



A Comparison Study for PSD Performance in OFDM Systems Based on Using Autocorrelation Techniques

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Abstract - Cooperative wireless communication relay networks in OFDM-based systems have been shown to improve performance in various systems using digital modulation techniques as a form of spatial diversity, OFDM, a multicarrier modulation technique, is widely adopted because of its many advantages. Most of the time, in-band spectrum of the multicarrier signal is assumed to be flat. A recent literature analyzes the PSD and concludes that the assumption of flat spectrum is not completely correct. This paper analyzes the PSD with different approach. The modulated carriers are cropped in an interval of symbol length and viewed as a modulated pulse. An expression for the PSD with the necessary and sufficient conditions is revealed. The validity of the flat PSD assumption of multicarrier signals is also suggested. This method indeed improves the performance in comparison in various aspects of PSD with fixed and random values of input applied on functions on FFT, IFFT etc. to existing when filtering is necessary for band limited conditions.

Keywords: Orthogonal Frequency Division Multiplexing, Orthogonal Pulse, Power Spectral Density, Spectrum Analysis

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a digital modulation technique who consists of transmitting a data stream using a large number of parallel narrow-band sub carriers instead of a single wide-band carrier. Orthogonal frequency division multiplexing (OFDM), a form of multicarrier modulation techniques, is one of the most widely-adopted modulation techniques because of its many advantages, for example, the ease of implementation using discrete Fourier transform and the immunity to multipath fading gained from the extended symbol length [1]. Figuratively speaking, OFDM partially overlays the spectra of a number of narrow band modulated signals and makes use of the orthogonality of the carriers to extract the data from the combined waveform. In some literatures, e.g., [2], OFDM power spectral density (PSD) is treated as a flat spectrum. An intensive analysis in a very recent work shows that the assumption of flat PSD is not completely true, especially for the case of the OFDM adopted by the 802.11a standard [3].

In this paper, a different approach is used to analyze the PSD of OFDM. The orthogonal carriers are cropped at a size of the symbol length and they are viewed as an orthogonal symbol. The results are identical to the result found in [3] but in this work, the remark is that orthogonality is a not required condition. The result is then taken to analyze the PSD of OFDM signals. A conclusion on the validity of the flat PSD is drawn, finally.

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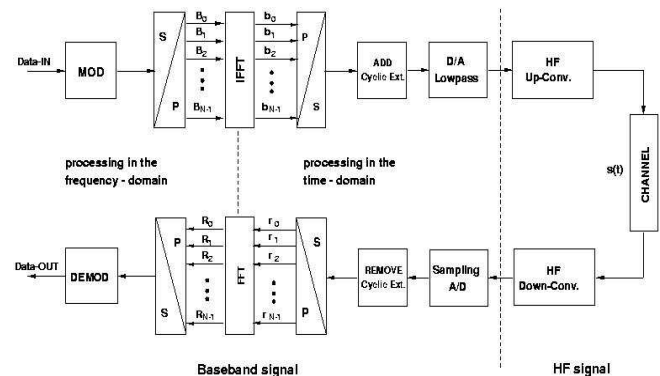


Fig. 1 Block diagram of OFDM Transmitter receiver for PSD optimization

2. PSD OF COMBINED WAVEFORMS

A set of functions, $\{f_0(t), f_1(t), f_2(t), \dots\}$ is said to be orthogonal over a symbol interval $[-Ts/2, Ts/2]$ if

$$\int_{-Ts/2}^{Ts/2} f_p(t) f_q(t) dt = \begin{cases} 0, & p \neq q \\ \delta_{pp}, & p = q \end{cases} \quad (1)$$

where δ_{pp} is the energy of the function $f_p(t)$. Let $s(t)$ be a transmit waveform of which digital information is kept on the coefficients of the orthogonal basis pulse $f_p(t)$. Such a modulation technique is named a P -dimensional orthogonal scheme. The transmit waveform $s(t)$ can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{p=0}^{P-1} a_{n,p} f_p(t - nT_s) \quad (2)$$

where $a_{n,p}$ denotes the digital data of the n th symbol carried on the p th pulse (dimension). It is shown in [4] that $P_s(f)$ the power spectral density of one-dimensional $s(t)$, which only $f_0(t)$ is available, can be written as

$$P_s(f) = \frac{1}{T_s} |F_0(f)|^2 \left[\sum_{k=-\infty}^{\infty} R_0(k) e^{jk2\pi T_s} \right] \quad (3)$$

where $F_0(f)$ is the spectrum of the basis pulse $f_0(t)$ and $n, n, k, R, k, a, a, +, () = 0$. More specifically, the signal is said to be antipodal if the binary data is used to sign the basis pulse $f_0(t)$, i.e., $a_{n,p} \in \{-1, 1\}$. Assuming uncorrelated binary data, $R_0(k) = 1$ for only $k = 0$ and zero elsewhere. The summation term in (3) is removed. As a result, the PSD of the antipodal signal is just the magnitude squared of the basis pulse Fourier transform divided by the bit period. It can also find out through the channel using in device in transmitter and receiver sides.

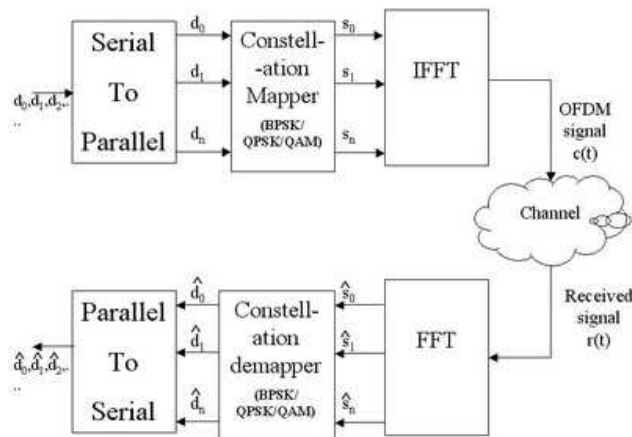


Fig. 2 Block diagram of OFDM using Constellation mapper device (for BPSK/QAM/QPSK) Network System.

Now, let us generalize the results for the PSD of $s(t)$. PSD can be determined either directly by its definition or indirectly using Wiener-Khinchine theorem. The choice of the following analysis is based on the direct method. By definition, the power spectral density (PSD) of a random process $s(t)$ is given by

$$P_s(f) = \lim_{T \rightarrow \infty} \left(\frac{|S_T(f)|^2}{T} \right) \quad (4)$$

where $S_T(f)$ is Fourier transform of the truncated version of infinite waveform $s(t)$.

Let us observe the original signal $s(t)$ for $2N+1$ symbol periods, from $-N$ th to N th symbol. The truncated signal $s_T(t)$ can be written as

$$s_T(t) = \sum_{n=-N}^N \sum_{p=0}^{P-1} a_{n,p} f_p(t - nT_s) \quad (5)$$

Its Fourier transform is then

$$S_T(f) = \sum_{n=-N}^N \sum_{p=0}^{P-1} a_{n,p} F_p(f) e^{-j\omega n T_s} \quad (6)$$

Where $F_p(f)$ is Fourier transform of the basis pulse $f_p(t)$. Thus

$$|S_T(f)|^2 = S_T(f) S_T^*(f) \quad (7)$$

$$\begin{aligned} &= \left(\sum_{n=-N}^N \sum_{p=0}^{P-1} a_{n,p} F_p(f) e^{-j\omega n T_s} \right) \left(\sum_{m=-N}^N \sum_{q=0}^{P-1} a_{m,q} F_q^*(f) e^{-j\omega m T_s} \right) \\ &= \sum_{n=-N}^N \sum_{m=-N}^N \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} a_{n,p} F_p(f) e^{-j\omega n T_s} a_{m,q} F_q^*(f) e^{+j\omega m T_s} \end{aligned} \quad (8)$$

In (8) n and m represent the time index while p and q identify the basis pulse numbers. Thus, the PSD for $s_T(t)$ becomes

$$P_s(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \left(\sum_{n=-N}^N \sum_{m=-N}^N \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} a_{n,p} F_p(f) e^{-j\omega n T_s} a_{m,q} F_q^*(f) e^{+j\omega m T_s} \right) \quad (9)$$

where $T = T_s(2N+1)$. Since only $a_{n,p}$ and $a_{m,q}$ are random, the ensemble averaging operator affects only on them. Equation (9) reduces to

$$\begin{aligned} P_s(f) &= \lim_{N \rightarrow \infty} \frac{1}{T_s(2N+1)} \times \\ &\sum_{n=-N}^N \sum_{m=-N}^N \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} (a_{n,p} a_{m,q}) e^{-j\omega(n-m)T_s} F_p(f) F_q^*(f) \end{aligned} \quad (10)$$

The term $n, p, m, q, a, a, ,$ is in fact a cross correlation function of the n th and m th data bits carried on the p th and q th pulses, respectively. It is referred by $R_{pq}(m, n)$. Assuming the random processes associated with p th and q th dimensions are jointly wide sense stationary, $R_{pq}(m, n)$ can be written as $R_{pq}(m-n) = R_{pq}(k)$ where $k = m-n$.

Let us define a cross spectral function $G(k, f)$ by

$$G(k, f) = \sum_{p=0}^{P-1} \sum_{q=0}^{P-1} R_{pq}(k) e^{-j\omega(k)T_s} F_p(f) F_q^*(f) \quad (11)$$

$G(k, f)$ can be decomposed into co-spectral component $G_{co}(k, f)$ and cross-spectral component $G_{cross}(k, f)$ where

$$G_{co}(k, f) = \sum_{p=0}^{P-1} R_{pp}(k) e^{-j\omega(k)T_s} |F_p(f)|^2 \quad (12)$$

$$G_{cross}(k, f) = \sum_{p=0}^{P-1} \sum_{q=0, q \neq p}^{P-1} R_{pq}(k) e^{-j\omega(k)T_s} F_p(f) F_q^*(f) \quad (13)$$

Thus PSD becomes

$$P_s(f) = \lim_{N \rightarrow \infty} \frac{1}{T_s(2N+1)} \sum_{n=-N}^N \sum_{m=-N}^N G(k, f) \quad (14)$$

For a given N , the outer and the inner summations together produce a total of $(2N+1)2$ adding terms. The values of k range from $-2N$ to $2N$. For a given k , (14) evaluates $G(k,f)$ in total of $(2N+1) - |k|$ times. By letting $m = k-n$ and summing over the outer summation results in

$$P_z(f) = \frac{1}{T_s} \lim_{N \rightarrow \infty} \frac{(2N+1)}{(2N+1)} \sum_{k=-N-n}^{N-n} G(k,f) \tag{15}$$

Moreover, assuming the digital data on the same pulse, $a_{n,p}$ and $a_{m,p}$, are independent, $R_{pp}(k) = 0$ for $k \neq 0$. Therefore the power spectral density of such a signal is just the sum of the magnitudes of the Fourier transform squared of the basis pulses.

$$P_z(f) = \sum_{p=0}^{P-1} R_{pp}(0) |F_p(f)|^2 \tag{16}$$

This result is a generalized version of the PSD formula for multi-dimensional antipodal waveforms developed in [5]. An essential remark is that *the derivation does not require the orthogonality of the basis pulses at all*. Therefore orthogonality is not a necessary condition. The sufficient condition for the PSD is that the data must be uncorrelated. Note that a sequence with channel coding does not satisfy this condition. Another note is that a pulse train modulated by random data appears orthogonal to another modulated pulse train when observed for a sufficiently long time.

3. COMPARISON OF SIMULATION RESULTS

Available transmission bandwidth. OFDM is the modulation technique used in many new broadband communication schemes, including digital television, digital audio broadcasting, ADSL and wireless LANs. It also allows digital data to be efficiently and reliably transmitted over a radio channel, even in multipath environments [11-13]. In OFDM, although sub-carriers overlap, this does not create any problem since they are orthogonal, that is, the peak of one occurs when that of others are at zero. This is achieved by realizing all the subcarriers together using the inverse fast Fourier transform (IFFT). The analysis of BER performances have suggested that OFDM is better than CDMA which is currently incorporated in most existing 3G systems [4, 5]. A major problem in most wireless systems is the presence of a multipath channel. In this environment, the transmitted signal reflects off several objectives and a result, multiple delayed versions of the transmitted signal arrive at the receiver which causes the received signal to be distorted. A multipath channel will cause two problems for an OFDM system. The first is ISI which occurs when the received OFDM symbol is distorted by the previously transmitted OFDM symbol and has a similar effect to the ISI that occurs in a single-carrier system. However, in such systems, the interference is typically due to several symbols other than only the previous ones; and the symbol period is typically much shorter than the time span of the channel, whereas the typical OFDM symbol period is much longer than the time span of the channel.

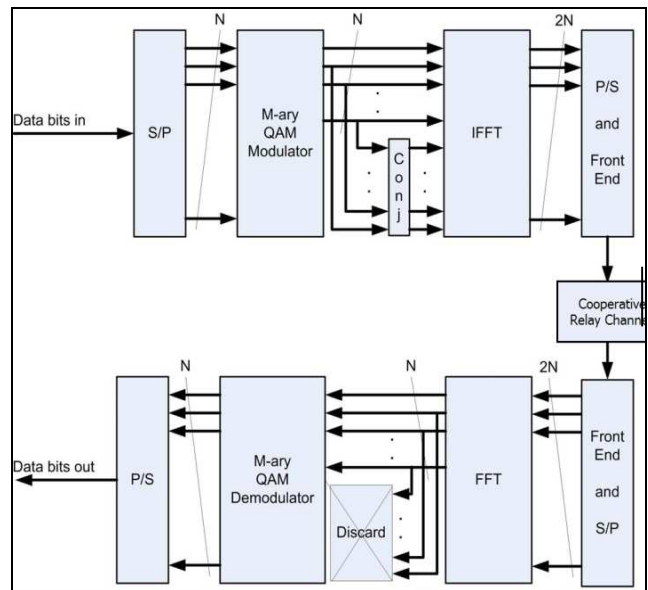


Fig. 3 Block diagram of M-ary QAM on OFDM Relay Network System.

Simulation of PSD is implemented based on a commonly known rectangular pulse, $\Pi(t/T) = 1$ for $|t| < T/2$ and zero elsewhere. The Fourier transform pair of the pulse is expressed as

$$\Pi\left(\frac{t}{T}\right) \leftrightarrow T_s \frac{\sin(\pi f T)}{\pi f T}$$

Then the results based on this time probability are

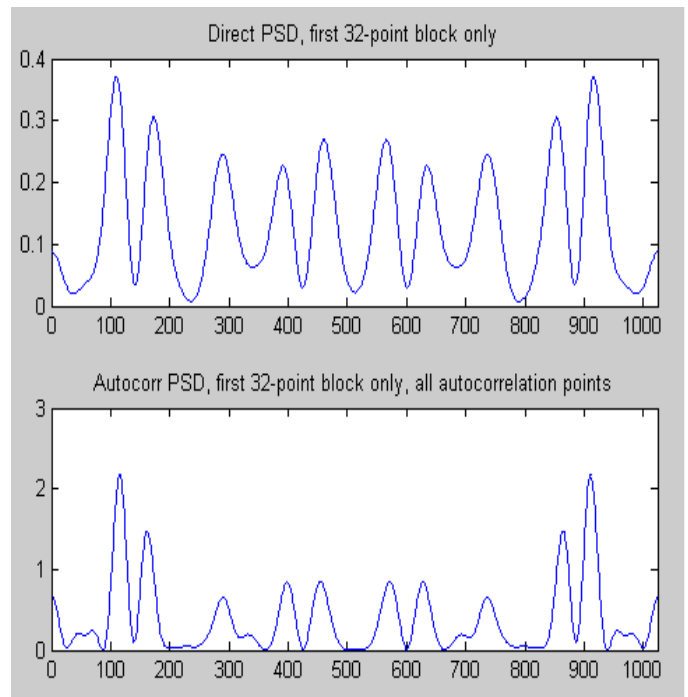


Fig. 3.1 Direct PSD & Autocorrelation for 32 point block only

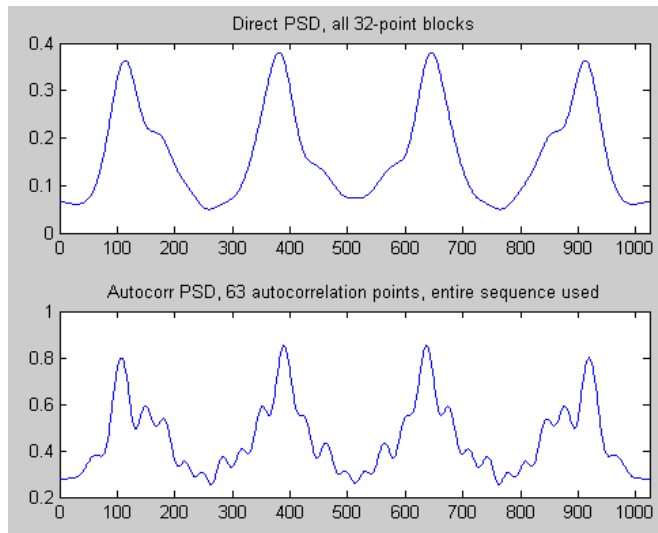


Fig. 3.2 Direct PSD for 32 point block & Autocorrelation For entire sequence of 63 points

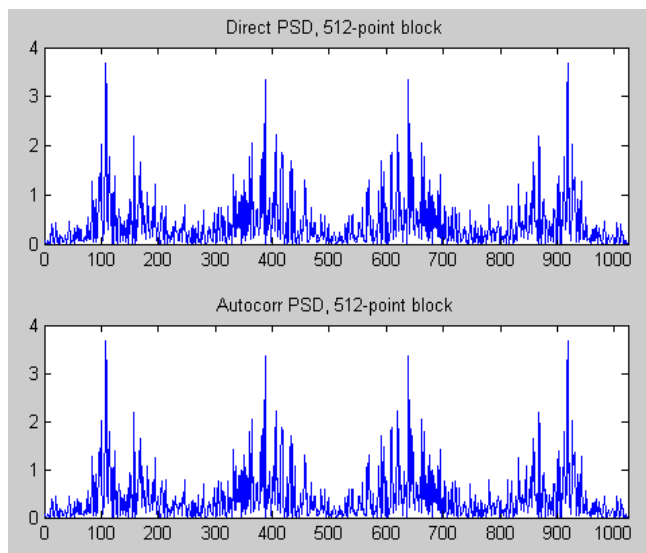


Fig. 3.3 Direct PSD & Autocorrelation for 512 point block only

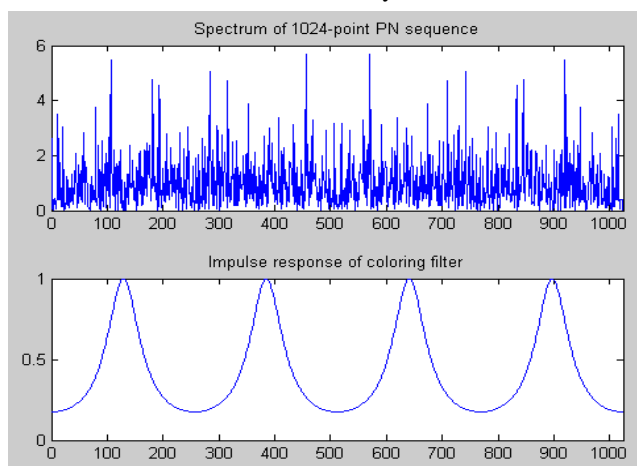


Fig. 3.4 Spectrum of 1024 point PN sequence & Impulse Response of coloring filter

4. CONCLUSION

The analysis shows that the PSD of such a digitally modulated signal can be evaluated from the sum of the Fourier magnitude squared. Independent random data is a sufficient condition whereas the basis pulse orthogonality is not necessary. It is shown that PSD of an OFDM with rectangular time window is not perfectly flat but reasonably considered flat. The future works will focus on the analysis of the PSD of the 802.11 a/g standard, in which a raised-cosine based time window is used. If M-ary QAM/BPSK/QPSK OFDM-based system with Best Relay Selection the best-relays selection cooperative has PSD better than the regular cooperative diversity. In OFDM, although sub-carriers overlap, this does not create any problem since they are orthogonal, that is, the peak of one occurs when that of others are at zero [18]. This is achieved by realizing all the subcarriers together using the inverse fast Fourier transform (IFFT). The analysis of PSD performances have suggested that OFDM is better than CDMA which is currently incorporated in most existing 3G systems [4, 5, 19]. The major problem is resolved by using 512/1024 points for block & autocorrelation for different sequences in OFDM scheme in most wireless systems is the presence of a multipath channel for getting good PSD for various systems and broadband services of [19] TV transmission and mobile channel operations. In this environment, the transmitted signal reflects off several objectives and a result, multiple delayed versions of the transmitted signal arrive at the receiver which causes the received signal to be distorted. Many wired systems also have a similar problem with reflection occurring due to impedance mismatches in the transmission line

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