0.268	0.277	0.277	0.292	0.301	0.311	0.314
40	$\hat{\rho}$ (dif)	0.434	0.481	0.526	0.57	0.616	0.661	0.704	0.748	0.786	0.827	0.849
	$\hat{\rho}$ (lev)	0.69	0.709	0.726	0.748	0.768	0.791	0.813	0.844	0.873	0.915	0.939
	$\hat{\rho}$ (ols)	0.435	0.482	0.527	0.57	0.616	0.662	0.705	0.749	0.787	0.828	0.849
	Bias(dif)	-0.066	-0.069	-0.074	-0.08	-0.084	-0.089	-0.096	-0.102	-0.114	-0.123	-0.131
	Bias(lev)	0.19	0.159	0.126	0.098	0.068	0.041	0.013	-0.006	-0.027	-0.035	-0.041
	Bias(ols)	-0.065	-0.068	-0.073	-0.08	-0.084	-0.088	-0.095	-0.101	-0.113	-0.122	-0.131
	RMSE(dif)	0.159	0.157	0.159	0.16	0.158	0.158	0.157	0.158	0.163	0.165	0.169
	RMSE(lev)	0.22	0.195	0.168	0.146	0.128	0.113	0.105	0.096	0.095	0.087	0.087
	RMSE(ols)	0.159	0.156	0.158	0.159	0.157	0.157	0.156	0.157	0.162	0.164	0.168

Table 2: Simulation results of comparison of the Bias and RMSE of $\hat{\rho}$ (dif), $\hat{\rho}$ (lev) and $\hat{\rho}$ (ols) (T = 50,100 and 200).

T		ρ										
		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	0.98
50	$\hat{\rho}$ (dif)	0.445	0.494	0.541	0.587	0.633	0.678	0.724	0.769	0.81	0.852	0.874
	$\hat{\rho}$ (lev)	0.694	0.713	0.733	0.752	0.775	0.796	0.822	0.852	0.881	0.921	0.947
	$\hat{\rho}$ (ols)	0.446	0.495	0.542	0.588	0.635	0.679	0.725	0.771	0.811	0.853	0.875
	Bias(dif)	-0.055	-0.056	-0.059	-0.063	-0.067	-0.072	-0.076	-0.081	-0.09	-0.098	-0.106
	Bias(lev)	0.194	0.163	0.133	0.102	0.075	0.046	0.022	0.002	-0.019	-0.029	-0.033
	Bias(ols)	-0.054	-0.055	-0.058	-0.062	-0.065	-0.071	-0.075	-0.079	-0.089	-0.097	-0.105
	RMSE(dif)	0.139	0.14	0.136	0.137	0.134	0.133	0.131	0.13	0.135	0.134	0.138
	RMSE(lev)	0.217	0.19	0.164	0.14	0.12	0.103	0.09	0.082	0.082	0.073	0.071
	RMSE(ols)	0.139	0.139	0.135	0.136	0.133	0.132	0.13	0.129	0.133	0.132	0.136
100	$\hat{\rho}$ (dif)	0.474	0.52	0.569	0.617	0.665	0.713	0.762	0.809	0.856	0.9	0.925
	$\hat{\rho}$ (lev)	0.705	0.723	0.745	0.766	0.788	0.812	0.837	0.865	0.899	0.936	0.963
	$\hat{\rho}$ (ols)	0.476	0.522	0.571	0.619	0.668	0.716	0.764	0.812	0.859	0.904	0.928
	Bias(dif)	-0.026	-0.03	-0.031	-0.033	-0.035	-0.037	-0.038	-0.041	-0.044	-0.05	-0.055
	Bias(lev)	0.205	0.173	0.145	0.116	0.088	0.062	0.037	0.015	-0.001	-0.014	-0.017
	Bias(ols)	-0.024	-0.028	-0.029	-0.031	-0.032	-0.034	-0.036	-0.038	-0.041	-0.046	-0.052
	RMSE(dif)	0.093	0.092	0.09	0.088	0.085	0.082	0.079	0.076	0.074	0.073	0.074
	RMSE(lev)	0.215	0.185	0.159	0.132	0.107	0.086	0.068	0.056	0.048	0.043	0.039
	RMSE(ols)	0.092	0.091	0.089	0.087	0.083	0.08	0.077	0.074	0.071	0.069	0.07
200	$\hat{\rho}$ (dif)	0.483	0.534	0.581	0.632	0.68	0.73	0.779	0.826	0.876	0.923	0.95
	$\hat{\rho}$ (lev)	0.709	0.73	0.75	0.773	0.795	0.819	0.845	0.874	0.907	0.945	0.971
	$\hat{\rho}$ (ols)	0.487	0.537	0.585	0.635	0.684	0.734	0.783	0.831	0.881	0.929	0.955
	Bias(dif)	-0.017	-0.016	-0.019	-0.018	-0.02	-0.02	-0.021	-0.024	-0.024	-0.027	-0.03
	Bias(lev)	0.209	0.18	0.15	0.123	0.095	0.069	0.045	0.024	0.007	-0.005	-0.009
	Bias(ols)	-0.013	-0.013	-0.015	-0.015	-0.016	-0.016	-0.017	-0.019	-0.019	-0.021	-0.025
	RMSE(dif)	0.064	0.063	0.061	0.058	0.057	0.054	0.051	0.049	0.045	0.042	0.041
	RMSE(lev)	0.214	0.185	0.156	0.13	0.104	0.081	0.059	0.043	0.033	0.026	0.022
	RMSE(ols)	0.063	0.062	0.060	0.057	0.055	0.052	0.049	0.045	0.041	0.037	0.036

$\hat{\rho}$ (dif)= Difference GMM estimator, $\hat{\rho}$ (lev)= Level GMM estimator, $\hat{\rho}$ (ols)= Ordinary Least Square estimator, Bias(dif)= Bias of Difference GMM estimator, Bias(lev)= Bias of Level GMM estimator, Bias(ols)= Bias of OLS estimator, RMSE= Root Mean Square Error, RMSE(dif)= RMSE of Difference GMM estimator, RMSE(lev)=RMSE of Level GMM estimator, RMSE(ols)=RMSE of OLS estiator

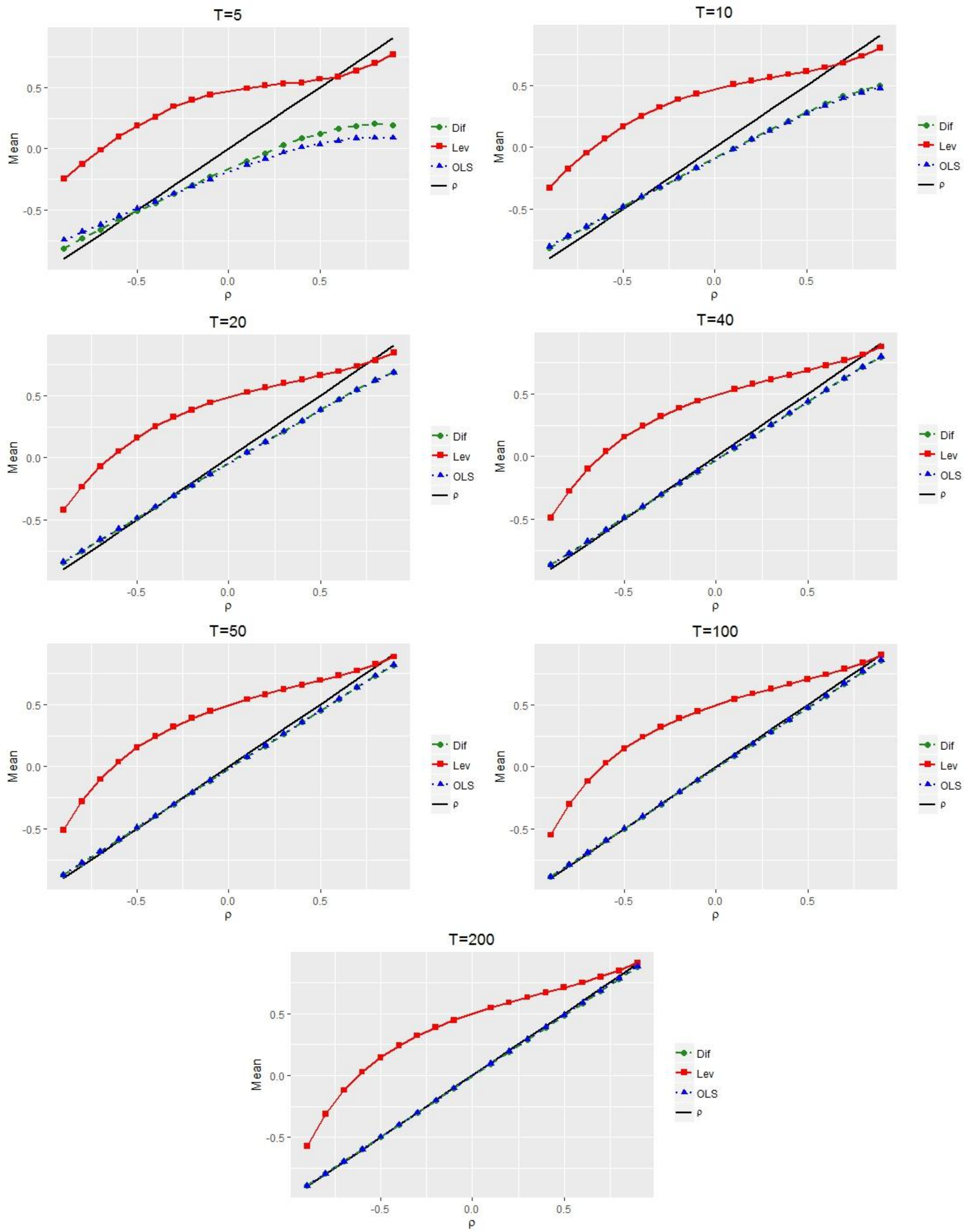

Figure 1: Means of Difference GMM, Level GMM and OLS estimators ($-1 < \rho < 1$).

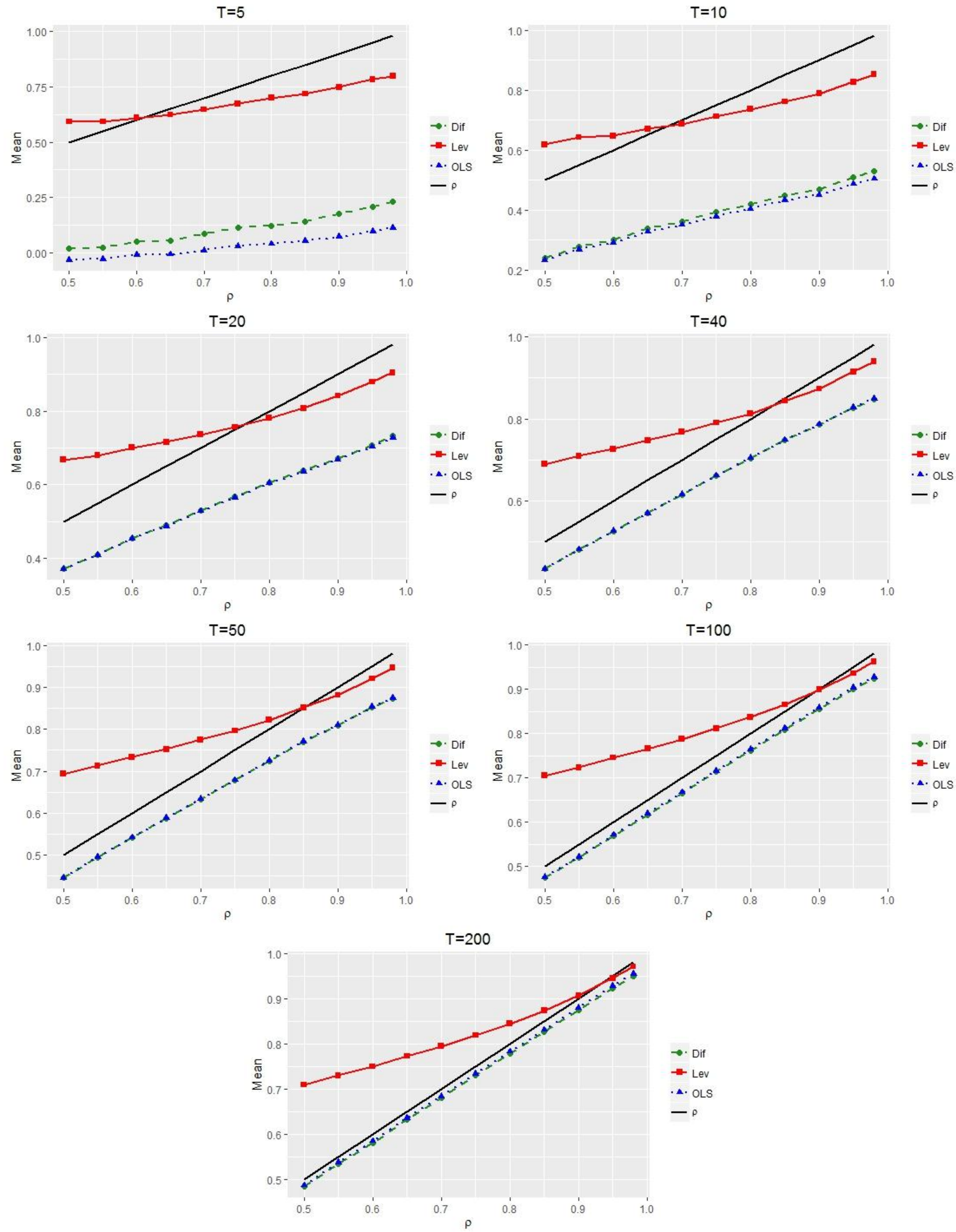

Figure 2: Means of Difference GMM, Level GMM and OLS estimators ($\rho \geq 0.5$).

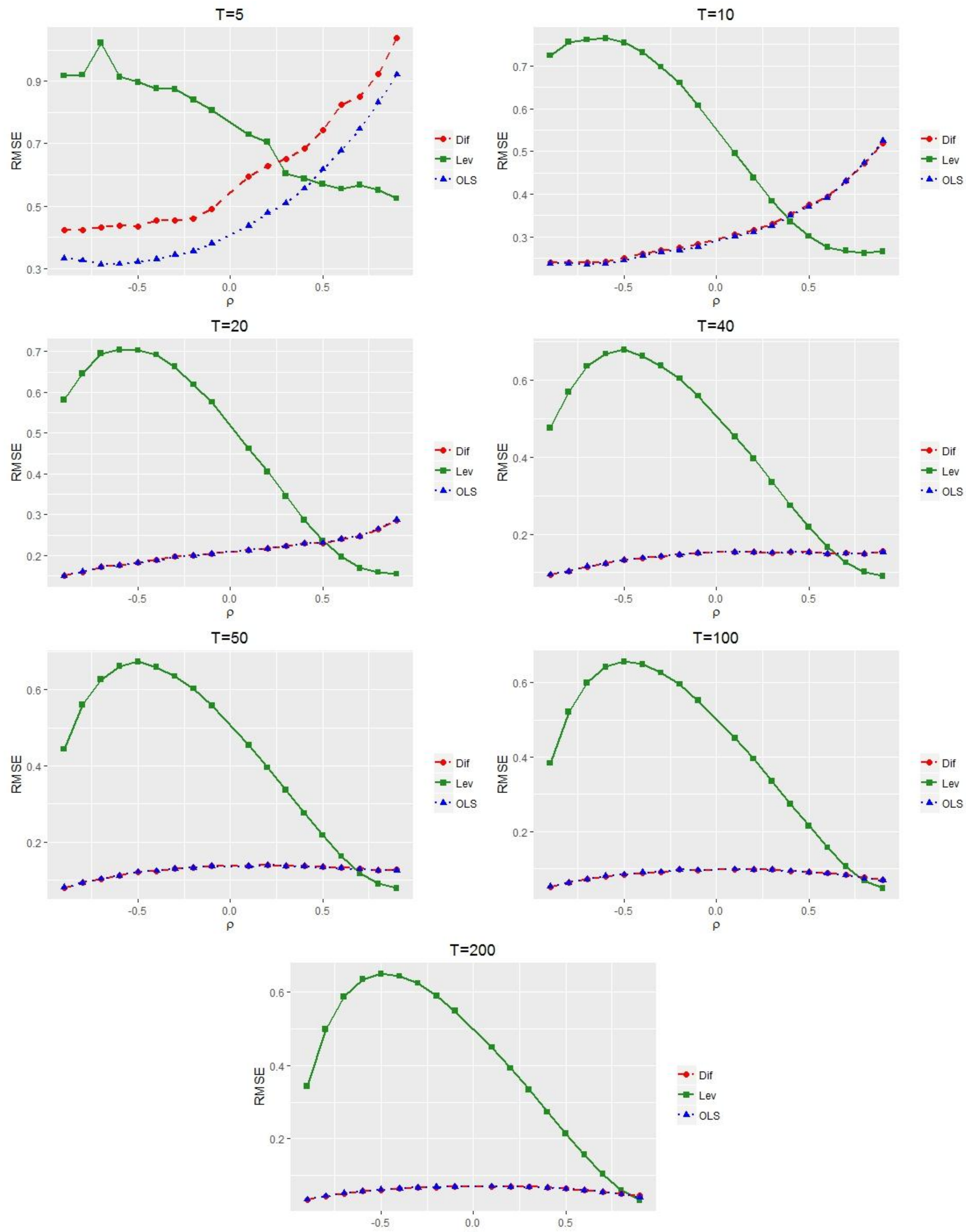

Figure 3: RMSEs of Difference GMM, Level GMM and OLS estimators ($-1 < \rho < 1$ *).*

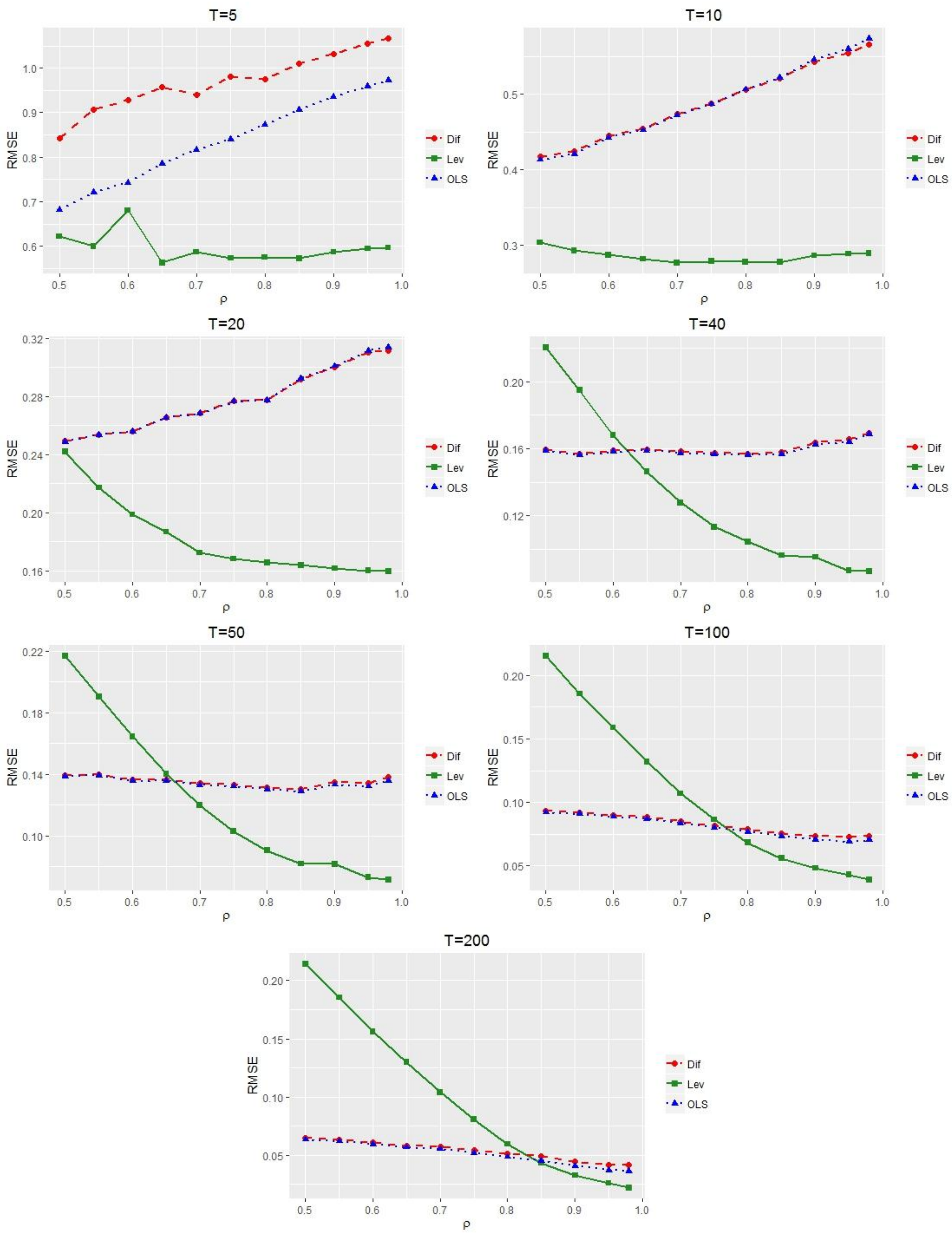

Figure 4: RMSEs of Difference GMM, Level GMM and OLS estimators ($\rho \geq 0.5$).

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